Space Math @ NASA - Searching for Goldilocks Planets in the Milky Way

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NASA is perhaps one of the most famous resources for downloading spectacular pictures of the universe. It also issues dozens of press releases each month about fantastic astronomical discoveries from black holes and dark matter, to the search for Earth-like planets. But more than being just 'eye candy' these pictures and press releases provide fodder for student inquiry using mathematics.

In 2004 I created the Space Math @ NASA program at NASA to show the many behind-the-scenes ways in which simple mathematics can enhance our understanding of what these pictures and press releases are saying. I had none of these resources available to me in grade school as I struggled with math, while passionately interested in astronomy So I decided as an astronomer to create a multitude of examples that modeled the explicit connection between the tedium of math and the excitement of space science. And no, you do not have to be a Rocket Scientist or use calculus all the time!

For most of the really cool discoveries, all you need are basic skills working with percentages, volumes of simple solids, and a bit of scientific notation. The problems posted at SpaceMath@NASA, now exceeding well over 400, span a wide range of mathematical levels. The majority are targeted at on-grade-level middle school students, although there are plenty that are adaptable for advanced high school students in Algebra 2, Geometry and Calculus. Many problems ask the student to use a ruler to determine the scale of an astronomical image, and then use this to create a histogram of, for example, crater diameter frequency. Astronomers use this data to estimate an age for the body. There are problems requiring geometric analysis, such as the paths taken by shock waves reverberating inside the sun. There are problems involving statistics, probability, Venn diagramming, algebraic manipulation and numerical substitution. There are also a number of calculus-level problems for determining the mass of Comet Tempel-1 (a volume integration), or the number of stars in the sky (integrating a power-law function). With over 400+ problems and 20 problem books to choose from, the SpaceMath@NASA is quite vast in scope and content. The possibilities for creating new problems using the existing ones as templates are, well, astronomical! Any student looking for a science fair project will surely find some promising leads with many of these problems.

In February, the 3 millionth problem was downloaded, and the traffic to this resource continues a healthy 5% monthly increase. A very short list of the Top-10 problems in April, 2011 is shown in the table. Most of them are suitable for pre-algebra students!

Title	Skill	Title	Skills
(71) Are the van Allen belts deadly?	Area of rectangle	(408) The speed of a Tsunami	D = V x T
(160) Relative sizes of stars	Proportions,scales	(185) The ISS - Follow that graph!	Graph analysis
(102) How fast does the sun spin?	Scale; d=Vx t	(67) Unit conversion exercises	Unit conversions
(157) The Space Shuttle Trajectory	Parametric eqns	(409) The 2011 Japan Earthquake	Algebra 1
(260) Famous unit conversion errors	Unit conversions	(172) The stellar magnitude scale	Logarithmic numbers

The most popular math problem books at SpaceMath@NASA are 'Algebra-2' a space math supplement, and perhaps not unexpectedly, 'Black Hole Math'. Now let's have a look at a series of problems that can be collected together to explore the theme of searching for Earth-like planets!

<u>Problem 1 - Habitable Zones</u>: When an astronomer has determined the distance, D, of a planet from its star, and the luminosity, L, of the star, they can easily determine what the equilibrium surface temperature, T, of the bare planet will be using a simple algebraic equation involving taking the fourth-root of a quantity. Students can manipulate this equation to model planets at differing distances and with different albedos, A, to discover where the 'liquid water' Habitable Zone lies for which surface temperatures are between 273 and 373 K. Here's what the equation looks like:

$$T=\left(\frac{(1\text{-}A)L}{16\ \sigma\pi D^2}\right)^{1/4}$$

So how hot would our Earth be using this model? By itself, the equation is pretty sterile, but when you put in the quantities that represent our Earth; D = 1.5×10^{11} meters, Albedo A = 0.30, solar luminosity L = 3.8×10^{26} watts, the Stefan-Boltzmann Constant $\sigma = 5.6\times10^{-8}$ and of course $\pi = 3.141$ you get T = 254 K. If Earth were a bare planet, its temperature would be about 20 degrees K below the freezing point of water! Thanks to the trace amount of CO_2 in our atmosphere, (currently about 380 PPM) we have a comfortable amount of Greenhouse Heating going on. This is a very important equation in astrobiology. Students would be encouraged to program it into an Excel Spreadsheet and research the following questions:

1) For a planet with a reflectivity like Earth of 30% (A = 0.30), orbiting a star identical to our sun in L, what would be the range of orbital distances for which $273 < T < 373$

2) What happens to the location and the width of this zone as the luminosity of the star increases?

3) What happens to the temperature of a planet as its reflectivity ,A, changes due to the appearance or disappearance of polar ice caps?

4) On Earth, the relationship between CO_2 in the atmosphere and global warming can be modeled by the linear equation T (K) = 283 + 0.01 (PPM). Students can derive a similar linear equation by comparing the Keeling Curve for atmospheric CO_2 with the amount of global warming during the last 50 years. How might you modify the equilibrium temperature equation, and the location of the Habitable Zone, to account for Greenhouse Heating by near-terrestrial levels of atmospheric CO_2 ?

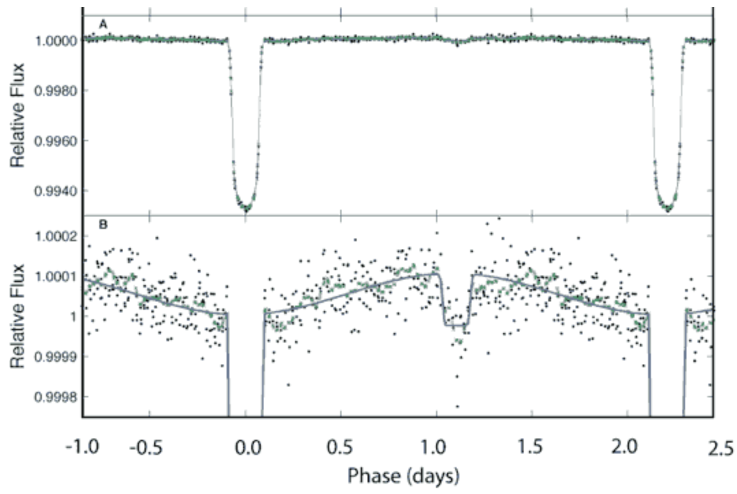
Problem 2 - How many Earth-Like Planets are there?

NASA's Kepler Mission uses a very sensitive instrument light meter called a photometer to measure the dimming of a star's light when a planet passes between the star and our vantage point here on Earth. These transit events not only tell us whether a star has a planet orbiting it, but we can also determine the planet's size, orbital period and distance using these measurements. The amount of dimming in the starlight is exactly determined by the simple equation:

$$\frac{I_s - I_m}{I_s} = \frac{\pi r_p^2}{\pi R_s^2}$$

This is nothing more than comparing the projected circular area of the planet with a radius of r_p to the area of the star whose radius is R_S. If your light meter says that the star light dimmed from I_S = 100% to I_m = 96%, then this equation says that $r_p/R_S = 0.2$, and so the planet's radius is 20% as large as the star's. Since astronomers can determine the radius of a star pretty well based on the star's distance, luminosity and surface temperature, we can determine the actual size of the planet from its transit! Have a look at the following transit light curve for the planet Hat-P7b from the Kepler mission:

The upper curve in the figure below shows that $I_S - I_m = 0.007$, so from our formula we get that r_p = 0.084 of the star's radius. Since R_S = 1.2 million km, the planet has a radius of about 100,000 km. This is 1.4 times the radius of Jupiter! We also see that from the repeated dips, the orbit period is about 2.25 days. By comparison, the orbit period of Mercury is 88 days! The orbit distance, for an assumed stellar mass equal to our sun, is given by $D^3 = T^2$ where D is in Astronomical Units (AUs) and T is in Earth years. (1 'AU' = 150 million km). For Hat-P7B, T = 2.25/265 = 0.0062 years, so T = 0.033 AU or 5 million km!



Students can find many of these transiting exoplanet light curves by doing a GOOGLE search, and work out the size, distance and period. Using the formula from Problem 1, they can determine whether the planet is orbiting in the Habitable Zone of its star.

The best place to get the relevant light curves and stellar data is at the Kepler Data Archive, **http://archive.stsci.edu/kepler/publiclightcurves.html**, which is relatively difficult for the novice to navigate. A helpful introduction for downloading and working with Kepler light curves can be found at **http://evildrganymede.net/2011/01/03/planethunters-download-kepler-lightcurve-data/**. If all you can get doing a GOOGLE search under 'Kepler light curves' are the light curves, it is relatively easy to estimate the planet radius by just using the solar radius of about 700,000 km as a reference. You will have to note the name of the star in the light curve caption, and then search the Kepler Archive to get the stellar data more exactly.

There are many questions one might consider asking using this data and analyzing it as indicated above. For example: 1) How much dimming would you expect to see for an Earth-like planet? 2) From the light curves you can find, what are the average orbital distances for the planets, and the planet diameters? 3) What would a light curve look like for three different planets orbiting the same star at different distances from their star? Again, students may program an Excel Spreadsheet to create their own light curves, and connect this information with Problem 1! The exciting thing is that you don't have to be a 'rocket scientist' to make your own discoveries about these transiting exoplanets! If you need more suggestions, additional exoplanet problems can be found at Space Math @ NASA.

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