

A very beautiful property of parabolas is that at a point called the FOCUS, all of the lines entering the parabola parallel to its axis are 'reflected' from the parabolic curve and intersect the focus. This property is used by astronomers to design telescopes, and by radio engineers to design satellite dishes.

The top figure to the left shows a satellite dish with a radio receiver located at the focus of the parabola. The radio rays are reflected from the parabolic surface and concentrated at the focus. This focusing and amplification property of parabolic reflectors is also used for solar heating and generating solar electricity.

The bottom figure defines the distance,  $f$ , of the focus from the bottom of the dish, and the diameter,  $D$ , of the dish.

Suppose you wanted to design a parabolic dish with a depth,  $d$ , of 1 meter and a radius of 5 meters. Where would the focus be located? If the basic equation of a parabola is  $y = ax^2$ . The location of the focus will be at  $f = 1/(4a)$ . Since we know that the point  $(5.0, 1.0)$  is on the curve of the parabola, that means that we can solve for  $a$  for this particular dish. We get  $a(5.0)^2 = 1.0$  so  $a = 1/25$ . Then the focus will be at  $f = 1/(4/25)$  so  $f = 25/4 = 6 \frac{1}{4}$  meters above the bottom of the dish. Let's design a few parabolic reflectors that we can use to reflect and concentrate sound waves, or sunlight!

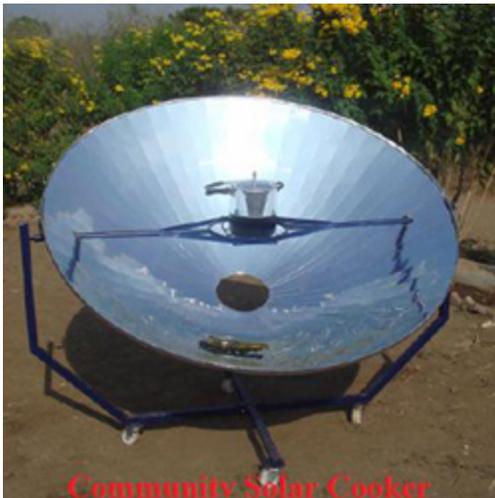
**Problem 1** – A bird watcher wants to record bird songs from a distance. He goes to the cooking store and finds a parabola-shaped bowl that is light-weight. It has a diameter of 12 inches and a depth of 5 inches. How far below the edge of the bowl does he have to mount the microphone to use this as a sound amplifier?

**Problem 2** – A hobbyist has several hundred small flat mirrors and wants to build a solar cooker by mounting the mirrors on the inside surface of a parabolic shaped form. The diameter of the parabola is 3 meters and the depth of the form is  $\frac{1}{2}$  meter. How far above the center of the form will the sunlight be the most concentrated?



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Answer: The radius of the bowl is  $x=6.0$  inches, and at this location  $y = 5.0$  inches. The equation is  $y = a x^2$ , and we know that the point  $(6.0, 5.0)$  is on this curve, so  $5.0 = a (6.0)^2$  so  $a = 0.139$ . The focus distance is then  $f = 1/(4 \times 0.139) = 1.8$  inches from the bottom of the bowl, or  $5.0 - 1.8 = \mathbf{3.2 \text{ inches}}$  from the top edge of the bowl.



**Problem 2** – A hobbyist has several hundred small flat mirrors and wants to build a solar cooker by mounting the mirrors on the inside surface of a parabolic shaped form. The diameter of the parabola is 3.0 meters and the depth of the form is  $1/2$  meters. How far above the center of the form will the sunlight be the most concentrated?

Answer: The diameter is 3.0 meters so the radius is 1.5 meters. The point  $(1.5, 0.50)$  is on the parabolic curve, so  $0.50 = a (1.5)^2$  and so  $a = 0.22$ . Then the focus is at  $f = 1/(4 \times 0.22) = \mathbf{1.1 \text{ meters from the bottom of the form.}}$

Note: The light from the sun falls on the parabola over an area of 2 square meters, and is concentrated into an area at the focus of  $1/100$  square meters, so the sunlight is amplified by  $2.0/0.01 = 200$  times, making this a very good solar cooker!