



Satellites are designed to optimize the number of experiments they can carry, while at the same time keeping the mass and power requirements at a minimum. The picture shows the IMAGE satellite with the dark solar cells attached to its surface.

Here is one example of a simple problem that can be encountered by a satellite designer.

A satellite is designed to fit inside the nose-cone (shroud) of a Delta II rocket. There is only enough room for a single satellite, and it cannot have deployable solar panels to generate electricity using solar cells. Instead, the solar cells have to be mounted on the exterior surface of the satellite. At the same time, the satellite configuration is that of a hexagonal prism. The total volume of the satellite is 10 cubic meters. The solar cells will be mounted on the hexagonal top, bottom, and the rectangular side panels of the satellite.

**Problem 1** - If the width of a panel is  $W$ , and the height of the satellite is  $H$ , what are the dimensions of the satellite that minimize the surface area and hence the available power that can be generated by the solar cells?

**Problem 2** - If only  $1/2$  of the solar cells receive light at any one time, and the power they can deliver is 100 watts per square meter, what is the power that this satellite can provide to the experiments and operating systems?

**Problem 3** - If the mass of the panels is 3 kg per square meter, what is the total mass of this satellite?

**Problem 4** – If the launch cost is \$10,000 per pound, how much will it cost to place this satellite into orbit? (Note, 1 pound = 0.453 kilograms)

**Problem 1** - If the width of a panel is  $W$ , and the height of the satellite is  $H$ , what are the dimensions of the satellite that minimize the surface area and hence the available power that can be generated by the solar cells?

Answer:

The volume of a hexagonal prism is given by

$$V = \frac{3 (3)^{1/2} H W^2}{2}$$

The surface area is given by

$$A = 6WH + 3 (3)^{1/2} W^2$$

Since  $V = 10$  cubic meters, we can solve for  $H$  to get

$$H = \frac{20}{3 (3)^{1/2} W^2}$$

Eliminate  $H$  from the equation for surface area to get

$$A = \frac{40}{(3)^{1/2} W} + 3 (3)^{1/2} W^2$$

Differentiate  $A$  with respect to  $W$ , set this equal to zero.

$$\frac{40}{(3)^{1/2} W^2} = 6 (3)^{1/2} W$$

A bit of algebra gives us  $W^3 = 40/(6 \cdot 3)$  so  **$W = 1.3$**  meters and from the definition for  $H$  we have  **$H = 2.3$  meters**.

**Problem 2** - If only 1/2 of the solar cells receive light at any one time, and the power they can deliver is 100 watts per square meter, what is the maximum power that this satellite can provide to the experiments and operating systems? **Answer: The total surface area is  $A = 6 (1.3) (2.3) + 3 (3)^{1/2} (1.3)^2 = 17.9 + 8.8 = 26.7$  square meters. Only  $\frac{1}{2}$  are available for sunlight, and so the total power will be about  $26.7 \times 0.5 \times 100 = 1,335$  watts.**

**Problem 3** - If the mass of the panels is 3 kg per square meter, what is the total mass of this satellite? **Answer:  $3 \times 26.7 = 80.1$  kilograms.**

**Problem 4** – If the launch cost is \$10,000 per pound, how much will it cost to place this satellite into orbit? (Note, 1 pound = 0.453 kilograms). **Answer: The cost to launch is  $80 \text{ kilograms} \times 10,000 \text{ dollars/pound} \times (1 \text{ pound}/0.453 \text{ kg}) = \$800,000/0.453 = \$1.8$  million dollars.**