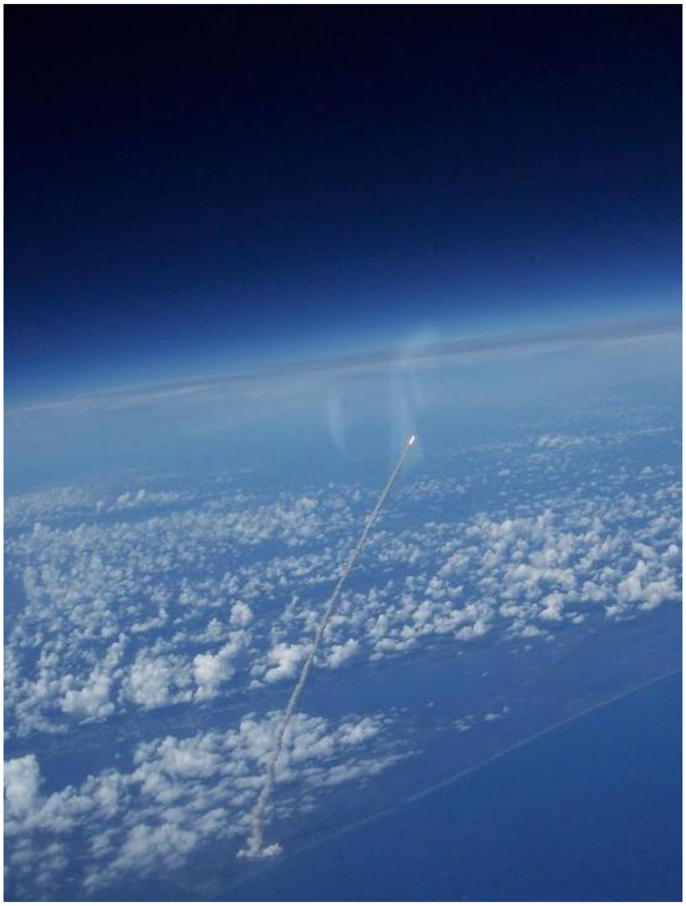
Space Shuttle Launch Trajectory - I



The trajectory of the Space Shuttle during the first 5 minutes of the launch of STS-30 can be represented by an equation for its altitude

$$h(T) = 2008 - 0.047\,T^3 + 18.3\,T^2 - 345T$$

and an equation for its down-range distance due-east

$$R(T) = 4680\,e^{0.029T}$$

where the distances are provided in units of feet commonly used by NASA engineers for describing trajectories near Earth. The problems below will use these 'parametric equations of motion' to determine the time of the highest acceleration.

Image: Shuttle launch as seen from a NASA aircraft.

Problem 1 - Use the parametric equations for h(T) and R(T) to determine the equation for the speed, S, of the Shuttle along its trajectory where $dS/dt = ((dh/dt)^2 + (dR/dt)^2)^{1/2}$

Problem 2 - Determine the formula for the magnitude of the acceleration of the Shuttle using the second time derivatives of the parametric equations.

Problem 3 - From your answer to Problem 2, A) find the time at which the acceleration is an extremum, and specifically, a maximum along the modeled trajectory. B) What is the acceleration in feet/sec^2 at this time? C) If the acceleration of gravity at the earth's surface is 32 feet/sec^2, how many 'Gs' did the astronauts pull at this time?

Problem 1 - Use the parametric equations for h(T) and R(T) to determine the equation for the speed, S, of the Shuttle along its trajectory where $dS/dt = ((dh/dt)^2 + (dR/dt)^2)^{1/2}$

$dh/dt = - 0.142 T^2 + 36.6 T - 345$

$dR/dt = 135.7 e^{0.029T}$

Then

$$dS/dt = ((- 0.142 T^2 + 36.6 T - 345)^2 + (135.7 e^{0.029T})^2)^{1/2}$$

Problem 2 - The components of the acceleration vector, a, are given by $a_h = d^2h/dT^2$ and $a_R = d^2R/dT^2$

so $D^2h/dt^2 = 36.6 - 0.284T$ $d^2R/dt^2 = 3.93 e^{0.029T}$

Then the magnitude of the acceleration vector is just

$|a| = (a_h^2 + a_R^2)^{1/2} = (0.08 T^2 - 20.8 T + 1339 + 15.4 e^{0.058 T})^{1/2}$

Problem 3 - A) The minimum or maximum acceleration is found by solving for T when $d|a|/dT = 0$

$0 = \mathbf{1/2} (0.08 T^2 - 20.8 T + 1339 + 15.4 e^{0.058 T})^{-1/2} (0.16T - 20.8 + 0.89e^{0.058T})$

Then $0 = 0.16T - 20.8 + 0.89e^{0.058T}$ or $20.8 = 0.16 T + 0.89e^{0.058T}$

A calculator can be used to plot this curve and determine that for **T = 46.7 seconds** the condition d |a|/dT = 0 is obtained. This is near the time usually recorded by engineers as the Maximum Dynamic Pressure point 'Max-Q'.

B) Evaluating the answer to Problem 2 at T=46.7 seconds we get $|a| = (773)^{1/2} = 28$ **feet/sec**^{2}.

C) The number of Gs = 28/32 = **0.9 Gs**.