



Thousands of years ago, a star reached the end of its life and began to eject its outer layers forming one of the most complex 'planetary nebulae' in the sky. Known as NGC 6543, it is located 3,300 light years from Earth in the constellation Draco. Its stunning appearance has been known to astronomers for a century as the Cat's Eye Nebula. Detailed telescopic studies of this nebula by astronomers using the Hubble Space Telescope (top left) and recently the Nordic Optical Telescope (top right) have uncovered a wealth of detail caused by the ejected gas and dust from the star (white spot at the center of the Hubble image). Careful measurements allow us to estimate how this nebula has evolved over the centuries.

Problem 1 – The gases that make up the bright central nebula are traveling at 16.4 km/sec. If the radius of the nebula is about 0.25 light years. If one light year = 9.3×10^{12} km, about how long did it take, in years to 2 significant figures, for the nebular shell to form?

Problem 2 – The eleven equally-spaced outer dust shells that have been detected in the Hubble image extend to a maximum radius of 2.5 light years. If the expansion speeds of the shells is 16.4 km/sec, and to two significant figures, how long ago was the outermost shell ejected? About how often did the central star eject these shells?

Problem 3 – The dying star was ejecting about 20 trillion tons of matter every second (2×10^{16} kg). If the estimated mass of the star was 5.0 times our sun, about how many percent of the stars mass was ejected to make the planetary nebula with the age calculated in Problem 2? (1 solar mass = 3.0×10^{30} kg).

http://www.nasa.gov/mission_pages/hubble/science/dark-matter-survey.html
Ambitious Hubble Survey Obtaining New Dark Matter Census
Oct 13, 2011

<http://apod.nasa.gov/apod/ap020904.html>

HST ACS image http://w3.iaa.es/xpn/pn_files/NGC6543/optical.html

Problem 1 – The gases that make up the bright central nebula are traveling at 16.4 km/sec. If the radius of the nebula is about 0.25 light years. If one light year = 9.3×10^{12} km, about how long did it take, in years, for the nebular shell to form?

Answer: Time = Distance/speed, so

$$\text{Time} = 0.25 \times 9.3 \times 10^{12} \text{ km} / 16.4 \text{ km/sec}$$

$$\text{Time} = 1.4 \times 10^{11} \text{ seconds. } \times (1 \text{ year} / 3.1 \times 10^7 \text{ sec})$$

$$\text{Time} = 4,500 \text{ years.}$$

Problem 2 – The eleven equally-spaced outer dust shells that have been detected in the Hubble image extend to a maximum radius of 2.5 light years. If the expansion speeds of the shells is 16.4 km/sec, how long ago was the outermost shell ejected? About how often did the central star eject these shells?

Answer:

Time = Distance/speed

$$= 2.5 \times 9.3 \times 10^{12} \text{ km} / (16.4 \text{ km/sec})$$

$$= 46,000 \text{ years.}$$

The shell spacing is about 2.5 light years/(11 shells) so the ejection interval is 46,000 years/11 shells= **4,200 years per shell ejection event.**

The dying star ejected its first detectable shell about 46,000 years ago, and every 4,000 years would eject a new shell until about 4,500 years ago when it ejected a massive cloud of gas and dust in a giant explosion.

Problem 3 – The dying star was ejecting about 20 trillion tons of matter every second (2×10^{16} kg/sec). If the estimated mass of the star was 5.0 times our sun, about how many percent of the stars mass was ejected to make the planetary nebula with the age calculated in Problem 2? (1 solar mass = 3.0×10^{30} kg).

Answer:

$$\text{Mass} = (2 \times 10^{16} \text{ kg/sec}) \times 46000 \text{ years} \times (3.1 \times 10^7 \text{ sec} / 1 \text{ year})$$
$$= 2.8 \times 10^{28} \text{ kg}$$

$$\text{Fraction} = 2.8 \times 10^{28} \text{ kg} / (5.0 \times 3.0 \times 10^{30} \text{ kg}) = 0.0018$$

$$\text{Percentage} = 100\% \times 0.0018 = \mathbf{0.18 \%}.$$