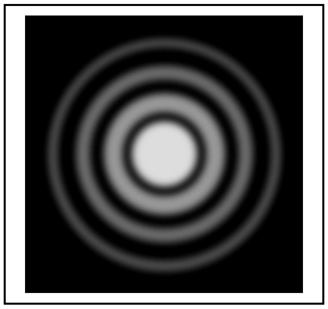
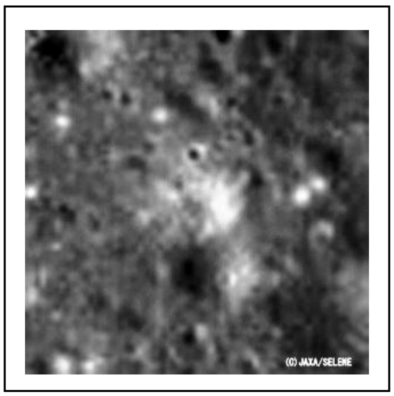
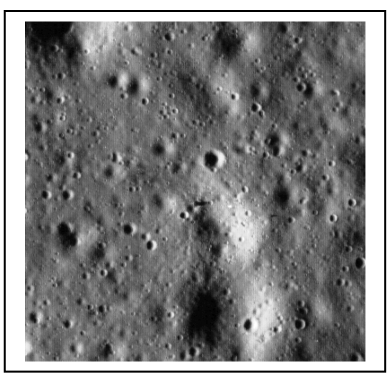
$$R = 1.22 \frac{L}{D}$$







There are many equations that astronomers use to describe the physical world, but none is more important and fundamental to the research that we conduct than the one to the left! You cannot design a telescope, or a satellite sensor, without paying attention to the relationship that it describes.

In optics, the best focused spot of light that a perfect lens with a circular aperture can make, limited by the diffraction of light. The diffraction pattern has a bright region in the center called the Airy Disk. The diameter of the Airy Disk is related to the wavelength of the illuminating light, L, and the size of the circular aperture (mirror, lens), given by D. When L and D are expressed in the same units (e.g. centimeters, meters), R will be in units of angular measure called radians (1 radian = 57.3 degrees).

You cannot see details with your eye, with a camera, or with a telescope, that are smaller than the Airy Disk size for your particular optical system. The formula also says that larger telescopes (making D bigger) allow you to see much finer details. For example, compare the top image of the Apollo-15 landing area taken by the Japanese Kaguya Satellite (10 meters/pixel at 100 km orbit elevation: aperture = about 15cm) with the lower image taken by the LRO satellite (0.5 meters/pixel at a 50km orbit elevation: aperture =). The Apollo-15 Lunar Module (LM) can be seen by its 'horizontal shadow' near the center of the image.

Problem 1 - The Senator Byrd Radio Telescope in Green Bank West Virginia with a dish diameter of D=100 meters is designed to detect radio waves with a wavelength of L= 21-centimeters. What is the angular resolution, R, for this telescope in A) degrees? B) Arc minutes?

Problem 2 - The largest, ground-based optical telescope is the D = 10.4-meter Gran Telescopio Canaris. If this telescope operates at optical wavelengths (L = 0.00006 centimeters wavelength), what is the maximum resolution of this telescope in A) microradians? B) milliarcseconds?

Problem 3 - An astronomer wants to design an infrared telescope with a resolution of 1 arcsecond at a wavelength of 20 micrometers. What would be the diameter of the mirror?

Answer Key

Problem 1 - The Senator Byrd Radio Telescope in Green Bank West Virginia with a dish diameter of D=100 meters is designed to detect radio waves with a wavelength of L= 21-centimeters. What is the angular resolution, R, for this telescope in A) degrees? B) Arc minutes?

Answer: First convert all numbers to centimeters, then use the formula to calculate the resolution in radian units: L = 21 centimeters, D = 100 meters = 10,000 centimeters, then R = 1.22 x 21 cm /10000 cm so R = 0.0026 radians. There are 57.3 degrees to 1 radian, so A) 0.0026 radians x (57.3 degrees/ 1 radian) = 0.14 degrees. And B) There are 60 arc minutes to 1 degrees, so 0.14 degrees x (60 minutes/1 degrees) = 8.4 arcminutes.

Problem 2 - The largest, ground-based optical telescope is the D = 10.4-meter Gran Telescopio Canaris. If this telescope operates at optical wavelengths (L = 0.00006 centimeters wavelength), what is the maximum resolution of this telescope in A) microradians? B) milliarcseconds?

Answer: R = 1.22 x (0.00006 cm/10400 cm) = 0.000000069 radians. A) Since 1 microradian = 0.000001 radians, the resolution of this telescope is 0.069 microradians. B) Since 1 radian = 57.3 degrees, and 1 degree = 3600 arcseconds, the resolution is 0.000000069 radians x (57.3 degrees/radian) x (3600 arcseconds/1 degree) = 0.014 arcseconds. One thousand milliarcsecond = 1 arcseconds, so the resolution is 0.014 arcsecond x (1000 milliarcsecond / arcsecond) = 14 milliarcseconds.

Problem 3 - An astronomer wants to design an infrared telescope with a resolution of 1 arcsecond at a wavelength of 20 micrometers. What would be the diameter of the mirror?

Answer: From R = 1.22 L/D we have R = 1 arcsecond and L = 20 micrometers and need to calculate D, so with algebra we re-write the equation as D = 1.22 L/R.
Convert R to radians:

R = 1 arcsecond x (1 degree/3600 arcsecond) x (1 radian / 57.3 degrees) = 0.0000048 radians.

L = 20 micrometers x (1 meter/1,000,000 micrometers) = 0.00002 meters.

Then D = 1.22 (0.00003 meters)/(0.0000048 radians) = 5.1 meters.