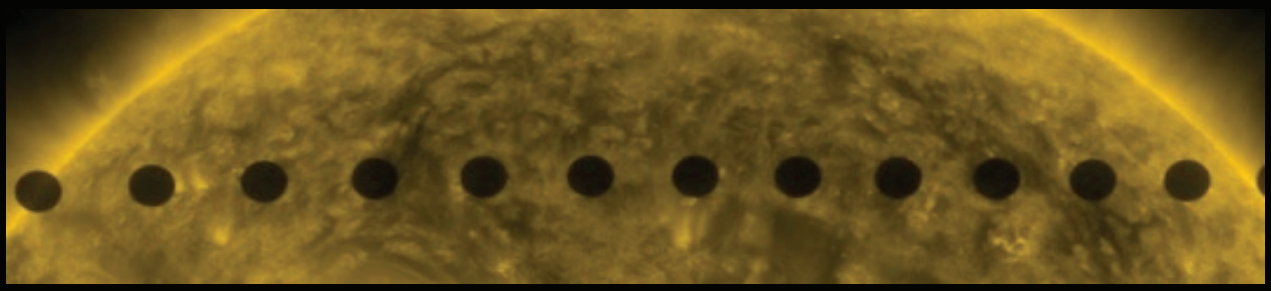
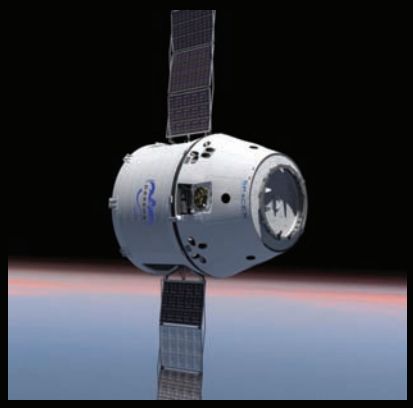
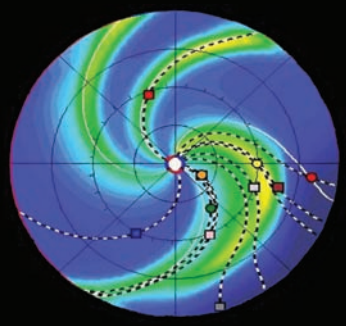
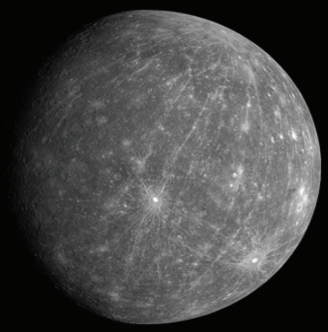
National Aeronautics and Space Administration

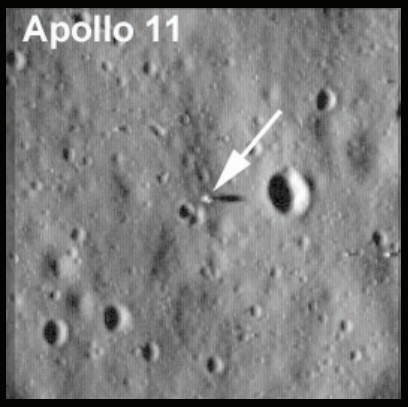
Space Math VIII

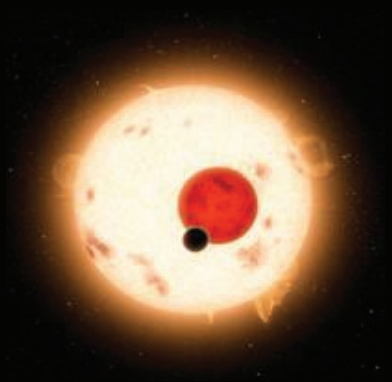


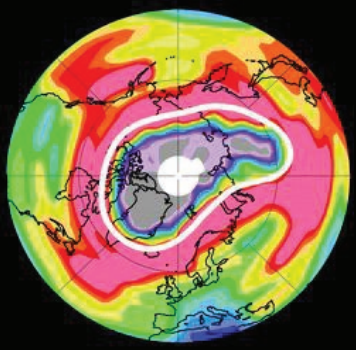












This collection of activities is based on a weekly series of space science problems distributed to thousands of teachers during the 2011-2012 school year. They were intended for students looking for additional challenges in the math and physical science curriculum in grades 5 through 12. The problems were created to be authentic glimpses of modern science and engineering issues, often involving actual research data.

The problems were designed to be 'one-pagers' with a Teacher's Guide and Answer Key as a second page. This compact form was deemed very popular by participating teachers.

For more weekly classroom activities about astronomy and space visit the NASA website,

http://spacemath.gsfc.nasa.gov

Add your email address to our mailing list by contacting Dr. Sten Odenwald at

Sten.F.Odenwald@nasa.gov

Front and back cover credits: Front) SDO montage of Transit of Venus June 5, 2012; SpaceX Dragon capsule (NASA); Solar wind model (CCMC; NASA Goddard); Mercury (MESSENGER); Cats Eye Nebula (Hubble); Apollo 11 landing area (LRO); Kepler 16 a and b (Kepler); Arctic ozone hole (AURA); Asteroid Vesta (Dawn); STEREO launch (Courtesy Dominic Agostini) Back) SDO image of the sun during transit of venus, June 5, 2012, and composite UV image of sun.

Interior Illustrations: Planet Sizes (Kepler); Fuel tank (NASA); Dione (Cassini); Fermi Skymap (Fermi); Shuttle tiles (NASA); Leatherback Turtle (Courtesy NOAA: Scott R. Benson, NMFS Southwest Fisheries Science Center); Kepler 16 ab (Kepler); CoRot-2a (NASA M. Weiss); Mercury interior (MESSENGER); Space X Capsule (NASA; Kennedy SC); Kepler 22b (NASA Kepler); Black hole tidal shredding (NASA, Gezari and Guillochon); STEREO night launch (Dominic Agostini). Unless otherwise indicated, all additional illustrations are courtesy NASA and its scientific missions.

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Mathematics Topic Matrix

Topic	Problem Numbers																													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
Inquiry																															
Technology, rulers									X			X	X	X			X				X	X	X								X
Numbers, patterns, percentages	X	X			X																		X	X							
Averages																															
Time, distance, speed									X	X	X	X	X	X	X	X	X	X	X	X	X	X	X							X	X
Density, mass, volume						X																									
Areas and volumes			X																						X						
Scale Drawings, proportions				X			X		X	X	X	X	X	X		X	X	X	X		X	X	X					X	X	X	X
Geometry, Pythagorean Theorem											X													X				X	X		
Scientific Notation																															
Unit Conversions			X			X																									
Fractions	X	X																													
Graph or Table Analysis							X	X		X	X			X	X		X	X	X	X						X	X				
Solving for X																								X							
Evaluating Fns																															
Modeling																										X	X				
Probability																															
Rates/Slopes																										X	X				
Logarithmic Fns								X																							
Polynomials																															
Power Fns																															
Exponential Fns																				X											
Conics																															
Piecewise Fns																															
Trigonometry																															
Integration																															
Differentiation																															
Limits																															

Mathematics Topic Matrix (cont'd)

Topic	Problem Numbers																	
	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49
Inquiry																		
Technology, rulers																		
Numbers, patterns, percentages	X																	
Averages	X																	
Time, distance, speed		X													X			X
Density, mass, volume			X	X				X										
Areas and volumes				X				X		X								
Scale drawings																X	X	X
Geometry, Pythagorean Theorem					X	X												
Scientific Notation		X	X	X			X	X				X	X	X				
Unit Conversions			X															
Fractions																		
Graph or Table Analysis	X																	
Solving for X						X			X			X	X	X				
Evaluating Fns																		
Modeling																		
Probability																		
Rates/Slopes		X		X			X								X			
Logarithmic Fns																		
Polynomials															X			X
Power Fns																		
Exponential Fns																		
Conics											X					X	X	
Piecewise Fns																		
Trigonometry					X	X					X							X
Integration																		
Differentiation																		
Limits																		

How to use this book

Teachers continue to look for ways to make math meaningful by providing students with problems and examples demonstrating its applications in everyday life. Space Math offers math applications through one of the strongest motivators-Space. Technology makes it possible for students to *experience* the value of math, instead of just reading about it. Technology is essential to mathematics and science for such purposes as “access to outer space and other remote locations, sample collection and treatment, measurement, data collection and storage, computation, and communication of information.” 3A/M2 authentic assessment tools and examples. The NCTM standards include the statement that "Similarity also can be related to such real-world contexts as photographs, models, projections of pictures" which can be an excellent application for all of the Space Math applications.

This book is designed to be used as a supplement for teaching mathematical topics. The problems can be used to enhance understanding of the mathematical concept, or as a good assessment of student mastery.

An integrated classroom technique provides a challenge in math and science classrooms, through a more intricate method for using **Space Math VIII**. Read the scenario that follows:

Ms. Green decided to pose a new activity using Space Math for her students. She challenged each student team with math problems from the **Space Math VIII** book. She copied each problem for student teams to work on. She decided to have the students develop a factious space craft. Each team was to develop a set of criteria that included reasons for the research, timeline and budget. The student teams had to present their findings and compete for the necessary funding for their space craft. The students were to use the facts available in the **Space Math VIII** book.

Space Math VIII can be used as a classroom challenge activity, assessment tool, enrichment activity or in a more dynamic method as is explained in the above scenario. It is completely up to the teacher, their preference and allotted time. What it does provide, regardless of how it is used in the classroom, is the need to be proficient in math. It is needed especially in our world of advancing technology and physical science.

Alignment with Standards

AAAS: Project:2061 Benchmarks

(3-5) - Quantities and shapes can be used to describe objects and events in the world around us. 2C/E1 --- Mathematics is the study of quantity and shape and is useful for describing events and solving practical problems. 2A/E1 **(6-8)** Mathematicians often represent things with abstract ideas, such as numbers or perfectly straight lines, and then work with those ideas alone. The "things" from which they abstract can be ideas themselves; for example, a proposition about "all equal-sided triangles" or "all odd numbers". 2C/M1 **(9-12)** - Mathematical modeling aids in technological design by simulating how a proposed system might behave. 2B/H1 ---- Mathematics provides a precise language to describe objects and events and the relationships among them. In addition, mathematics provides tools for solving problems, analyzing data, and making logical arguments. 2B/H3 ----- Much of the work of mathematicians involves a modeling cycle, consisting of three steps: (1) using abstractions to represent things or ideas, (2) manipulating the abstractions according to some logical rules, and (3) checking how well the results match the original things or ideas. The actual thinking need not follow this order. 2C/H2

NCTM: Principles and Standards for School Mathematics

Grades 6–8 :

- work flexibly with fractions, decimals, and percents to solve problems;
- understand and use ratios and proportions to represent quantitative relationships;
- develop an understanding of large numbers and recognize and appropriately use exponential, scientific, and calculator notation; .
- understand the meaning and effects of arithmetic operations with fractions, decimals, and integers;
- develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios.
- represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules;
- model and solve contextualized problems using various representations, such as graphs, tables, and equations.
- use graphs to analyze the nature of changes in quantities in linear relationships.
- understand both metric and customary systems of measurement;
- understand relationships among units and convert from one unit to another within the same system.

Grades 9–12 :

- judge the reasonableness of numerical computations and their results.
- generalize patterns using explicitly defined and recursively defined functions;
- analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior;
- understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions;
- draw reasonable conclusions about a situation being modeled.

Teacher Comments

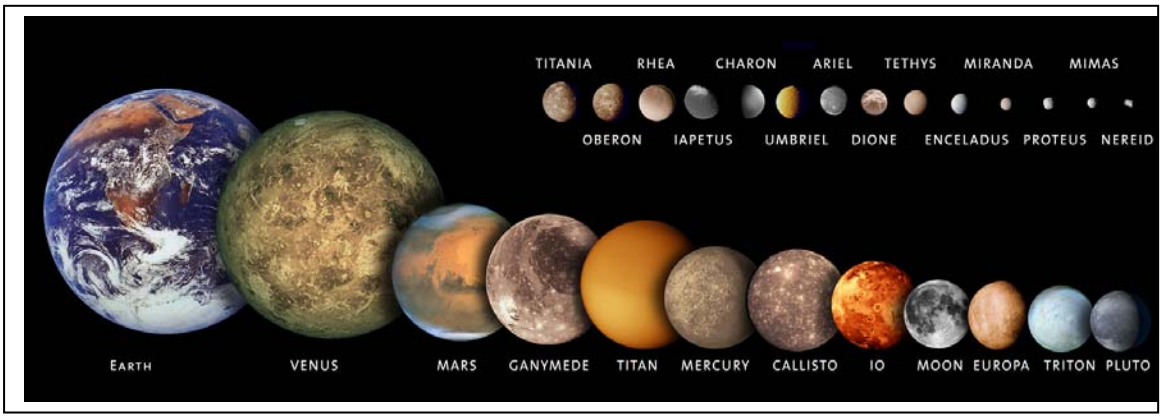
"Your problems are great fillers as well as sources of interesting questions. I have even given one or two of your problems on a test! You certainly have made the problems a valuable resource!" (Chugiak High School, Alaska)

"I love your problems, and thanks so much for offering them! I have used them for two years, and not only do I love the images, but the content and level of questioning is so appropriate for my high school students, they love it too. I have shared them with our math and science teachers, and they have told me that their students like how they apply what is being taught in their classes to real problems that professionals work on." (Wade Hampton High School ,SC)

"I recently found the Space Math problems website and I must tell you it is wonderful! I teach 8th grade science and this is a blessed resource for me. We do a lot of math and I love how you have taken real information and created reinforcing problems with them. I have shared the website with many of my middle and high school colleagues and we are all so excited. The skills summary allows any of us to skim the listing and know exactly what would work for our classes and what will not. I cannot thank you enough. I know that the science teachers I work with and I love the graphing and conversion questions. The "Are U Nuts" conversion worksheet was wonderful! One student told me that it took doing that activity (using the unusual units) for her to finally understand the conversion process completely. Thank you!" (Saint Mary's Hall MS, Texas)

"I know I'm not your usual clientele with the Space Math problems but I actually use them in a number of my physics classes. I get ideas for real-world problems from these in intro physics classes and in my astrophysics classes. I may take what you have and add calculus or whatever other complications happen, and then they see something other than "Consider a particle of mass 'm' and speed 'v' that…" (Associate Professor of Physics)

Big Moons and Small Planets!



This diagram shows the Top-26 moons and small planets in our solar system, and drawn to the same scale.

Problem 1 – What fraction of the objects are smaller than our moon?

Problem 2 – What fraction of the objects are larger than our moon but are not planets?

Problem 3 – What fraction of the objects, including the moon, are about the same size as our moon?

Problem 4 – If Saturn's moon Titan is ½ the diameter of Earth, and Saturn's moon Dione is 1/6 the diameter of Titan, how large is Dione compared to Earth?

Problem 5 – Oberon is 1/7 the diameter of Earth, Io is 1/3 the diameter of Earth, and Titania is 4/9 the diameter of Io. Which moon is bigger: Oberon or Titania?

Problem 1 – What fraction of the objects are smaller than our moon?
Answer: **17/26**

Problem 2 – What fraction of the objects are larger than our moon but are not planets?

Answer: Io, Callisto, Titan and Ganymede : 4/26 or **2/13**

Problem 3 – What fraction of the objects, including the moon, are about the same size as our moon?

Answer: Moon, Europa, Triton and Pluto so 4/26 = **2/13**.

Problem 4 – Saturn's moon Titan is ½ the diameter of Earth, and Saturn's moon Dione is 1/6 the diameter of Titan, how large is Dione compared to Earth?

Answer: ½ x 1/6 = **1/12 the size of Earth**.

Problem 5 – Oberon is 1/7 the diameter of Earth, Io is 1/3 the diameter of Earth, and Titania is 4/9 the diameter of Io. Which moon is bigger: Oberon or Titania?

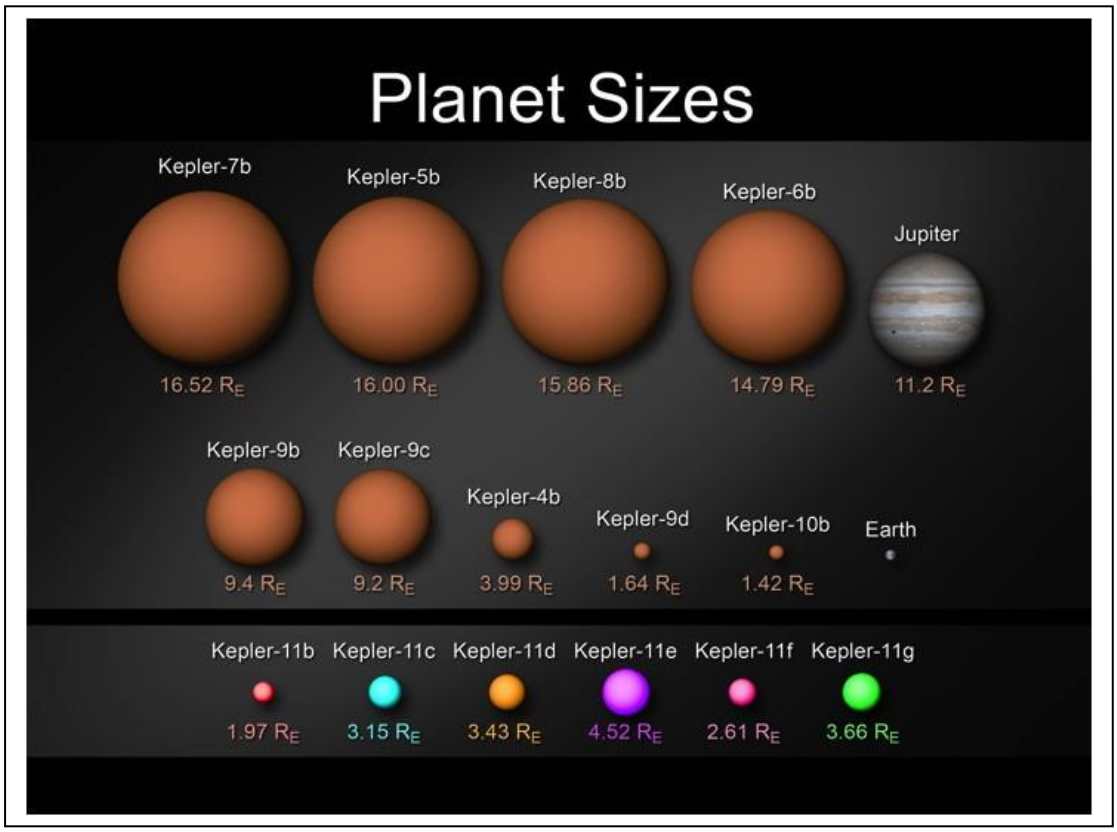
Answer: Oberon is 1/7 the diameter of Earth.
Titania is 4/9 the diameter of Io, and Io is 1/3 the diameter of Earth
So Titania is (4/9) x (1/3) the diameter of Earth
So Titania is 4/27 the diameter of Earth.

Comparing Oberon, which is 1/7 the diameter of Earth with Titania, which is 4/27 the diameter of Earth, which fraction is larger: 1/7 or 4/27?

Find the common denominator 7 x 27 = 189, then cross-multiply the fractions:

Oberon: 1/7 = 27/189 and Titania: 4/27 = (4x7)/189 = 28/189 so

Titania is 28/189 Earth's diameter and Oberon is 27/189 Earth's diameter, and so **Titania is slightly larger!**



This diagram shows disks representing the planets discovered in orbit around 8 different stars all drawn to the same scale. Earth and Jupiter are also shown so that you can see how big they are in comparison. Solve the problems below using fraction arithmetic to find out how big these new planets are compared to Earth and Jupiter.

Problem 1 – Kepler-5b is 8 times the diameter of Kepler-11b .Kepler-11b is twice the diameter of Earth. How big is the planet Kepler-5b compared to Earth?

Problem 2 – The planet Kepler-9c is 9./11 the diameter of Jupiter, and Kepler-11e is 1/2 the diameter of Kepler-9c. How big is the planet Kepler-11e compared to Jupiter?

Problem 3 – The planet Kepler-10b is 1/10 the diameter of Kepler-6b, and Kepler-9b is 9/15 the diameter of Kepler-6b. If Kepler-11g is 4/9 the diameter of Kepler-9b, how big is Kepler-11g compared to Kepler-10b?

Problem 1 – Kepler-5b is 8 times the diameter of Kepler-11b .Kepler-11b is twice the diameter of Earth. How big is the planet Kepler-5b compared to Earth?

Answer: It helps to set up these kinds of problems as though they were unit conversion problems, and then cancel the planet names to get the desired ratio:

$$\frac{1 \times \text{Kepler5b}}{8 \times \text{Kepler11b}} \times \frac{1 \times \text{Kepler11b}}{2 \times \text{Earth}} = \frac{1\ \text{Kepler5b}}{8 \times \text{Earth}}$$ so Kepler 59 is **8x Earth in diameter**

Note how the 'units' for Kepler-11b have canceled out.

Problem 2 – The planet Kepler-9c is 9./11 the diameter of Jupiter, and Kepler-11e is 1/2 the diameter of Kepler-9c. How big is the planet Kepler-11e compared to Jupiter?

$$\frac{11 \times \text{Kepler9c}}{9 \times \text{Jupiter}} \times \frac{2 \times \text{Kepler11e}}{1 \times \text{Kepler9c}} = \frac{22\text{x Kepler9c}}{9 \times \text{Jupiter}}$$ so Kepler 11e = **9/22 Jupiter**

From the figure we see Kepler11e = 4.52 Re and Jupiter = 11.2 Re so Kepler11e = 4.5/11 = 9/22 Jupiter.

Problem 3 – The planet Kepler-10b is 1/10 the diameter of Kepler-6b, and Kepler-9b is 9/15 the diameter of Kepler-6b .If Kepler-11g is 4/9 the diameter of Kepler-9b, how big is Kepler-11g compared to Kepler-10b?

$$\frac{10\text{xKepler10b}}{1\text{xKepler6b}} \times \frac{9\ \text{Kepler6b}}{15\ \text{Kepler9b}} \times \frac{4\ \text{Kepler9b}}{9\ \text{Kepler11g}} = \frac{40\ \text{Kepler10b}}{15\ \text{Kepler11g}}$$

So 15 x the diameter of Kepler 11g is equal to 40x the diameter of Kepler 10b
And so, Kepler 11g is 40/15 times the diameter of Kepler 10b

From the figure we see that Kepler 11g is 3.66 Re and Kepler 10b is 1.4 Re which is in about the same ratio as our fractions. 3.66/1.4 = 2.6 and 40/15 = 2.7.

A Simple Gauge in a Fuel Tank - I

This is a photo of the Space Shuttle main fuel tank just after being jettisoned at an altitude of 50 miles. The liquid hydrogen is approximately shaped like a cylinder with a diameter of 8.4 meters and a length of 29.6 meters.

The formula for the volume of a cylinder is given by

$$V = \pi R^2 h$$

where **R** is the radius of the cylinder and **h** is its length.

Problem 1 – To two significant figures, what is the volume of the fuel tank in:

A) Cubic meters?

B) Cubic centimeters?

C) Liters (1 liter = 1000 cm^3)?

D) Gallons? (1 liter = 0.26 gallons)

Problem 2 – For safety, engineers want to install a gauge that will indicate when there is only 1/8 of the original fuel volume remaining in the tank. How many meters from the bottom of the tank must the gauge be located to be triggered?

Problem 3 – The rate at which fuel is burned by the Space Shuttle engines is about 1000 gallons per second. From ignition, how many minutes (to the nearest tenth) will it take for the fuel to reach the level of the fuel gauge?

Problem 1 – What is the volume of the fuel tank for R = 8.4/2 = 4.2 meters and h = 29.6 meters.

A) V = 3.14 $(4.2)^2$ (29.6) = 1639 **= 1600 meters^3**

B) V = 1600 x (100 cm / 1 m)^3 = **1.6 billion cm^3**

C) V = **1.6 million liters**

D) V = 1.6 million liters x (0.26 gallons/1 liter)
= 416,000 = **420,000 gallons**

Problem 2 – For safety, engineers want to install a gauge that will indicate when there is only 1/8 of the original fuel volume remaining in the tank. How many meters from the bottom of the tank must the gauge be located to be triggered?

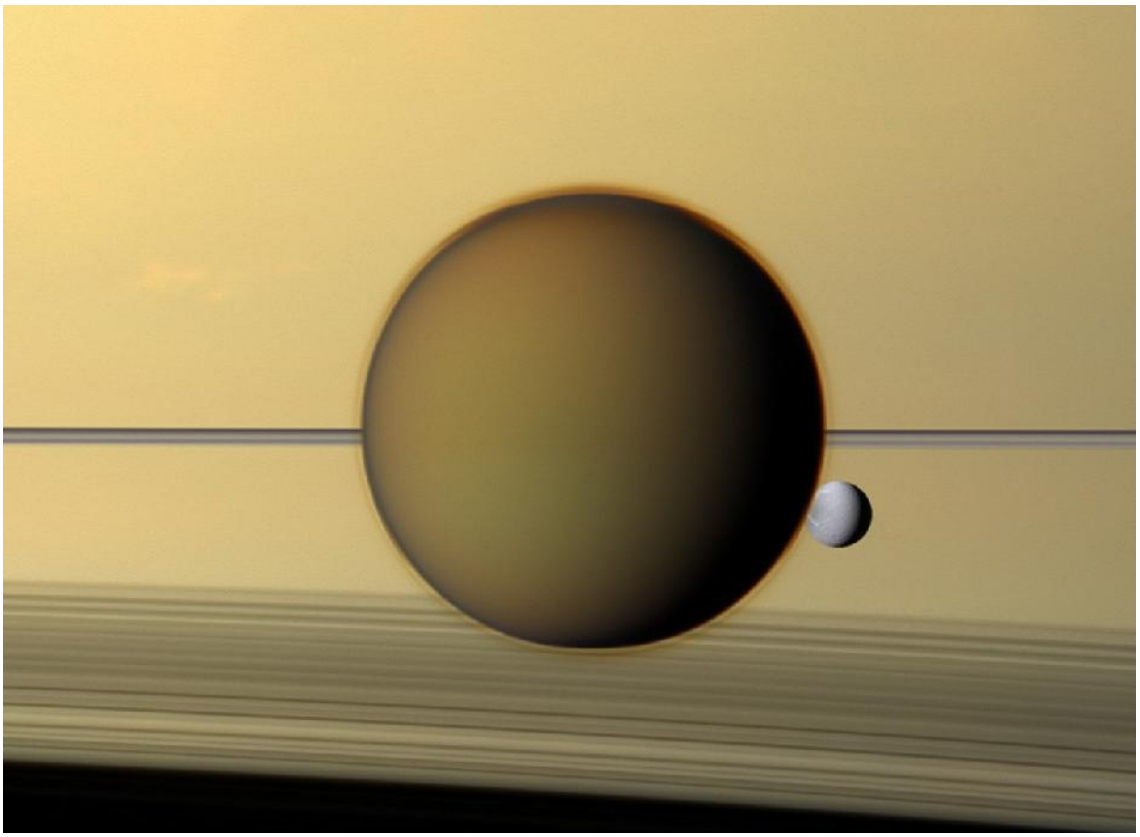
Answer: Method 1: V = 1/8 (1600) = 200 meters^3. Then since R = 4.2 meters, from the formula we get

200 = 3.14$(4.2)^2$ h and solving for h we get **3.6 meters** from the bottom of the tank.

Method 2: From the formula for the cylinder, since R remains the same, only h will vary and so the gauge will be located 1/8(29.6 meters) = **3.7 meters** from the bottom of the tank.

Problem 3 – The rate at which fuel is burned by the Space Shuttle engines is about 1000 gallons per second. From ignition, how many minutes (to the nearest tenth) will it take for the fuel to reach the level of the fuel gauge?

Answer: The tank must lose 7/8 of its original volume in gallons which equals 7/8 (420,000) = 367,500 gallons. At a rate of 1000 gallons per second, this will take 367.5 seconds or **6.1 minutes**.



Saturn's third-largest moon Dione can be seen through the haze of its largest moon, Titan, in this view of the two posing before the planet and its rings from NASA's Cassini spacecraft. This view looks toward Titan (about 5000 kilometers across) with Saturn in the background, and the smaller moon Dione as it is about to be eclipsed by Titan. The images were obtained with the Cassini spacecraft narrow-angle camera on May 21, 2011 at a distance of approximately 1.4 million miles (2.3 million kilometers) from Titan.

Problem 1 – From the image above, what is the ratio of the apparent diameter of Dione to the diameter of Titan rounded to the nearest, simple fraction (example: 0.34 = 1/3)?

Problem 2 - Suppose that Dione were exactly the same diameter, in kilometers, as Titan. From the vantage point of Cassini, about what would be the distance to Dione in kilometers?

Problem 3 – The actual diameter of Dione is known to be about 1000 kilometers. What is its actual distance from Cassini at the time this image was taken? Explain your reasoning.

. Press Release: **NASA's Cassini Delivers Holiday Treats From Saturn**

December 22, 2011

http://www.nasa.gov/mission_pages/cassini/whycassini/cassini20111222.html

Problem 1 – From the image above, what is the ratio of the apparent diameter of Dione to the diameter of Titan rounded to the nearest, simple fraction (example: 0.34 = 1/3)?

Answer: Use a millimeter ruler to measure the apparent diameters of the two moons. Titan is about 60 millimeters and Dione is about 9 millimeters in diameter. The ratio is 9/60 or 3/20. Students should use **1/7** as 'simplest fraction' approximation.

Problem 2 - Suppose the Dione were exactly the same diameter, in kilometers, as Titan. From the vantage point of Cassini, about what would be the distance to Dione in kilometers?

Answer: To appear as large as it seems to Cassini, Titan is at a distance of 2.3 million kilometers. If Dione were the same physical diameter as Titan (5000 kilometers) it would have to be 7 times farther from Cassini in order for the apparent diameters to be in the ration 1/7. The distance to Dione is therefore 7 x 2.3 million km or **16 million km**.

Problem 3 – The actual diameter of Dione is known to be about 1000 kilometers. What is its actual distance from Cassini at the time this image was taken? Explain your reasoning.

Answer: If Titan and Dione were of comparable physical size, Dione would have to be 16 million km from Cassini in order for Dione to appear to be 1/7 the diameter of Titan.

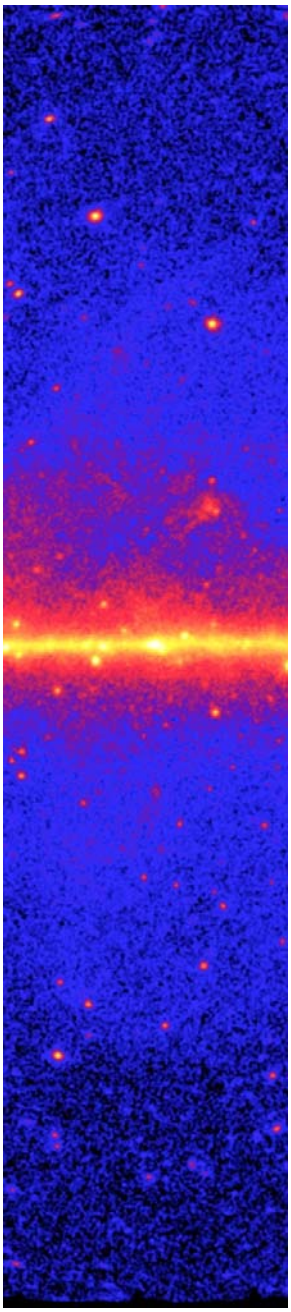
But, Dione's physical diameter compared to Titan is 1000 km compared to 5000 km for Titan, to their actual diameter ratio when seen at the same distance from Cassini is 1000/5000 = 1/5. If Dione and Titan were sitting next to each other, the ratio of their physical diameters alone would make the disk of Dione appear to be 1/5 the diameter of Titan or 12 millimeters instead of the 9 millimeters we measured.

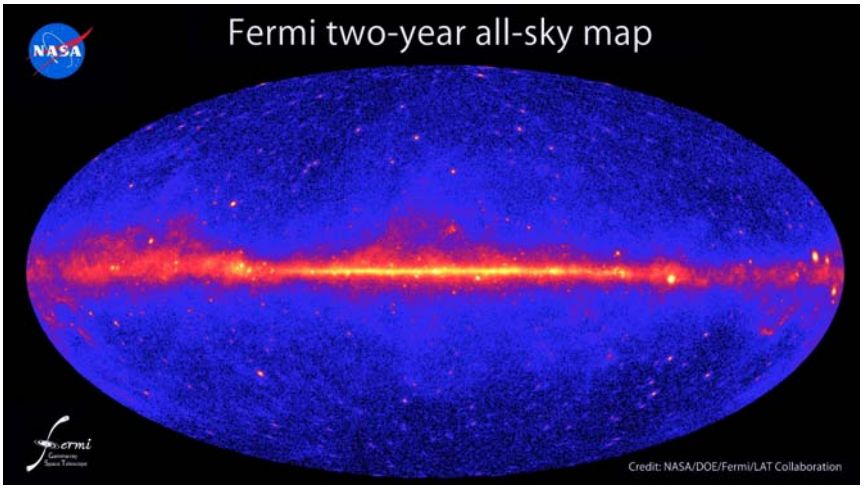
To make the apparent ratio equal the slightly smaller ratio of 1/7 at a distance of 2.3 million km, we have to move Dione slightly further away from Cassini than Titan by an amount equal to the proportion:

$$\frac{(1/7)}{2.3 million} = \frac{(1/5)}{X}$$

so x = 2.3 million x (1/5)/(1/7) = 2.3 million x 7/5 = **3.2 million km**

Note: even though Titan and Dione <u>appear</u> close together in the picture, they are actually over (3.2-2.3) million = 900,000 kilometers apart...about 3 times the distance from Earth to the Moon!





This all-sky image, constructed from two years of observations by NASA's Fermi Gamma-ray Space Telescope, shows how the sky appears at light energies greater than 1 billion electron volts (1 GeV). As a comparison, the x-rays used by your dentist to search for cavities have energies of only about 5,000 electron volts (5 KeV).

In the false-color diagram above, brighter colors like red orange and yellow, indicate brighter gamma-ray sources. A diffuse glow fills the sky and is brightest along the plane of our galaxy (middle). The point-like gamma-ray 'spots' are believed to be pulsars and supernova remnants within our galaxy, as well as distant galaxies powered by supermassive black holes. (Credit: NASA/DOE/Fermi LAT Collaboration)

Earlier this year, the Fermi team released its second catalog of sources detected by the satellite's Large Area Telescope (LAT), producing an inventory of 1,873 gamma-ray point-sources found in their survey. The resulting classifications of the sources are shown in the lower-left table.

A much smaller study of 11 of these 572 unidentified Fermi/LAT sources by the Japanese Suzaku X-ray Observatory revealed that 6 could be identified as pulsars, 3 were unknown, 1 was a normal flaring star and 1 was a blazar-type galaxy.

Type of Object	Number
Blazar galaxy	1069
Pulsars	115
Supernovae	77
Active Galaxies	20
Normal galaxies and stars	20
Unknown objects	572

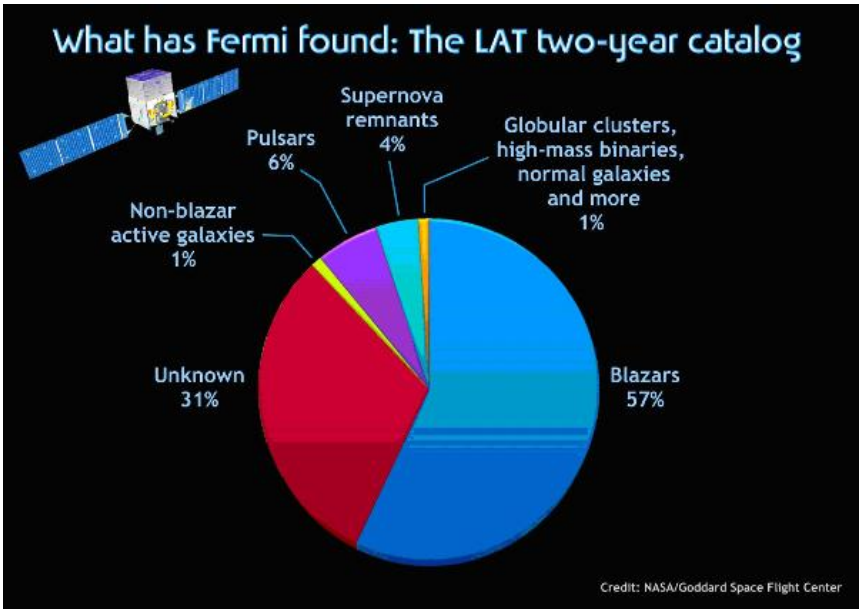
Problem 1 – Create a pie graph showing the percentage of gamma-ray sources in each of the six different categories listed in the original Fermi/LAT survey.

Problem 2 – Based upon the results from the second Suzaku Survey of the 'unknown' objects, and assuming that the Suzaku objects were randomly selected from the Fermi/LAT 'unknowns' , what are the new percentages for the 6 categories?

Problem 3 – What is the probability that the remaining unknown Fermi/LAT survey sources are very faint blazer galaxies and pulsars?

Problem 1 –Answer: See the percentages listed in the table below and the actual pie graph provided by the Fermi/LAT research report.

Type of Object	Number	Percentage
Blazar galaxy	1069	**57%**
Pulsars	115	**6%**
Supernovae	77	**4%**
Active Galaxies	20	**1%**
Normal galaxies and stars	20	**1%**
Unknown objects	572	**31%**



This pie graph is from
http://www.nasa.gov/mission_pages/GLAST/news/gamma-ray-census.html

Problem 2 – Answer: The 11 Suzaku survey says that the 572 Fermi/LAT unknowns should be distributed as follows among the Fermi/LAT source types:

Pulsars = 6/11 x 572 = 312 sources
Unknown = 3/11 x 572 = 156 sources
Normal star = 1/11 x 572 = 52 sources
Blazar = 1/11 x 572 = 52 sources.

Adding these to the already identified Fermi/LAT source types we get the new distribution:

Type of Object	Number (N=1873)	Percentage
Blazar galaxy	1069 +52 = 1121	**60%**
Pulsars	115 + 312=427	**23%**
Supernovae	77=77	**4%**
Active Galaxies	20=20	**1%**
Normal galaxies and stars	20 + 52=72	**4%**
Unknown objects	156	**8%**

Approximately 24,300 tiles were installed on each space shuttle and each tile was designed to survive 100 trips to space and back. Varying in thickness from one inch (2.54 cm) to five inches (12.7 cm) depending on the heating they will be subjected to, the tiles collectively protected the orbiter from temperatures as high as 2,300 degrees Fahrenheit during its reentry into the Earth's atmosphere.

The silica tile material is referred to as LI-900. They insulate heat so well that tiles can be held bare-handed on one side even while the opposite side is still red hot. Educators can demonstrate that ability in the classroom, substituting a blow torch for the re-entry-generated heating.

LI-900 has a density of 9 pounds per cubic foot (144.2 kg/m^3). It is made from pure silica glass fibers, but 94% of the volume of each tile is pure air, making each tile incredibly light and strong!

Problem 1 – If the dimensions of an average tile are 15cm x 15 cm x 6cm, what is the total volume of the Space Shuttle heat shield provided by the 24,300 tiles in cubic meters?

Problem 2 – About what is the mass, in grams, of one average tile?

Problem 3 – What is the total mass of the Space Shuttle heat shield in

A) kilograms?

B) pounds ? (1 pound = 0.453 kg)

Problem 1 – If the dimensions of an average tile are 15cm x 15 cm x 6cm, what is the total volume of the Space Shuttle heat shield provided by the 24,300 tiles in cubic meters?

Answer: A single average tile has a volume of
V = 0.15 m x 0.15m x 0.06 m
= 0.00135 meters^3,

so the total volume occupied by 24,300 tiles is about

V = 24,300 x 0.00135 = **32.8 cubic meters.**

Problem 2 – About what is the mass, in grams, of one average tile?

Answer: Mass = Volume x Density
= 0.00135 m^3 x 144.2 kg/m^3
= 0.195 kilograms.

Since 1 kilogram = 1000 grams, we have a mass per tile of about **195 grams**.

Problem 3 – What is the total mass of the Space Shuttle heat shield in A) kilograms? B) pounds is 1 pound = 0.453 kg?

Answer: A) Mass = volume x density
= 32.8 cubic meters x 144.2 kg/m^3 = **4,730 kg**.

B) 4,630 kg x (1 pound / 0.453 kg) = 10,441 pounds (or about 5 tons!)

Note to Teachers: **Free Space Shuttle Tiles for your Classroom!**

You can still get your own free NASA tiles from the Space Shuttle program!!! Schools may request a tile at the "NASA Space Programs - Historic Artifacts Prescreening" Web site. **http://gsaxcess.gov/NASAWel.htm**

Once at the site, go to the "NASA Artifacts Prescreening Register" block of information to register and receive your login ID and password.

There is no charge for the Shuttle Tiles and Space Food Kits . However, the recipient is responsible for the shipping and handling fee of $23.40 for Shuttle Tiles and $28.03 for the Space Food Kits. Payment must be made to the shipping agent with a credit card via a web link provided in the module.

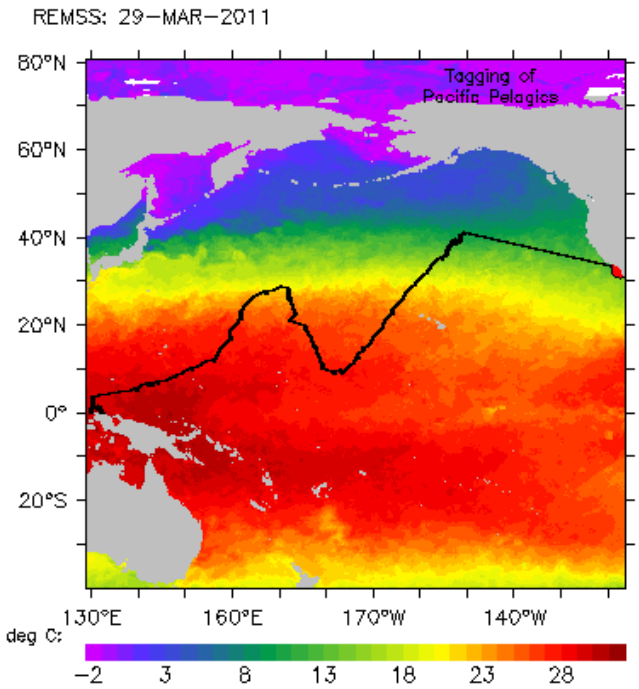
	Day Number	Latitude	Longitude	Distance (km)
1	0	+5° North	130° East	0
2	249	+10° North	155° East	2813
3	488	+30° North	170° East	2707
4	705	+10° North	180° East	2446
5	1099	+40° North	150° West	4453
6	1347	+32° North	120° West	2831

Plot the points listed in the table above to track the sea turtle from the start of its journey to its end. Use your map to answer these questions:

Question 1 - In what part of the world did it begin its trip?
Question 2 - Where did the turtle end up?
Question 3 - How many total kilometers did the turtle travel?

Note to teacher: The data was obtained from the Census of Marine Life's Tagging of Pacific Predators (TOPP) website at http://www.topp.org/

The data is for a Leatherback Sea Turtle Tag Number 2607020, PTT number 23662, between July 22, 2007 to March 30, 2011. The actual data is shown in the image below, which superimposes the track onto a satellite map of the ocean temperature in degrees Celsius (see color bar).
http://las.pfeg.noaa.gov/TOPP_recent/TOPP_tracks180.html?species=0&zone=10



Suggested answers:

Question 1 - In what part of the world did it begin its trip? - **New Guinea**

Question 2 - Where did it end up? **California**

Question 3 - How many total kilometers did it travel? (Hint: use a piece of string on a globe). 2813 + 2707 + 2446 + 4453 + 2831 = **15,250 kilometers**.

Extension Problem: Students can be asked to estimate the speed of the turtle in kilometers per day (about 11.3 km/d) or other units (miles per hour or miles per year etc..) where 1 km = 0.62 miles.
MPH = 11.3 km/day x (1 day / 24 h) x (1 mile / 0.62 km) = 0.76 mph.
Miles per year = 0.76 mph x (24 h/ 1 day) x (365 days/yr) = 6,658 miles/year.

The Juno spacecraft was launched on August 5, 2011 on a 5 year journey to Jupiter. This image was taken 120 seconds after launch and shows one of the solid rocket boosters being jettisoned. The camera is on the Atlas booster and looks down on the engines and the distant arc of a cloudy Earth. Scenes from the launch can be found on *YouTube*, and show a dazzling launch from multiple viewing locations on Earth and in space.

During the launch, and the boost to orbit, the altitude of the rocket changes continuously as the engines provide thrust, eventually lifting the entire payload into orbit at a planned altitude of 420 kilometers (261 miles). A short table of the rocket's altitude and times is provided below.

Elapsed Time (seconds)	Altitude (miles)	Altitude (kilometers)
160	46	
192	60	
268	81	
274	83	
315	95	
319	97	
339	112	

Problem 1 - American engineers use English units for all measurements including the details of the rocket launch where 1 mile = 1.6 kilometers. Use this information to complete the table above in metric units rounded to the nearest kilometer.

Problem 2 - From the tabular data, graph the altitude of the rocket in time.

Problem 3 - From the data, find the function A(t) that predicts the altitude of the rocket at future times. The function will be of the form

$$A(t) = a + b\ln(t)$$

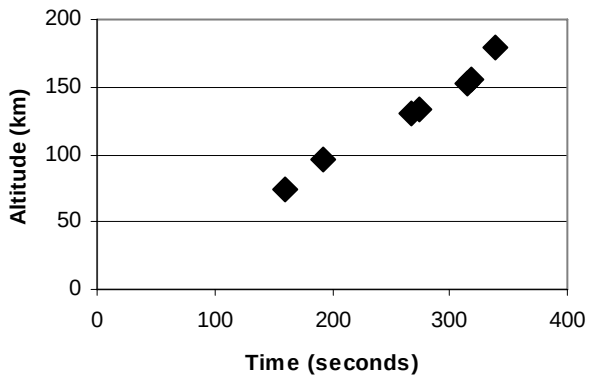
Find the constants a and b for A(t) in kilometers and A(t) in miles.

Problem 4 - How many seconds after launch will it take for the rocket to reach orbit altitude at 420 kilometers?

Problem 1 - American engineers use English units for all measurements including the details of the rocket launch where 1 mile =1.6 kilometers. Use this information to complete the table above in metric units rounded to the nearest kilometer.

Elapsed Time (seconds)	Altitude (miles)	**Altitude (kilometers)**
160	46	**74**
192	60	**96**
268	81	**130**
274	83	**133**
315	95	**152**
319	97	**155**
339	112	**179**

Problem 2 - From the tabular data, graph the altitude of the rocket in time.



Problem 3 - From the data, find the function A(t) that predicts the altitude of the rocket at future times. The function will be of the form

$$A(t) = a + b\ln(t)$$ Find the constants a and b.

Answer: Choose (160,74) and (319,155) then
74 = a + 5.1b and 155 = a + 5.8b
Solve by substitution: a = 74 - 5.1b then 155 =(74-5.1b) + 5.8b
So 81 = 0.7b and b = 116. Then a = -518
So with A(t) in kilometers, and t in seconds, we have:

$$A(t) = -518 + 116\ln(t)$$

In miles this becomes $$A(t) = -324 + 73\ln(t)$$

Problem 4 - How many seconds after launch will it take for the rocket to reach orbit altitude at 420 kilometers?

Answer: $420 = -518 + 116\ln(t)$ so t = **3,248 seconds or about 54 minutes**.

Note: The actual time is about 3,229 seconds.

This sequence of images was taken of the launch of the Juno spacecraft on August 5, 2011 from Cape Canaveral. The images were taken, from left to right, at T+21, T+23 and T+25 seconds after launch, which occurred at 12:25:00 pm EDT. The original video can be found on *YouTube*. The distance from the base of the Atlas-Centaur rocket to its top is 45 meters (148 feet). As the video was produced, the camera zoomed-out between the T+21 image and the T+23 image. Both the T+23 and T+25 images were taken at exactly the same zoom scale.

Problem 1 - From the information given, find the speed of the rocket in meters/sec and kilometers/hr between A) 21 and 23 seconds after launch and B) 23 to 25 seconds after launch.

Problem 2 - What is the average acceleration of the rocket in meters/sec^2 between 21 and 25 seconds after launch?

Problem 3 - At the average acceleration of this rocket, about when will it be traveling faster then the speed of sound (Mach 1) which is 340 meters/sec?

Problem 1 - From the information given, find the speed of the rocket in meters/sec and kilometers/hr between A) 21 and 23 seconds after launch and B) 23 to 25 seconds after launch.
Answer: Students will need to determine the scale of each image by using a millimeter ruler to measure the length of the rocket body, which is known to be 45 meters. When printed using a regular laser printer, the lengths of the rockets are about 21) 5.5mm 23) 4.0 mm and 25) 3.0 mm
The image scales are therefore 8.2 meters/mm, 11.3 meters/mm and 15 meters/mm

To measure speed, all we need to do is measure the height of the bottom of the rocket vertically from a well-defined point near the bottom of the image away from the exhaust cloud. The horizontal band of water just below the exhaust plume provides a good reference. Using the millimeter ruler we get 21) 35 mm 23) 36 mm and 25) 43 mm Converting this in to meters using the three scales we get 21) 287 meters 23) 407 meters and 25) 645 meters

Speed: 21 to 23 seconds; s1 = (407-287) / 2 sec so s1 = **60 meters/sec**
23 to 25 seconds: s2 = (645 - 407)/ 2 sec so s2 = **119 meters/sec**

In km/h we get s1 = **216 km/hour** and s2 = **428 km/hr**.

Students estimates will vary depending on the method and measuring accuracy used.

Problem 2 - What is the average acceleration of the rocket in meters/sec^2 between 21 and 25 seconds after launch?

Answer: acceleration = difference in speed/difference in time so
Acc = (119 meters/sec - 60 meters/sec) / (4 seconds)
= **15 meters/sec^2**

Problem 3 - At the average acceleration of this rocket, about when will it be traveling faster then the speed of sound (Mach 1) which is 340 meters/sec?

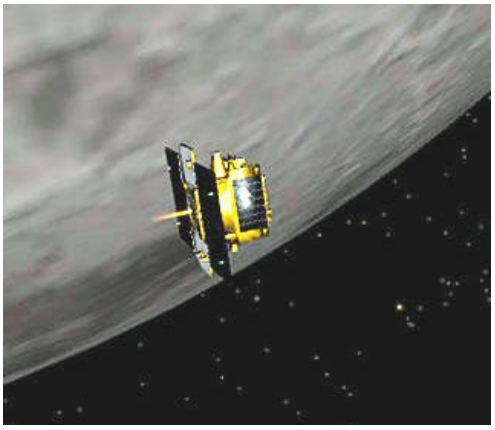
Answer: speed = initial speed + acceleration x time

340 = 119 + 15 x T

So **T = 15 seconds after the initial speed** of 119 m/s was reached. This occurs about T = 25 sec + 15 sec = **40 seconds after launch**.

According to actual flight information, Mach 1 was reached a bit later at T+ 51 sec.

The Grail and LRO Spacecraft Encounters in Lunar Orbit



On May 31, 2012 the Grail 'Ebb' spacecraft and the Lunar Reconnaissance Orbiter (LRO) will come very close to each other in their orbits around the moon. LRO is in a polar orbit, while Grail Ebb is in an equatorial orbit. Although there is no scientific value in the encounter, it does represent one of the first times that two NASA spacecraft orbiting the same astronomical body have passed so close to each other, and with the capability of actually seeing each other.

The table to the left gives the encounter times in the afternoon (Eastern Standard Time in hours, minutes and seconds) and distances (in kilometers) between the spacecraft.

Time	Distance	Time	Distance
4:00:38	190	4:01:23	110
4:00:42	180	4:01:35	105
4:00:46	170	4:01:46	110
4:00:51	160	4:01:56	120
4:00:56	150	4:02:03	130
4:01:01	140	4:02:09	140
4:01:07	130	4:02:14	150
4:01:14	120	4:02:20	160

Problem 1 – At what time were the spacecraft at their closest distances from one another?

Problem 2 – About how fast, in kilometers/hour was the distance between them changing just before closest approach?

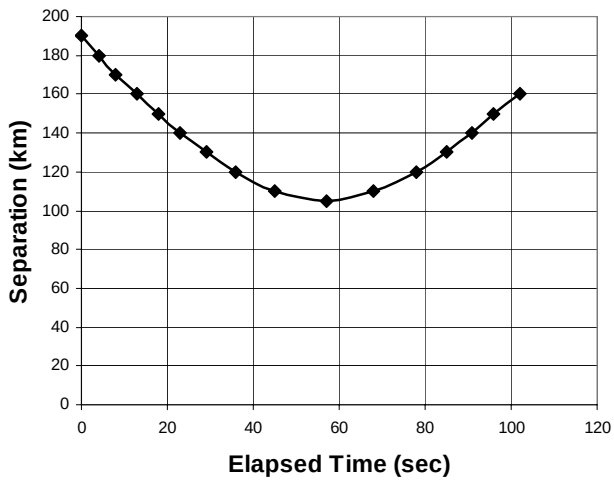
Problem 3 - Calculate the elapsed time of the encounter since 4:00:38 in seconds. Graph the tabular data in terms of elapsed time in seconds and distance in kilometers. What shape does the plotted curve resemble?

Problem 4 - The Grail 'Ebb' spacecraft will attempt to take a picture of LRO. At a distance of 50 kilometers, Grail/Ebb can just resolve an object if it is 8-meters across. That means that the angle corresponding to 8 meters at a distance of 50 kilometers is just large enough to be discerned by Grail. The LRO spacecraft is about 4 meters across. Using simple proportions, and the fact that the angular size of an object is inversely proportional to its distance, will Grail be able to see any details on the LRO spacecraft at the time of closest approach?

Problem 1 – At what time were the spacecraft at their closest distances from one another? Answer: This would be at **4:01:35** at a distance of 105 kilometers.

Problem 2 – About how fast, in kilometers/hour was the distance between them changing just before closest approach? Answer: Speed = distance/time. Distance = 110-105 = 5 km; time = 4:01:35 – 4:01:23 = 12 seconds so speed = 5km/12sec = 0.417 km/sec. In terms of km/hr, speed = 0.41 km/s x (3600 sec/1 hr) = **1500 km/hr**.

Problem 3 - Calculate the elapsed time of the encounter since 4:00:38 in seconds. Graph the tabular data in terms of elapsed time in seconds and distance in kilometers. What shape does the plotted curve resemble? Answer: **A parabola!**



Problem 4 - The Grail 'Ebb' spacecraft will attempt to take a picture of LRO. At a distance of 50 kilometers, Grail/Ebb can just resolve an object if it is 8-meters across. That means that the angle corresponding to 8 meters at a distance of 50 kilometers is just large enough to be discerned by Grail. The LRO spacecraft is about 4 meters across. Using simple proportions, and the fact that the angular size of an object is inversely proportional to its distance, will Grail be able to see any details on the LRO spacecraft at the time of closest approach?

Answer: At their closest separation, 105 kilometers is about twice as large as 50 km, and so for the same angle as seen by an 8 meter object at 50 km, the object would have to be about 8 x 105 km/50 km = 16.8 meters long in order to be seen by Grail. But LRO has a maximum size of only 4 meters, so that means Grail will only see LRO as an unresolved 'dot' of light as it passes by.

Note: Explore the encounter by using NASA's Eyes on the Solar System orbit simulator. Select the Moon, and the date and time of the encounter, then manipulate the scene until the two spacecraft orbits are highlighted. Step the time forward until you come to the encounter scene. To jump to this scene from here click on this link after first setting up EOSS to run on your computer: **http://1.usa.gov/LoD2YE**

The table below provides the altitude, range and times for the Space Shuttle Atlantis after its launch at 11:29:00 a.m. EDT from NASA's Cape Canaveral Space Center, Launch Pad 39A.

Time (minutes)	Altitude (km)	Range (km)
0	0	0
0.7	6	5
1.2	17	10
1.5	27	20
2.5	61	80
2.7	67	104
3.0	76	126
3.3	84	172
3.8	94	224
4.3	101	297
5.1	107	413
5.4	108	486
6.0	108	600
6.4	107	821
7.0	105	1040
7.6	103	1245
8.0	103	1474
8.6	106	1859
9.0	108	2006

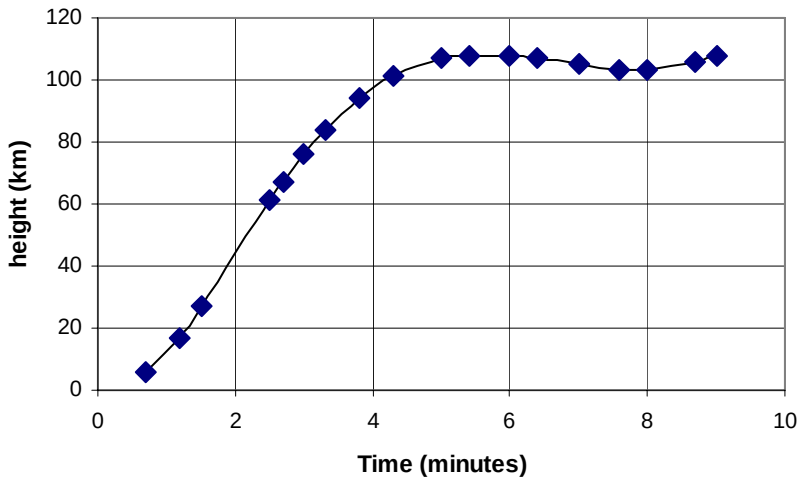
Problem 1 - Plot the altitude versus time for the launch.

Problem 2 - Plot the down-range distance versus time for this launch.

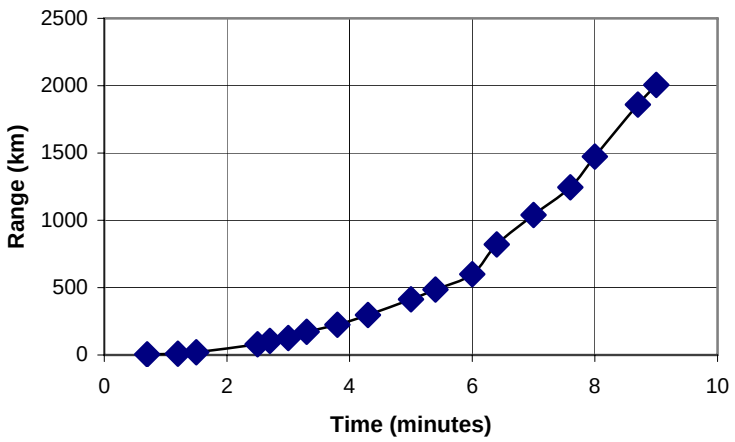
Problem 3 - The actual distance traveled by the Shuttle can be found using the Pythagorean Theorem where the hypotenuse of the right triangle is formed from the distance traveled in altitude (vertical 'y' direction) and the distance traveled in range (horizontal 'x' direction). How far did Atlantis travel between 8.0 and 9.0 minutes after launch?

Problem 4 - What was the average speed of Atlantis between 8.0 and 9.0 minutes after the launch in: A) kilometers/minute? B) miles per hour?

Problem 1 - Plot the altitude versus time for the launch.



Problem 2 - Plot the down-range distance versus time for this launch.



Problem 3 - The actual distance traveled by the Shuttle can be found using the Pythagorean Theorem where the hypotenuse of the right triangle is formed from the distance traveled in altitude (vertical 'y' direction) and the distance traveled in range (horizontal 'x' direction). How far did Atlantis travel between 8.0 and 9.0 minutes after launch?
Answer: Y = altitude difference = 108 - 103 = 5 kilometers .Y = range difference = 2006 - 1474 = 532 km, so the distance traveled = $(5^2 + 532^2)^{1/2}$ = **532 km**.

Problem 4 - What was the average speed of Atlantis between 8.0 and 9.0 minutes after the launch in: A) kilometers/minute? B) miles per hour?

Answer: A) speed = distance/time so speed = 532 km/1 minute, speed = **532 km/minute**.
B) 532 km/minute x (60 minutes/1 hr) x (0.62 miles / 1 km) so **speed = 19,790 miles/hr**.

The data were obtained from the GOOGLE Earth tracking data using the application file available at: **http://www.nasa.gov/mission_pages/shuttle/shuttlemissions/shuttle_google_earth.html**

Space Shuttle Atlantis (STS-135) Launch Speed

This sequence of images shows the historic launch of the Space Shuttle Atlantis (STS-135) on July 8, 2011 at 11:29 a.m. EDT, from launch pad 39A at the NASA Cape Canaveral Space Center.

From bottom to top, the image times are 11:29:15.0, 11:29:16.0, 11:29:17.0, 11:29:18.0, and 11:29:19.0. The length of the space shuttle orbiter (not the red fuel tank) is 37 meters.

The launch sequence can be seen in the video located at:

Problem 1 - Using a millimeter ruler, what is the scale of an individual image in meters/mm?

Problem 2 - Measure the height in meters between the tip of the red shuttle fuel tank and a fixed location near the bottom of each frame.

Problem 3 - Graph the height of the fuel tank versus elapsed time beginning at T=0 in the bottom (first) image.

Problem 4 - About what was the average speed of the Shuttle in the top image in A) meters/sec? B) miles per hour?

Problem 1 - Using a millimeter ruler, what is the scale of an individual image in meters/mm?

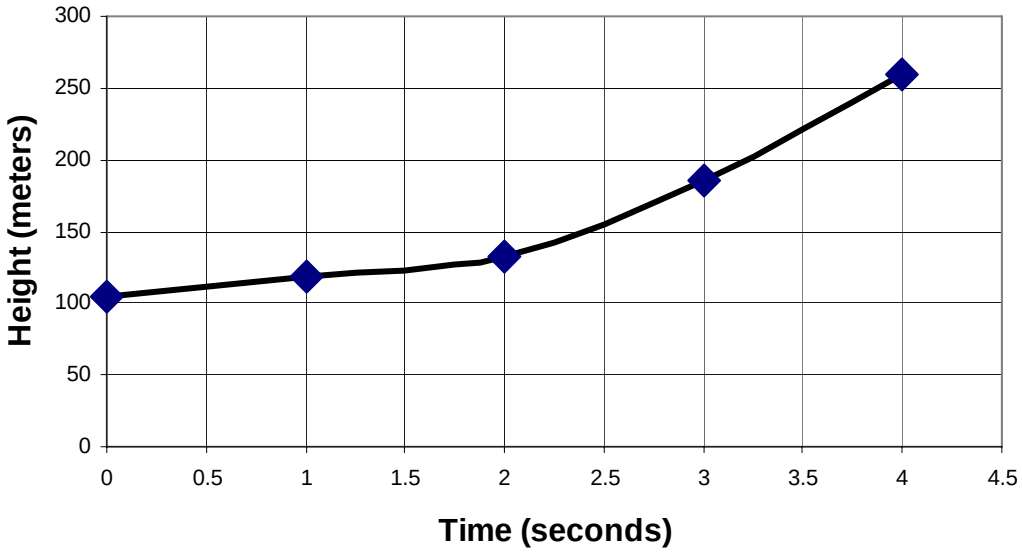
Answer: When this page is reproduced at normal scale, the length of the Orbiter is about 5 millimeters ,which corresponds to 37 meters, so the scale is 37/5 = **7.4 meters/mm.**

Problem 2 - Measure the height in meters between the tip of the red shuttle fuel tank and a fixed location near the bottom of each frame.

Answer; Students need to select a feature on the ground that is visible in each of the frames .One such feature is the top of the black rectangle 'status board' directly below the launch gantry. Measured from the top of this board to the tip of the red fuel tank, the values are as follows: 14 mm, 16mm, 18mm, 25mm, 35mm which correspond to heights of 104m ,118m, 133m, 185m and 259m.

Problem 3 - Graph the height of the fuel tank versus elapsed time beginning at T=0 in the bottom (first) image.

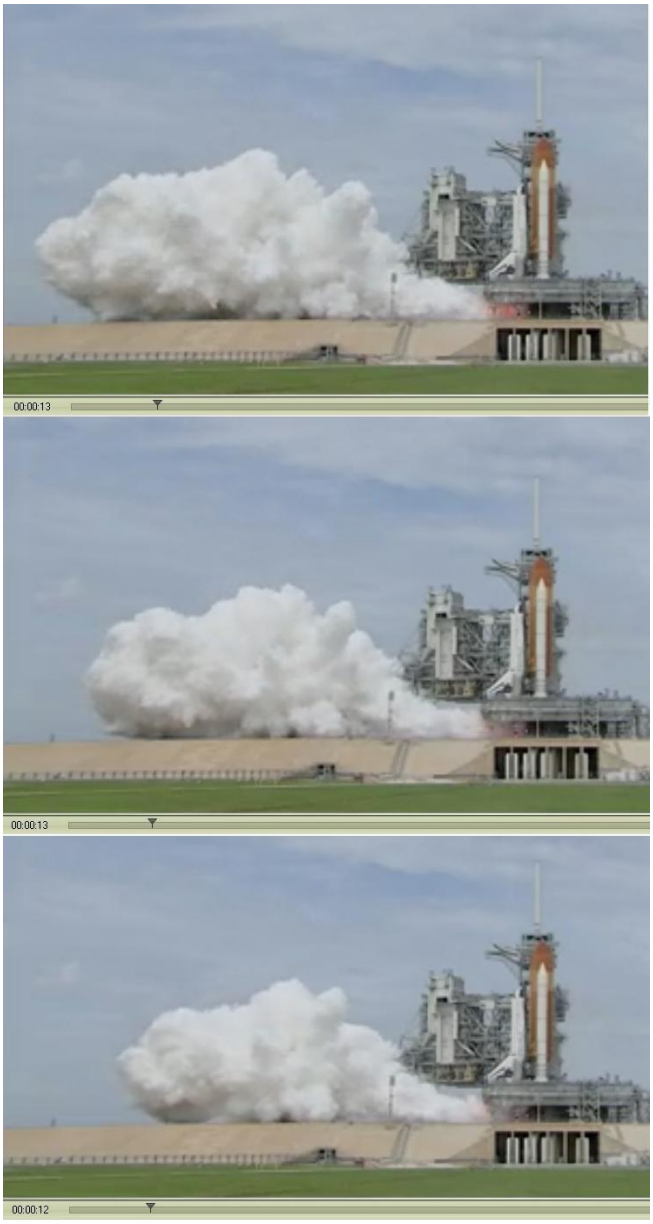
Answer: The elapsed times for each of the frames are 0s, 1s, 2s, 3s and 4s. The height graph is as follows:



Problem 4 - About what was the average speed of the Shuttle in the top image?

Answer: A) Between 3 and 4 seconds, the height changed from 185 to 259 meters, so the speed was about (259-185)/ (4 - 3) = **74 meters/sec** B) about **165 mph**.

Space Shuttle Atlantis (STS-135) - Exhaust speed



This sequence of images shows the historic launch of the Space Shuttle Atlantis (STS-135) on July 8, 2011 at 11:29 a.m. EDT, from launch pad 39A at the NASA Cape Canaveral Space Center.

From bottom to top, the image times are 11:29:12.0, 11:29:12.5, and 11:29:13.0. The length of the space shuttle is 37 meters from its pointed top end to the base of its rocket nozzles.

A rocket moves forward by throwing mass out its rocket engines as fast as possible. It does NOT move forward by 'pushing against' the ground as a popular misconception might suggest.

By ejecting thousands of pounds of gas every second, a rocket motor produces the thrust needed to lift a payload and move it in the opposite direction to its exhaust.

The plume of gas is ejected at high speed from the Shuttle main engines and makes a right-angle turn as it is vented horizontally across the gantry platform.

Problem 1 - Using a millimeter ruler, what is the scale of each image in meters/mm?

Problem 2 - How far did the leading edge of the exhaust plume travel between the top and bottom images?

Problem 3 - What was the speed of the exhaust plume in A) meters/sec? B) kilometers/hr? C) miles/hr?

Problem 1 - Using a millimeter ruler, what is the scale of each image in meters/mm?

Answer: The Shuttle measures about 13 millimeters in length on an ordinary reproduction of this 8.5 x 11-inch page, so the scale is **2.8 meters/mm**.

Problem 2 - How far did the leading edge of the exhaust plume travel between the top and bottom images?

Answer: For example, students might measure the distance from the tip of the cloud and the left-edge of the image. Top Image = 5 mm. Bottom image = 15 mm ,so the distance traveled is 10 millimeters or from the scale factor, 10 x 2.8 = **28 meters**.

Problem 3 - What was the speed of the exhaust plume in A) meters/sec? B) kilometers/hr? C) miles/hr?

Answer: A) The time difference between the top and bottom images is 11:29:13.0 - 11:29:12.0 = 1 second. The average speed would be 28 meters/1 sec = **28 meters/sec.**

B) 28 meters/sec x (1 km/1000 meters) x (3600 sec / 1 hr) = **100 km/hr**.

C) 100 km/hr x (0.62 miles / 1 km) = **62 miles per hour**.

This pair of images shows the historic launch of the Space Shuttle Atlantis (STS-135) on July 8, 2011 at 11:29 a.m. EDT, from launch pad 39A at the NASA Cape Canaveral Space Center. It shows part of the exhaust plume cloud ejected by the shuttle engines as they were ignited.

The bottom image was taken at 11:29:14.0 a.m. EDT, and the top image was obtained at 11:29:15.0 a.m. EDT. The length of the space shuttle is 37 meters from its pointed top end to the base of its rocket nozzles.

The speed of the exhaust plume can be estimated from the clues in the two images.

Problem 1 - What is the scale of each image in meters/millimeter?

Problem 2 - How far did the exhaust plume travel in the time interval between the two images?

Problem 3 - What is the speed of the exhaust plume in A) meters/sec? B) kilometers/hr and C) miles per hour?

Problem 4 - Assuming that you could survive the noise of the rocket motors, would you be able to out-run the exhaust plume if you ran as fast as Olympic sprinter Usain Bolt in the 2008, 100-meter race whose time was 9.7 seconds?

Problem 1 - What is the scale of each image in meters/millimeter?
Answer: The shuttle measures about 12 millimeters so the scale is 37 meters/12 mm = **3 meters/mm**.

Problem 2 - How far did the exhaust plume travel in the time interval between the two images?
Answer: Students can measure the distance from the tip of the plume to the right-hand edge of the image to get 14mm (top) and 33mm (bottom) for a difference of 19 mm. Multiplying by the scale of 3 m/mm we get **57 meters**.

Problem 3 - What is the speed of the exhaust plume in A) meters/sec? B) kilometers/hr/ C) miles per hour?

Answer: The time between the two images is 1 second, then

A) 57 meters/1 sec = **57 m/sec**.
B) 57 m/s x (1 km/1000 m) x (3600 s/1 hr) = **205 km/hr**.
C) 205 km/hr x 0.62 miles / 1 km) = **127 mph**.

Problem 4 - Assuming that you could survive the noise of the rocket motors, would you be able to out-run the exhaust plume if you ran as fast as Olympic sprinter Usain Bolt in the 2008, 100-meter race whose time was 9.7 seconds?

Answer: Usains speed was 100 meters/9.7 sec = 10.3 meters/sec which is <u>much slower</u> than the 57 m/sec plume speed.

Saturn V Rocket Launch Speed and Height

Time (Sec)	Altitude (m)	Speed (m/s)
0	0	0
1	2	3
2	4	4
3	13	7
4	25	10
5	32	12
6	48	15
7	55	16
8	83	20
9	103	23
10	126	25
11	148	27
12	179	31
13	217	34
14	242	36
15	269	39
16	321	43
17	367	46
18	406	49
19	465	53
20	510	56

The Saturn V rocket carrying the Apollo-11 astronauts to the moon was launched from the Kennedy Space Center on July 16, 1969 at 9:32:00 a.m. (EDT) from launch pad 39A. It weighed 2,766,913 kg just before launch, and was 102 meters tall. The gantry was 106 meters tall. See the YouTube Video of the launch at
http://www.youtube.com/watch?v=F0Yd-GxJ_QM&feature=related

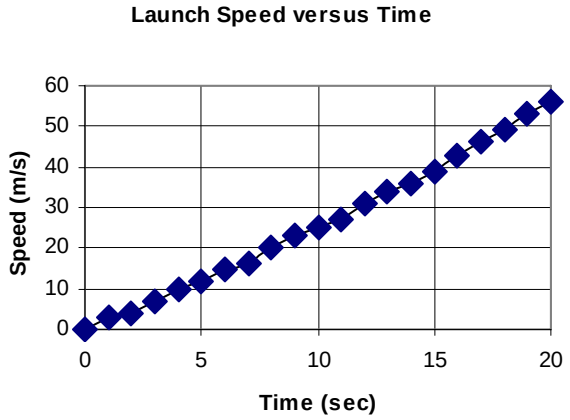
Problem 1 - Graph the speed of the rocket during the first 20 seconds after launch.

Problem 2 - Graph the height of the rocket above the launch pad during the first 20 seconds after launch.

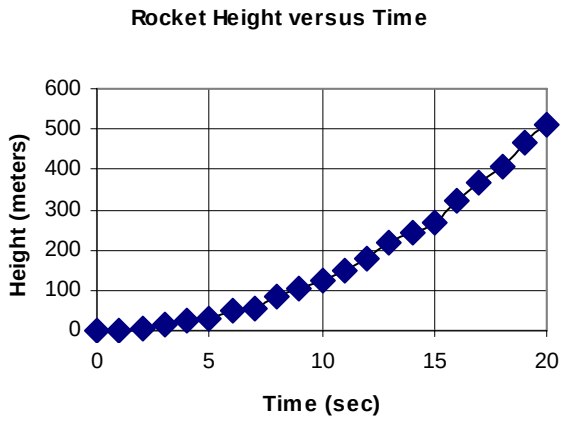
Problem 3 - At what time did the bottom of the rocket just clear the top of the launch gantry?

Problem 4 - How fast was the Saturn V traveling at the time the rocket engines just cleared the top of the gantry in A) meters/second? B) miles/hour?

Problem 1 - Graph the speed of the rocket during the first 20 seconds after launch.



Problem 2 - Graph the height of the rocket above the launch pad during the first 20 seconds after launch.



Problem 3 - At what time did the bottom of the rocket just clear the top of the launch gantry?

Answer: The gantry is 106 meters tall. The table, or graph, shows that at about **9 seconds**, the Saturn V rocket has an altitude of 103 meters, which is close to the gantry height.

Problem 4 - How fast was the Saturn V traveling at the time the rocket engines just cleared the top of the gantry in A) meters/second? B) miles/hour?

Answer: A) At 9 seconds, the rocket was traveling at about 23 meters/sec.
B) Converting 23 meters/sec to miles/hr:

23 meters/sec x (1 km / 1000 meters) x (0.62 miles/1 km) x (3600 sec/1 hr) = **51 miles per hour**.

Space Shuttle Endeavor - Last Flight

Time	Altitude (meters)	Speed (m/s)
0	0	0
1	3	5
2	7	12
3	17	17
4	42	24
5	69	30
6	94	36
7	134	42
8	186	50
9	247	59
10	283	63
11	334	69
12	429	79
13	523	87
14	614	95
15	665	100
16	755	106
17	888	117
18	1019	126
19	1158	136
20	1334	148

The final launch of NASA's space shuttle Endeavor (STS-134) occurred on May 16, 2011 at 8:56:28 a.m. EDT from launch pad 39A. The image above was taken 17 seconds after launch. See the launch video on YouTube at **http://www.youtube.com/watch?v=ShRa2RG2KDI**

This historic flight was watched by millions of people world-wide. The table above shows the speed and altitude data for the first 20 seconds after launch. The combined fuel tanks and Orbiter had a mass of 2,052,443 kg at launch. The launch gantry had a height of 106 meters.

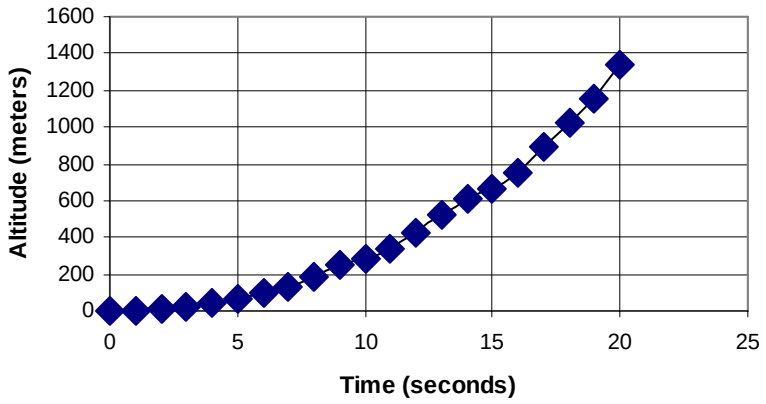
Problem 1 - Plot the altitude of Endeavor Shuttle versus time during the first 20 seconds of launch.

Problem 2 - Plot the speed of the Endeavor Shuttle versus time during the first 20 seconds of launch.

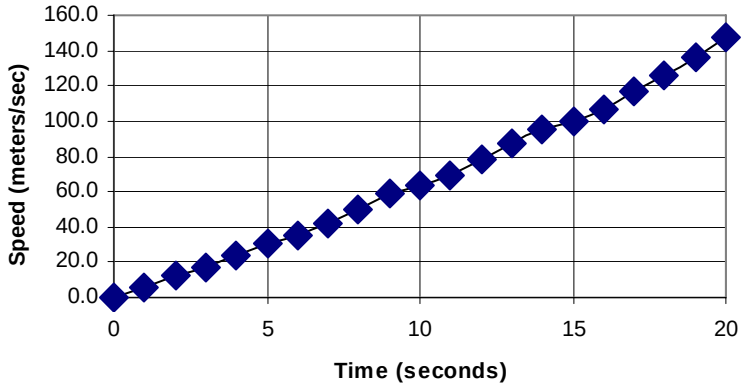
Problem 3 - About what is the speed of the Shuttle when it clears the gantry in A) meters/sec/ B) miles per hour?

Problem 4 - What is the average acceleration of the shuttle during its first 20 seconds of flight?

Problem 1 - Plot the altitude of Endeavor Shuttle versus time during the first 20 seconds of launch.



Problem 2 - Plot the speed of the Endeavor Shuttle versus time during the first 20 seconds of launch.



Problem 3 - About what is the speed of the Shuttle when it clears the gantry in A) meters/sec/ B) miles per hour?
Answer: A) The gantry height is 106 meters, so from Problem 1, we estimate that this speed occurred about 6 seconds after launch. The speed at this time is about **S=36 m/s.** B)
S = 36 m/s x (3600 s/1 hr) x (1 km/1000 meters) x (0.62 miles/km) = **80 mph**.

Problem 4 - What is the average acceleration of the shuttle during its first 20 seconds of flight?

Answer: Acceleration = velocity change/time, so between T=0 and T = 20 sec, the speed changed from 0 m/s to 148 m/s so A = 148/20 sec = **7.4 m/sec/sec**.

This sequence of stills was obtained from a YouTube.com video of the launch of MSL by United Space Alliance available at

http://www.youtube.com/watch?v=0cxsvVBemHY

This sequence shows the launch of the MSL mission from the Kennedy Space Center Launch Complex 49 on November 27, 2011 at 10:02 EST. The four images were taken, from bottom to top, at times 10:02:48 EST, 10:02:50 EST, 10:02:51 EST and 10:02:52 EST. At the distance of the launch pad, the width of each image is 400 meters.

Problem 1 - With the help of a millimeter ruler, what is the scale of each image in meters/mm?

Problem 2 - For each image, what is the distance between the bottom of the image and the base of the rocket nozzle for the Atlas V rocket in each scene?

Problem 3 – What is the estimated distance from the base of the launch pad to the rocket nozzle in each image?

Problem 4 – From the time information, what is the average speed of the rocket between A) Image 1 and 2? B) Image 2 and 3? C) Image 3 and 4?

Problem 5 – From the speed information in Problem 4, what is the average acceleration between A) Image 1 and Image 3? B) Image 2 and Image 4?

Problem 6 – Graph the height of the rocket versus the time in seconds since launch.

Problem 7 – Graph the speed of the rocket versus time in seconds after launch. For the time, use the midpoint time for each speed interval.

Problem 8 – Graph the acceleration of the rocket versus time in seconds after launch. For the time, use the midpoint time for each acceleration interval.

Problem 1 - Answer: Width = 69 mm, so scale = 400 m/69 mm = **5.8 meters/mm**

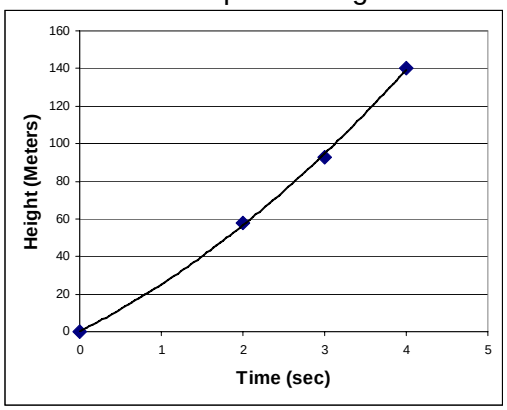
Problem 2 - Answer: 8mm, 18mm, 24mm and 32mm so using the scale of the image, the actual distances are **46m, 104m, 139m and 186 meters**.

Problem 3 – Answer: Take the differences in the measurements relative to the first image at the moment of launch to get h1 = 46m-46m = **0m**, h2=104m-46m = **58m**, h3=139m-46m = **93 m** and h4 =186m-46m = **140 m**.

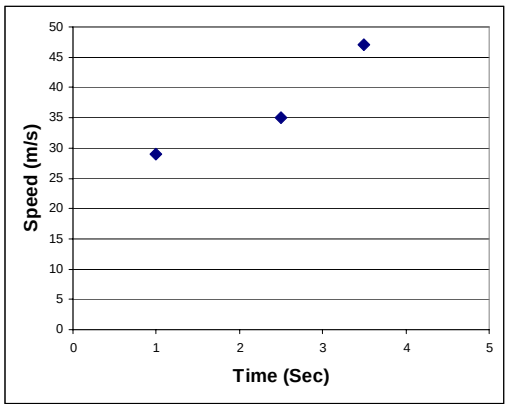
Problem 4 – Answer: A) v= distance/time, v1 = (58m-0m)/2sec = **29m/sec** B) v2 =(93m-58m)/1sec = **35 m/sec**, C) v3=(140m-93m)/1sec = **47 m/sec**.

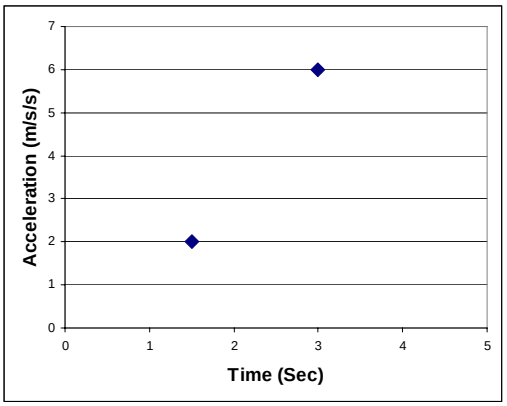
Problem 5 – Answer: A) a1 = (v2-v1)/3sec = (35-29)/3 = 6/3 = **2 m/sec^2**. B) a2 = (V3-v2)/2sec = (47-35)/2sec = **6 m/sec^2**.

Problem 6 – Graph the height of the rocket versus the time in seconds since launch.



Problem 7 – Graph the speed of the rocket versus time in seconds after launch. For the time, value, use the midpoint time for each speed interval. Answer: Left Above. For the first speed, the two height measurements are made at T=0 and T=2, so the speed V1 will be plotted at the midpoint time: T=(2-0)/2 = 1 sec





Problem 8 – Graph the acceleration of the rocket versus time in seconds after launch. For the time value, use the midpoint time for each acceleration interval. Answer: Right Above.

Apollo 17 - Launch from the Moon's Surface

On December 14, 1972 at 10:54:37 p.m. GMT, Astronauts Eugene Cernan and Harrison Schmidt blasted off from the lunar surface in the Lunar Module (LM). The launch was recorded by a camera left behind at the landing site in the Taurus-Litrow region. A sequence of images from this recording is shown to the left.

The sequence of images runs from the top to the bottom. The top image was taken at 10:54:37.00 p.m. and the bottom image was taken at 4.9 seconds later at 10:54:41.87 p.m. The width of the LM is 4.3 meters. See the YouTube video of the LM launch at
http://www.youtube.com/watch?v=iziumcklDbM&feature=related

Table of LM Heights and Times

Image	Time (seconds)	Height (meters)	Speed (m/s)
1	0	0	
2	1.8	2	
3	2.3	6	
4	2.9	10	
5	3.2	15	
6	3.7	18	
7	4.2	21	

Problem 1 - What is the average speed of the LM during the 4.87 seconds covered by this image sequence?

Problem 2 - What are the average speeds of the LM between Image 1 and A) Image 2? B) Image 3? C) Image 4? D) Image 5? E) Image 6? F) Image 7? Enter these speeds in the above table.

Problem 1 - What is the average speed of the LM during the 4.87 seconds covered by this image sequence?

Answer: The distance traveled was 21.0 meters, which took 4.9 seconds, so the average speed was S = 21.0 m/4.9s so **S = +4.3 meters/sec**.

Problem 2 - What are the average speeds of the LM between Image 1 and A) Image 2? B) Image 3? C) Image 4? D) Image 5? E) Image 6? F) Image 7? Enter these speeds in the table.

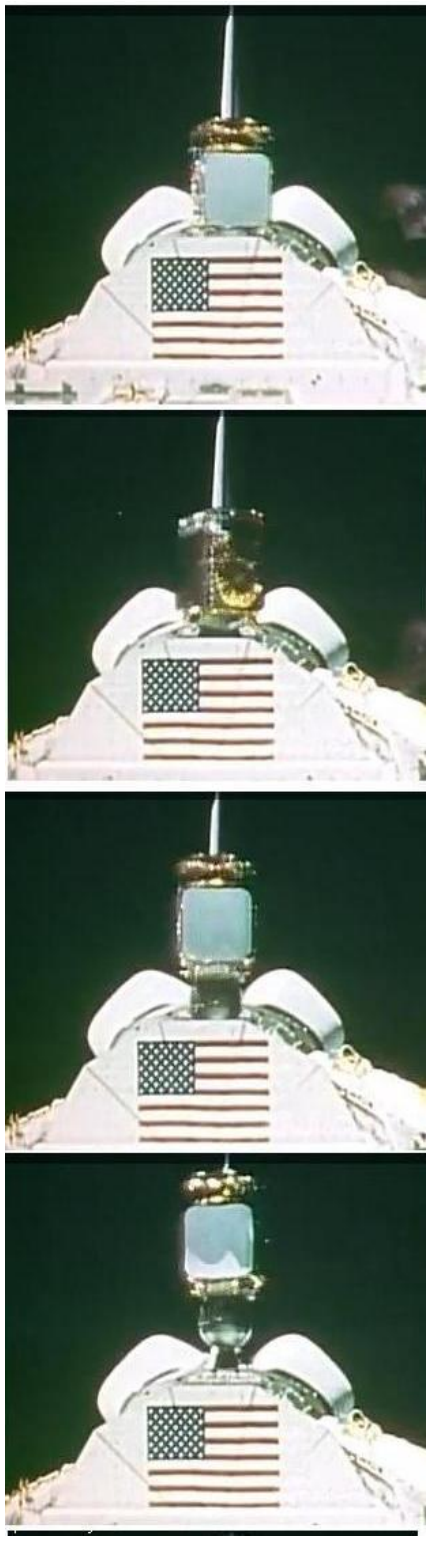
Answer: A) distance traveled = 2 meters, time = 1.8 seconds, so speed = 2/1.8 = **1.1 meters/sec**.
B) distance traveled = 6 meters, time = 2.3 seconds, so speed = 6/2.3 = **2.6 meters/sec.**
C) distance traveled = 10 meters, time = 2.9 seconds, so speed = 10 / 2.9 s = **3.4 meters/sec**.
D) distance traveled = 15.0 meters, time = 3.2 seconds, so speed = 15m / 3.2s = **4.7 meters/sec**.
E) distance traveled = 18.0 meters, time = 3.7 seconds, so speed = 18.0 m / 3.7 s = **4.9 meters/sec**.
F) distance traveled = 21.0 meters, time = 4.2 seconds, so speed = 21 m / 4.2 s = **5.0 meters/sec**.

Table of LM Heights and Times

Image	Time (seconds)	Height (meters)	Speed (m/s)
1	0	0	
2	1.8	2	1.1
3	2.3	6	2.6
4	2.9	10	3.4
5	3.2	15	4.7
6	3.7	18	4.9
7	4.2	21	5.0

Note: These speeds are approximate due to the quality of the video images, which had no time stamps to verify when the individual frames were taken. The times and heights were estimated from an approximate analysis of the video sequence.

Challenger (STS-8) Deploys the INSAT-1B Satellite



This sequence of images shows the launch of the INSAT-1B Indian communications satellite from the Challenger Space Shuttle cargo bay (STS-8) on August 30, 1983.

The sequence begins at the top frame and progresses downwards to the last frame showing the launch. The image times in seconds from top to bottom are: 36.3, 37.0, 37.7, and 39.2. The blue rectangular panel is 2 meters tall and is located on only one side of the satellite. The satellite is launched in a spinning mode to help stabilize it as it orbits Earth.

See the NASA video at the National Space Society:
http://www.nss.org/resources/library/shuttlevideos/shuttle08.htm

Table of Image Metrology

Image	Time (sec)	Height (meters)	Speed (m/s)
1	36.3	0.9	
2	37.0	1.5	
3	37.7	2.4	
4	39.2	3.9	

Problem 1 - From the table above, determine the average speed in meters/sec between: A) Image 1 and 2; B) Image 2 and 3; C) Image 3 and 4.

Problem 2 - What is the average speed of the satellite between Image 1 and Image 4?

Problem 3 - What is the rotation period of the satellite in seconds?

Problem 4 - How many revolutions per minute (RPM) was the satellite spinning after it was launched?

Problem 1 - From the table above, determine the average speed in meters/sec between: A) Image 1 and 2; B) Image 2 and 3; C) Image 3 and 4.

Table of Image Metrology

Image	Time (sec)	Height (meters)	Speed (m/s)
1	36.3	0.9	
2	37.0	1.5	**0.9**
3	37.7	2.4	**1.3**
4	39.2	3.9	**1.0**

Answer: Speed = distance/time. The time interval between Image 1 and 2 is 37.0 - 36.3 = 0.7 seconds. During this time, the satellite moved vertically a distance of 1.5 meters - 0.9 meters = 0.6 meters, so the speed was 0.6 meters / 0.7 seconds = 0.9 meters/sec. Similarly the answers for the other 2 time intervals are shown in the above table.

Problem 2 - What is the average speed of the satellite between Image 1 and Image 4?

Answer: Distance = 3.9 - 0.9 = 3.0 meters. Time interval = 39.2 - 36.3 = 2.9 seconds, so speed = **1.0 meters/sec**.

Problem 3 - What is the rotation period of the satellite in seconds?

Answer: We see from Image 3 and 4 that the satellite has made one full revolution. The time interval between the two frames is 39.2 - 37.7 = 1.5 seconds, so the period is **1.5 seconds**.

Problem 4 - How many revolutions per minute (RPM) was the satellite spinning after it was launched?

Answer: There are 60 seconds in 1 minute, so the RPM is just 60 seconds / 1.5 = 40 RPM.

1 revolution / 1.5 seconds x (60 seconds/1 minute) = **40 revolutions/minute**

The Upper Atmosphere Research Satellite (UARS) was launched in 1991. After 15 years of research, in 2005 its orbit was lowered from 550 km to 360 km as part of the orbital disposal of this retired satellite. Through atmospheric drag, the satellite began a slow continued descent from 352 km in 2008 to 320 km in 2011. By July, the pace quickened as the satellite steadily encountered a denser atmosphere. At the extrapolated rate of descent, it was predicted that the satellite would finally burn-up by September 23/24 - an event that intrigued and concerned millions of people who worried that some part of the satellite might strike them or do significant property damage.

The table below gives the perigee altitude in kilometers of the satellite at various dates in 2011 during its descent from orbit.

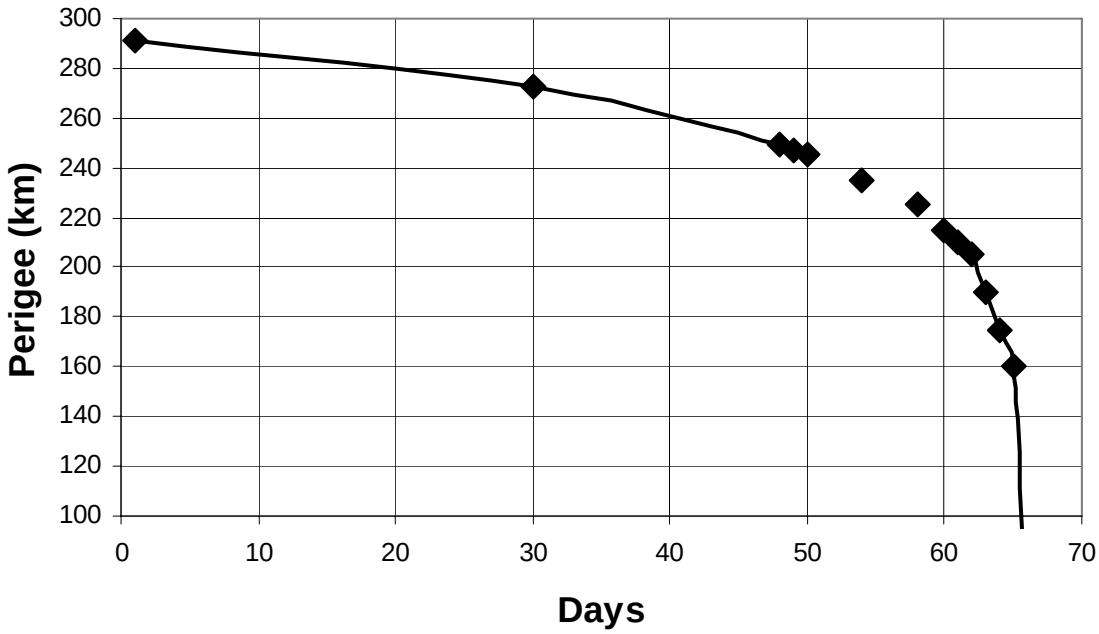
Date	Day	Altitude	Date	Day	Altitude
July 20	1	291	Sept. 17	59	220
August 19	30	273	Sept. 18	60	215
Sept. 6	48	249	Sept. 19	61	210
Sept. 7	49	247	Sept. 20	62	205
Sept. 8	50	245	Sept. 21	63	190
Sept. 12	54	235	Sept. 22	64	175
Sept. 15	57	230	Sept. 23	65	160
Sept. 16	58	225			

Problem 1 - Graph these points and connect the points with a line.

Problem 2 - What is the rate of altitude loss in meters per hour between A) August 19 and September 6? B) September 17 and 18? C) September 22 and 23?

Problem 3 - Can you construct a function that closely models the data and the shape of the plotted curve? When do you predict the satellite reaches zero altitude?

Problem 1 - Graph these points and connect the points with a line.



Problem 2 - What is the rate of altitude loss in meters per hour between A) August 19 and September 6? B) September 17 and 18? C) September 22 and 23?

Answer: A) R = (249 - 273)/(48 - 30)
= - 1.33 km/day x (1000 m/1 km)x(1 day/24h)
= **-56 meters/hour**.

B) R = **-208 meters/hour.**

C) R = **- 625 meters/hour.**

Problem 3 - Can you construct a function that closely models the data and the shape of the plotted curve? When do you predict the satellite reaches zero altitude?
Answer: This activity is facilitated by using Excel spreadsheets programmed with various 'guesses' until the modeled curve falls close to the actual curve. Students should quickly realize that because the rate of descent changes constantly, a linear equation using a constant 'slope' will not work. Hint: Divide each day number by 65 days to get x, then work with these 'normalized' day values. One function is

$$h = 291 - 6e^{3x^{3/2}}$$

Predicted date for h=0 is about x=1.18 or Day 77, which would be October 5th. More accurate modeling would account for the very rapid increase in the rates after September 20th which the model h(x) underestimates.

On July 15, 2011 the NASA spacecraft Dawn completed a 2.8 billion kilometer journey taking four years, and went into orbit around the asteroid Vesta. Vesta is the second largest asteroid in the Asteroid Belt. Its diameter is 530 kilometers. After one year in orbit, it will depart in 2012 for an encounter with asteroid Ceres. Meanwhile, from its orbit around Vesta, it will map the surface and see features less than 1 kilometer across.

Problem 1 - Use a millimeter ruler and the diameter information for this asteroid to determine the scale of this image in kilometers per millimeter.

Problem 2 - What is the diameter of the largest and smallest features that you can see in this image?

Problem 3 - Based on the distance traveled, and the time taken by the Dawn satellite, what was the speed of this spacecraft in A) kilometers per year? B) kilometers per hour?

Problem 4 - The Space Shuttle traveled at a speed of 28,000 km/hr in its orbit around Earth. How many times faster that the Shuttle does the Dawn spacecraft travel?

Problem 1 - Use a millimeter ruler and the diameter information for this asteroid to determine the scale of this image in kilometers per millimeter.

Answer: When printed using a standard printer, the width of the asteroid is about 123 millimeters. Since the true diameter of the asteroid is 530 km, the scale is then S = 530 km / 123 mm = **4.3 kilometers per millimeter**.

Problem 2 - What is the diameter of the largest and smallest features that you can see in this image?

Answer: **Students can find a number of small features in the image that are about 1 mm across, so that is about 4.3 kilometers. Among the largest features are the 9 large craters located along the middle region of Vesta from left to right. Their diameters are about 4 to 7 millimeters or 17 to 30 kilometers across. The large depression located in the upper left quadrant of the image is about 40 mm long and 15 mm wide in projection, which is equivalent to 172 km x 65 km in size.**

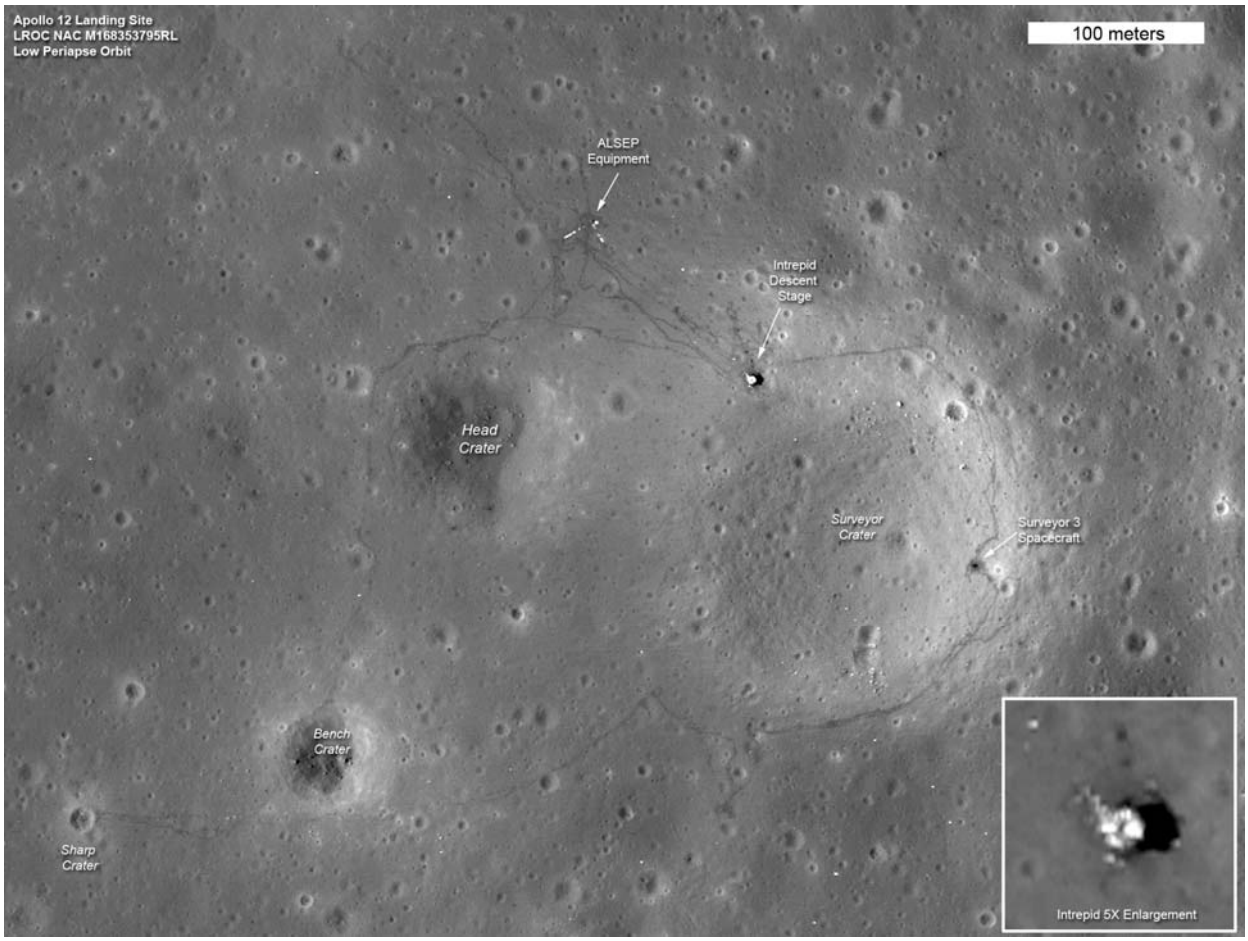
Problem 3 - Based on the distance traveled, and the time taken by the Dawn satellite, what was the speed of this spacecraft in A) kilometers per year? B) kilometers per hour?

Answer: Time = 4 years, distance = 2.8 billion km, so the speed is A) S = 2.8 billion km / 4 years = **700 million km/year**. B) Converting to an hourly rate, S = 700 million km/yr x (1 year/365 days) x (1 day/24 hours) = **79,900 km/hour**.

Problem 4 - The Space Shuttle traveled at a speed of 28,000 km/hr in its orbit around Earth. How many times faster that the Shuttle does the Dawn spacecraft travel?

Answer: Ratio = (79,900 km/hr) / (28,000 km/hr) = 2.9 . So Dawn is traveling at an average speed that is **2.9 times faster than the Space Shuttle!**

Note: The Space Shuttle is traveling 8 times faster that a bullet (muzzle velocity) from a high-powered M16 rifle! So Dawn is traveling 23 times faster than such a bullet!



NASA's Lunar Reconnaissance Orbiter (LRO) from a lunar orbit of 21 kilometers (13 miles) captured the sharpest images ever taken from space of the Apollo 12, 14 and 17 landing sites. Images show the twists and turns of the paths made when the astronauts explored the lunar surface. One of the details that shows up is a bright L-shape in the Apollo 12 image. It marks the locations of cables running from ALSEP's central station to two of its instruments. Although the cables are much too small for direct viewing, they show up because they reflect light very well.

Problem 1 – Following one of the walking paths, about how many meters did the astronauts have to walk from A) the ALSEP to the Descent Stage, and then around Surveyor Crater to finally reach the Surveyor spacecraft? B) The Surveyor spacecraft to Sharp Crater?

Problem 2 – Using your favorite method, about how many craters can you see across this entire area?

Problem 3 - If the craters were created over a period of about 3 billion years, about what may have been the average time between impacts to form the craters you see?

Problem 1 – Following one of the walking paths, about how many meters did the astronauts have to walk from A) the ALSEP to the Descent Stage, and then around Surveyor Crater to finally reach the Surveyor spacecraft? B) The Surveyor spacecraft to Sharp Crater?

Answer: Print out the problem on a typical laser printer and measure the '100 meter' bar with a millimeter ruler. An answer of about 23 millimeters yields an image scale of abouy 100 meters/23 mm = 4.3 meters/mm.

A) Using a piece of string or a millimeter ruler, measure the segments of the thin black 'track' that astronauts took. An answer of about 90 millimeters will be adequate. Using the image scale of 4.3 meters/mm you will get a distance of 90 mm x (4.3 m/mm) = 387 meters. This can be rounded to **390 meters**.

B) Measuring the track segments, a string length of about 200 millimeters is adequate. From the scale factor, this equals a physical distance of 200 x 4.3 = **860 meters** traveled.

Problem 2 – Using your favorite method, about how many craters can you see across this entire area?

Answer: Divide the area into a grid of squares. Count the number of craters you can see in one square, and multiply by the total number of squares. For example, if you make the squares 40mm x 40mm, you can fit 4 columns and 3 rows. Selecting the one in the second column, first row, you can count about 80 craters (from 0.5 to 2 millimeters across on the image) so the total number of craters is about 80 x 12 = 960 craters. **Answers between 800 and 1100 are also reasonable estimates.**

Note: From the image scale, the most common craters range in size from 0.2 millimeters to 2 millimeters, which corresponds to an actual size between 0.9 meters and 8.6 meters.

Problem 3 - If the craters were created over a period of about 3 billion years, about what may have been the average time between impacts to form the craters you see?

Answer: If we select 1000 craters as the average estimate, then the rate of cratering is about 1000 craters / 3 billion years or **1 crater every 3 million years**.

For more information about these images, see the NASA press release at:

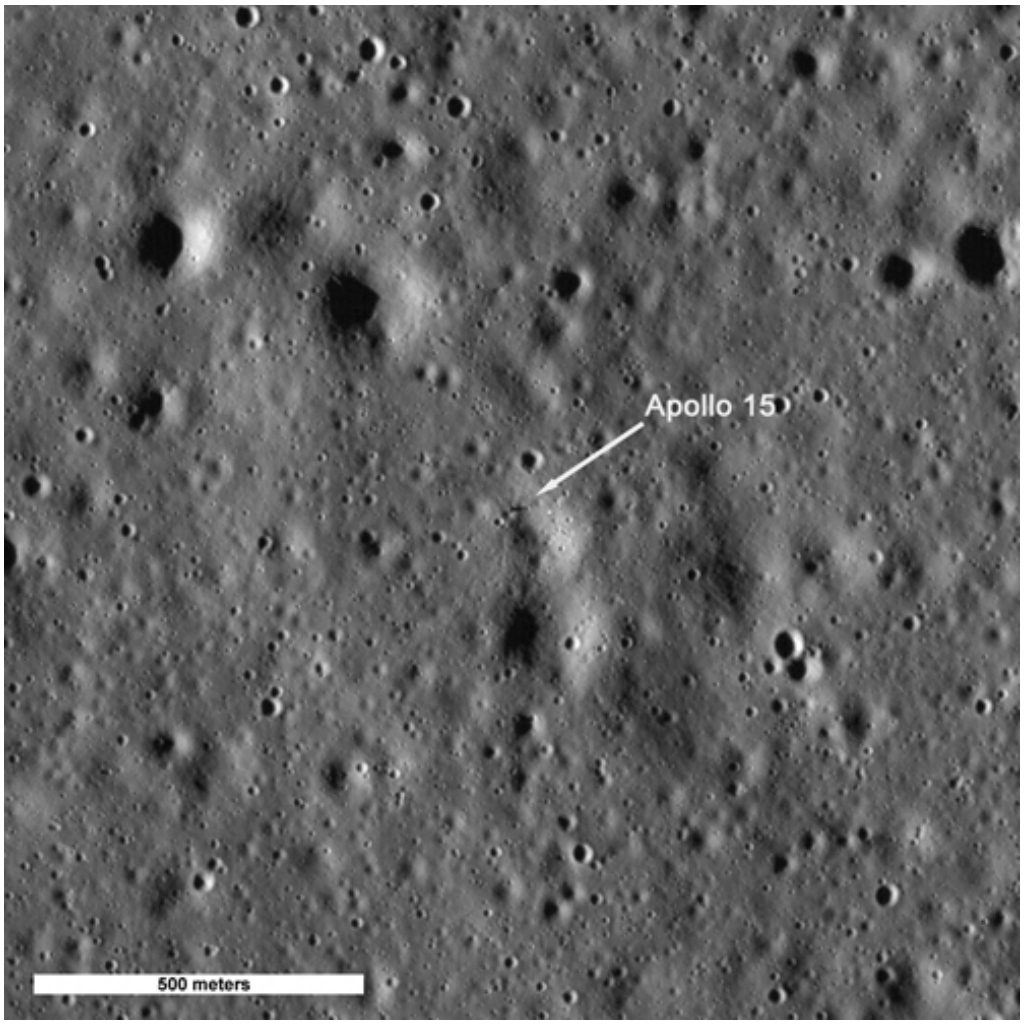
NASA Spacecraft Images Offer Sharper Views of Apollo Landing Sites
Sep 6, 2011

http://www.nasa.gov/mission_pages/LRO/news/apollo-sites.html

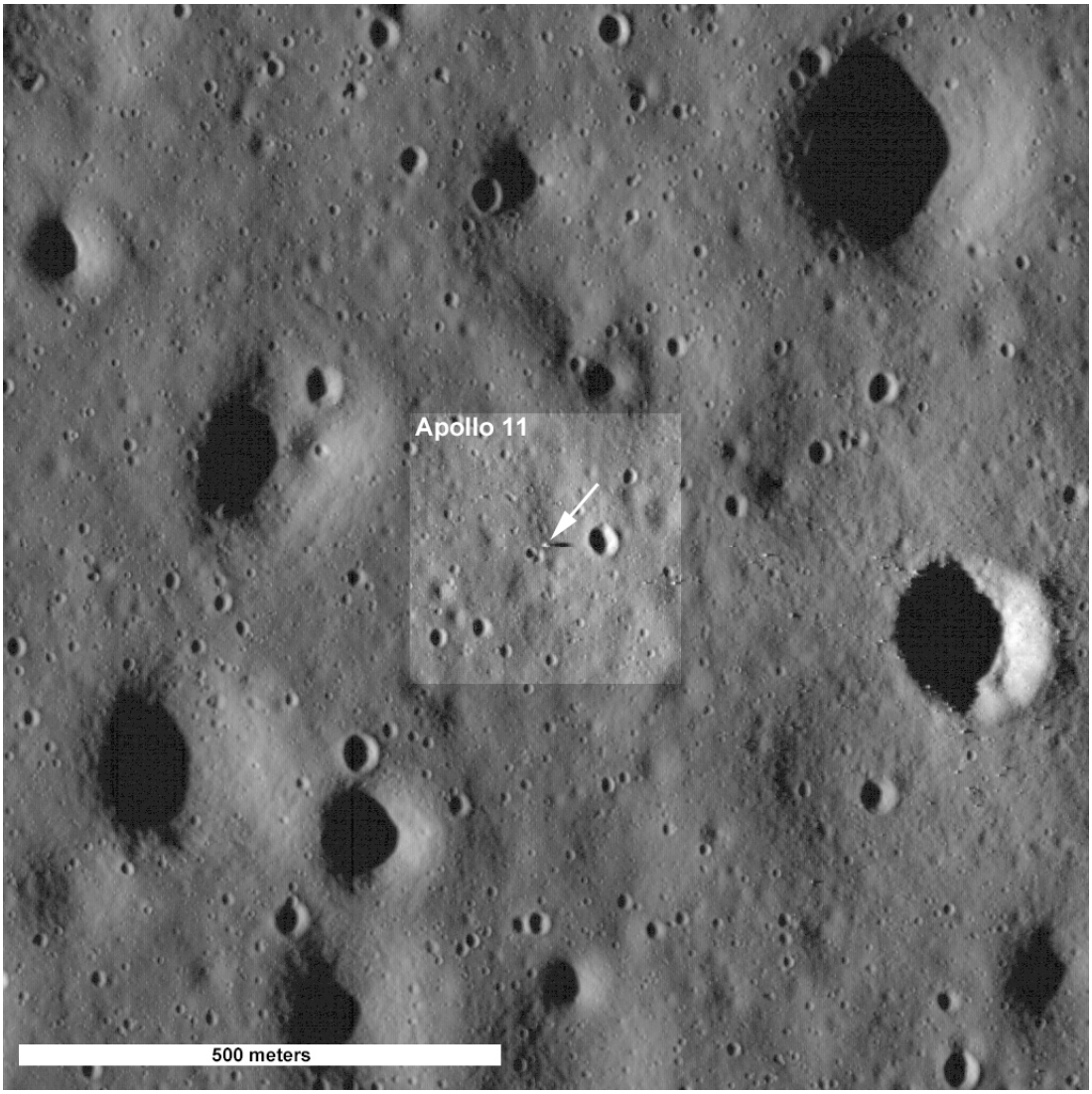
We have all sees pictures of craters on the moon. The images on the next two pages show close-up views of the cratered lunar surface near the Apollo 15 and Apollo 11 landing areas. They were taken by NASA's Lunar Reconnaissance Orbiter (LRO) from an orbit of only 25 kilometers!

Meteors do not arrive on the moon at the same rates. Very large meteors that produce the largest craters are much less common than the smaller bodies producing the smallest craters. That's because there are far more small bodies in space than large ones. Astronomers can use this fact to estimate the ages of various surfaces in the solar system by just comparing the number of large craters and small craters that they find in a given area.

Let's have a look the images below, and figure out whether Apollo-11 landed in a relatively younger or older region than Apollo 15!



This is the Apollo-15 landing area near the foot of the Apennine Mountain range. Note the bar indicating the 'scale' of the image. The arrow points to the location of the Lunar Descent Module.



This image taken by LRO is of the Apollo-11 landing area in Mare Tranquilitatus, with the arrow pointing to the Lunar Descent Module. The LDM was the launching platform for the Apollo-11 Lunar Excursion Module, which carried astronauts Neil Armstrong and Buzz Aldrin back to the orbiting Command Module for the trip back to Earth.

Note the length of the '500 meter' bar, which gives an indication of the physical scale of the image. How long would it take you to walk 500 meters?

Astronomers assume that during the last 3 billion years following the so-called 'Late Heavy Bombardment Era' the average time between impacts that created craters has been constant. That means that the more time that passes, the more craters you will find, and that they are produced at a more or less steady number for each million years that passes.

Dating a cratered surface.

For each of the above images, perform these steps.

Step 1 - With the help of a millimeter ruler, and the '500 meter' line in the image, calculate the scale factor for the image in terms of meters per millimeter.

Step 2 - Calculate the total area of the image in square kilometers.

Step 3 - Identify and count all craters that are bigger than 20 meters in diameter.

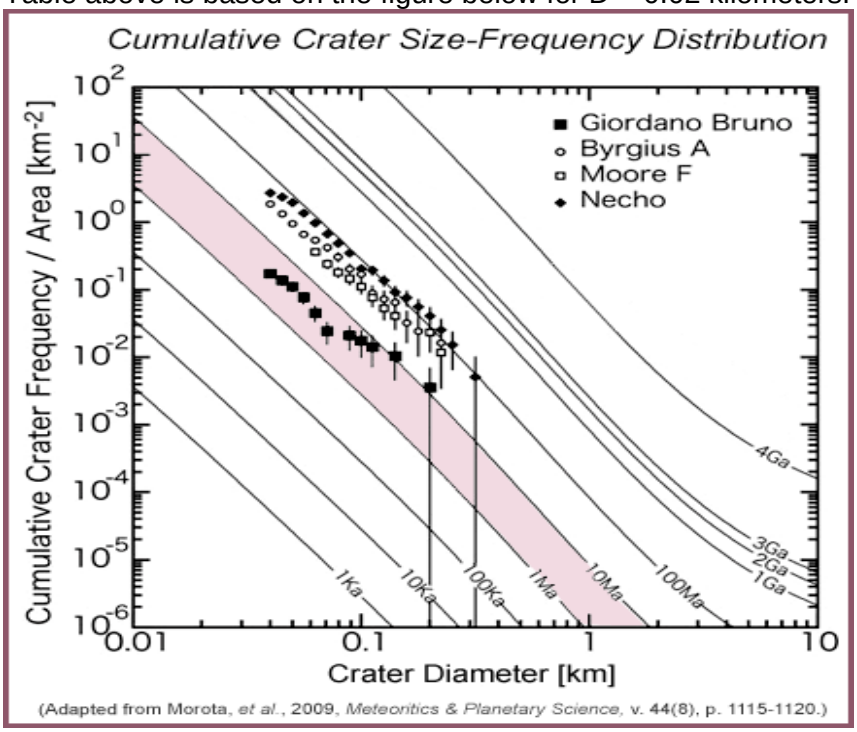
Step 4 - Divide your answer in Step 3 by the area in square kilometers in Step 2.

Step 5 - Look at the table below and estimate the average age of the surface.

Problem 1 - From your answer for each Apollo landing area in Step 5, which region of the moon is probably the youngest?

Estimated Age	Total number of craters per square kilometer
1000 years	0.0008
10000 years	0.008
100,000 years	0.08
1 million years	0.8
10 million years	8.0
100 million years	80.0
1 billion years	800.0

Table above is based on the figure below for D = 0.02 kilometers.



Another thing you might consider doing is to measure the diameters of as many craters as you can, and then plot a histogram (bar graph) of the number of craters you counted in a range of size intervals such as 5 - 10 meters, 11 to 20, 21 to 30 and so on. Because erosion (even on the moon!) tends to eliminate the smallest craters first, you can compare two regions on the moon in terms of how much erosion has occurred.

Problem 2 - Select the same-sized area on each of the Apollo images and count all the craters you can find within the size intervals you selected. How do the two landing areas compare to one another in terms of their crater frequency histograms?

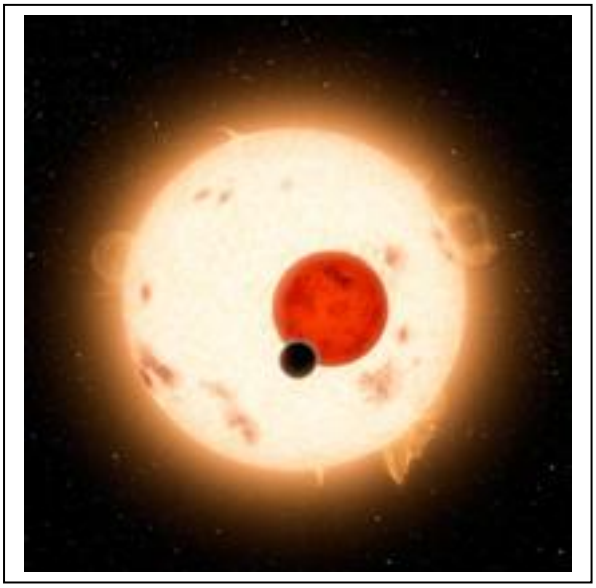
Problem 3 - Which surface do you think has experienced the most re-surfacing or erosion?

Problem 4 - Without an atmosphere, winds or running water, what do you think could have caused changes in the lunar surface after the craters were formed?

Note to Teachers: More technical information on crater dating can be found at

"How young is the Crater Giordano Bruno"

http://www.psrd.hawaii.edu/Feb10/GiordanoBrunoCrater.html



In 2011, the Kepler observatory detected a Saturn-sized planet orbiting the binary stars Kepler 16A and Kepler 16B. Nicknamed 'Tatooine', the view from this planet of its twin suns would be spectacular. The smaller star, Kepler-16B orbits once every 41 days at a distance of 30 million km from the larger star Kepler 16A, and the planet is in a circular orbit 108 million km from Kepler 16A which takes 229 days to complete. As seen from 'Tatooine', the small orange star Kepler-16B never gets more than 18 degrees from the much larger, yellow star Kepler 16A.

Predicting when Kepler 16B will pass across the face of Kepler 16A, called a transit, is made a bit more difficult because Tatooine is also moving along its orbit while Kepler 16B is in motion around the main star. To see a transit, Kepler 16B and Tatooine must be located in their orbits so that a line through their centers passes through the center of Kepler 16A at the center of the orbits. The time it takes this to happen is called the Synodic Period and is calculated using the formula

$$\frac{360}{t} - \frac{360}{T} = \frac{360}{P}$$

In our problem t = 41 days and T = 229 days so P = 50 days.

Step 1 - Draw a circle representing the orbit of Tatooine.

Step 2 - Place a dot on the circle to mark the location during the planet's orbit when the first transit was observed. Label this dot A

Step 3 - Place a second dot, B, at a position 50 days later where the next transit would be observed.

Problem 1 - How many days will elapse before someone on Tatooine sees a transit within 10 days of the first dot, A, that you placed on the circle?

Problem 2 - How many days will elapse before someone on Tatooine sees a transit within 5 days of the first dot that you placed on the circle?

Problem 3 - How many days will elapse before someone on Tatooine sees a transit within 1 day of the first dot that you placed on the circle?

Problem 4 - How many days will elapse before someone on Tatooine sees a transit on the same day as the first dot that you placed on the circle?

Problem 1 - How many days will elapse before someone on Tatooine sees a transit within 10 days of the first dot that you placed on the circle?

Answer: Students will need to compare two number series, one in intervals of 229 the orbit period of Tatooine, and one with a period of 50 days, which is when transits will occur.

229, **458**, **687**, 916, 1145, 1374, 1603, 1832, 2061
50, 100, 150, 200, 250, 300, 350, 400, **450**, 500, 550, 600, 650, **700**, 750, 800, 850, 900, 950

We see that 458 and 450 are within 8 days of each other, while the next match, 687 and 700 are 13 days apart, so after **458 days or two Tatooine 'years'** a transit will happen within 10 days of Point A.

Problem 2 - How many days will elapse before someone on Tatooine sees a transit within 5 days of the first dot that you placed on the circle?

Answer: Using the same method as in Problem 1, students should find the first such event after 1145 days when the two series yield 1150 and 1145. This will happen after 5 Tatooine years.

Problem 3 - How many days will elapse before someone on Tatooine sees a transit within 1 day of the first dot that you placed on the circle?

Answer: The first time this happens is after 4351 days, which is 19 Tatooine years.

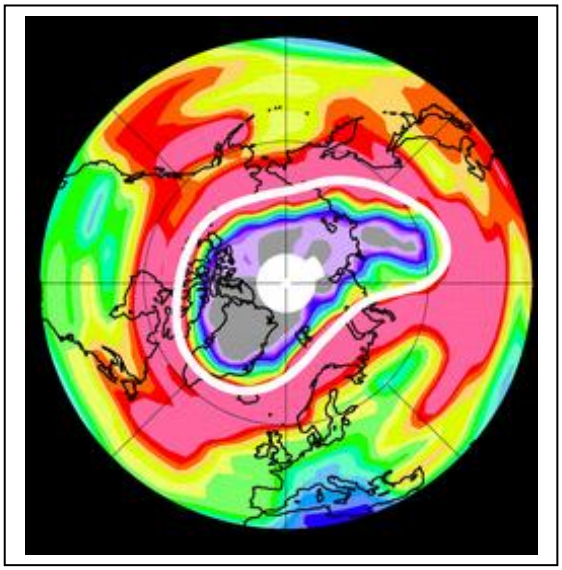
Problem 4 - How many days will elapse before someone on Tatooine sees a transit on the same day as the first dot that you placed on the circle?

Answer: This requires that students solve M x 50 = N x 229 to find N.

The first time this happens is after 11450 days where N = 50 and M = 229 transits. **So, a transit will occur on the same day of the year on Tatooine every 50 years!**

This can be done by finding the Least Common Multiple…since 229 is a prime number, the LCM is just 50 x 229 = 11450.

The Arctic Ozone Hole Continues to Grow in 2011



A NASA-led study has documented an unprecedented depletion of Earth's protective ozone layer above the Arctic during the winter and spring of 2011 caused by an unusually prolonged period of extremely low temperatures in the stratosphere.

The amount of ozone destroyed in the Arctic in 2011 was comparable to that seen in some years in the Antarctic, where an ozone "hole" has formed each spring since the mid 1980s. The stratospheric ozone layer, extending from about 10 to 20 miles (15 to 35 kilometers) above the surface, protects life on Earth from the sun's harmful ultraviolet rays.

Problem 1 - If the diameter of Earth is 12,700 km about what is the approximate rectangular area of the Arctic ozone hole interior to the white oval shown in the above satellite imagery in units of millions of square kilometers?

Problem 2 - The study by Dr. Gloria Manney and her colleagues, which was published in the journal Nature on October 2, 2011, stated that the loss of ozone was most severe between altitudes of 16 to 22 kilometers. What is the total volume of the ozone layer involved in the Arctic depletion in 2011 in millions of cubic kilometers?

Problem 3 – The normal Arctic concentration of ozone is about 4 parts-per-million by volume (ppmV), but during the Arctic depletion it fell to 1.5 ppmV. If 1 ppmV represents a concentration of 1 cm^3 of material in a 1 million cm^3 volume, what is the volume occupied by the lost ozone?

Problem 1 - Answer: The diameter of the Earth disk is 65 mm, so the scale of the image is 12,700 km/65 mm = 195 km/mm. The approximate rectangular shape that fits inside the oval has dimensions of 30 mm x 15 mm, so the actual dimensions are 5800 km x 3000 km. The area is then A = 5800x3000 = **17 million square kilometers.**

Problem 2 - What is the total volume of the ozone layer involved in the Arctic depletion in 2011 in millions of cubic kilometers? Answer: From Problem 1, the area of the region was A = 17 million km^2. The thickness is 22-16 = 6 kilometers, so the volume of this rectangular slab of atmosphere is V = 17 million km^2x 6 km, V = 102 million cubic kilometers.

Problem 3 –What is the volume occupied by the lost ozone? Answer: The ozone layer has a volume of 102 million km^3. The difference in ozone volumes before and after the loss is 4 ppmV – 1.5 ppmV = 2.5 ppmV, so V= 2.5 x 102 million km^3/1 million and so C = 255 km^3. To check, C(ppmV) = 255 km^3/102 million km^3 = 2.5 ppmV. So, the volume of disappeared ozone equals **255 km^3**.

For more information, read the press release:

NASA Leads Study of Unprecedented Arctic Ozone Loss

Oct 2, 2011

http://www.nasa.gov/topics/earth/features/arctic20111002.html

Table of Ozone Column Changes

Year	Column (Dobson Units)
1970	470
1972	466
1974	463
1976	460
1978	458
1980	455
1982	430
1984	445
1986	430
1988	445
1990	390
1992	395
1994	400
1996	375
1998	420
2000	370

This past spring, scientists using a variety of instruments witnessed an unprecedented depletion in Arctic stratospheric ozone levels. In a *Nature* paper published online on October 2, 2011, Dr. Gloria Manney of NASA's Jet Propulsion Laboratory in Pasadena and more than two dozen coauthors described the 2011 loss as "an Arctic ozone hole."

This table shows the changes in the Arctic ozone abundance for the month of March between 1970 and 2000 based on data provided by Dr. Paul Newman (NASA/GSFC). March is the coldest month of the Arctic year, and the time when the chemistry of the ozone layer favors the destruction of ozone molecules. The Antarctic ozone hole appears in September when it is mid-winter in the southern hemisphere.

Problem 1 - Graph the data in the table, and explain what kind of trend you see in the data. Is the amount of Arctic ozone detected in March generally increasing with time, decreasing with time, or remaining about the same?

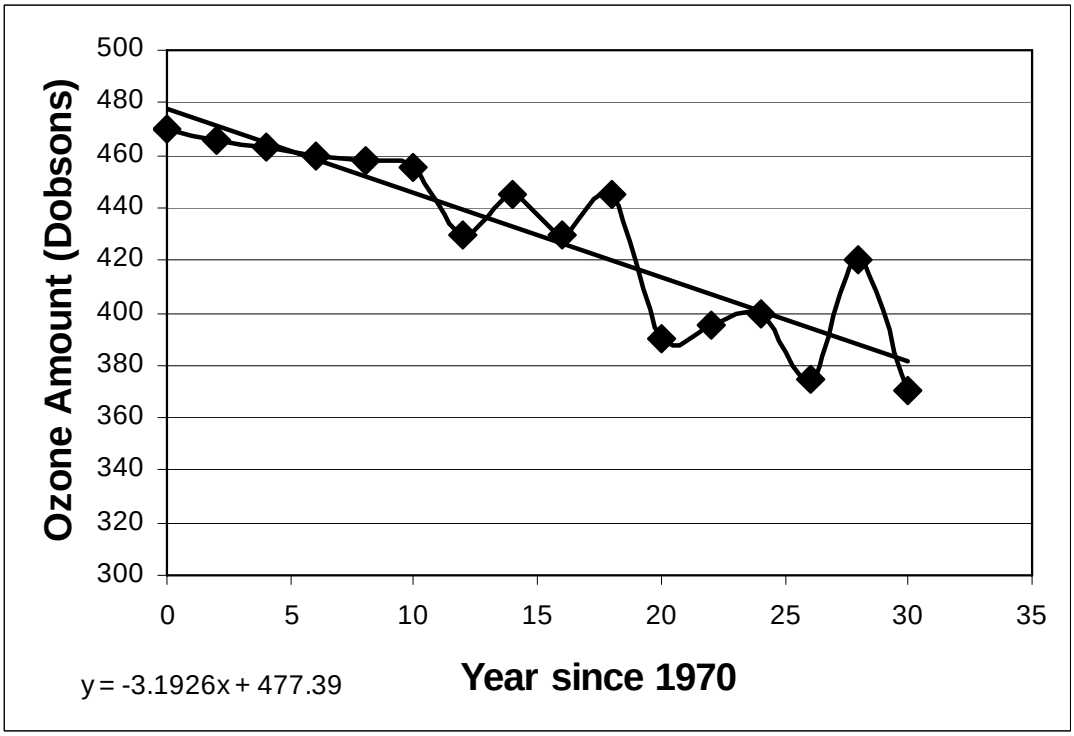
Problem 2 - Draw a slanted line through the plotted data so that half of the points are above the line and half are below the line. This represents a 'linear' regression model for the data. What does the slope of the line represent physically? What are the physical units for this slope?

Problem 3 - What is the formula for the line that you drew in Problem 2? What does the y-intercept for this line represent physically? What does the x-intercept for this line represent physically?

Problem 4 - From your linear model for the ozone data, what would you predict as the amount of ozone above the Arctic region in the year 2010?

Tabulated data from:
1970 - 1997: Paul Newman/NASA Goddard Space Flight Center. The figure is taken from P.A. Newman, J.F. Gleason, R.D. McPeters, and R.S. Stolarski, "Anomalously low ozone over the Arctic," Geophysical Research Letters, *Vol. 24, No. 22, pp. 2689-2692, November 15, 1997.*

1998-2010:



Problem 1 - Answer: See above. The amount of ozone is generally decreasing with time.

Problem 2 - Answer: Students can use a straight-edge and measure the slope of the line from its endpoints. In the example above the points are (30, 380) and (0, 480) so s = (380-480)/(30-0) = -3.3. The slope represents how fast the amount of ozone is decreasing over time. It is a negative number, so the amount is decreasing in time. The units for the slope are Dobsons/year.

Problem 3 - Answer: D = 477 - 3.3T where T is the year. The Y intercept, 477 Dobsons, is the amount of ozone predicted for 'Year 0', which is 1970, while the x-intercept, 145, (or 1970+145 = 2115 is the year when the amount of ozone will be 0.0 Dobsons.

Problem 4 - Answer: D = 477 - 3.3(41) so D = **342 Dobsons**.

Note: The actual amount of ozone detected in March, 2011 was found to be about 250 Dobsons! This is why it is called an ' Ozone hole' since similar readings are found over the Antarctic region.

The Declining Arctic Ice Cap during September

Survey Year	Ice area in millions of square km
1979	7.2
1980	7.9
1981	7.3
1982	7.5
1983	7.5
1984	7.2
1985	6.9
1986	7.5
1987	7.5
1988	7.5
1989	7.0
1990	6.2
1991	6.6
1992	7.6
1993	6.5
1994	7.2
1995	6.1
1996	7.9
1997	6.7
1998	6.6
1999	6.2
2000	6.3
2001	6.8
2002	5.9
2003	6.2
2004	6.1
2005	5.6
2006	5.9
2007	4.3
2008	4.7
2009	5.4
2010	4.9
2011	4.6

The data table to the left shows the minimum ice cap area for the Arctic during the month of September. At this time, the ice cap volume is at an annual minimum during the Arctic Summer. Global warming theory predicts that the polar regions will experience the largest impacts from continued planetary warming.

In this exercise, we will use regression techniques to examine the trends in this tabular data, and examine how reliable forcasts will be for the year 2030 given the current data.

In the problems below, perform the calculations using your choice of technology, or by hand.

Problem 1 - Graph the tabular data using convenient scaling of the horizontal (Year since 1979) and vertical (Area) axes.

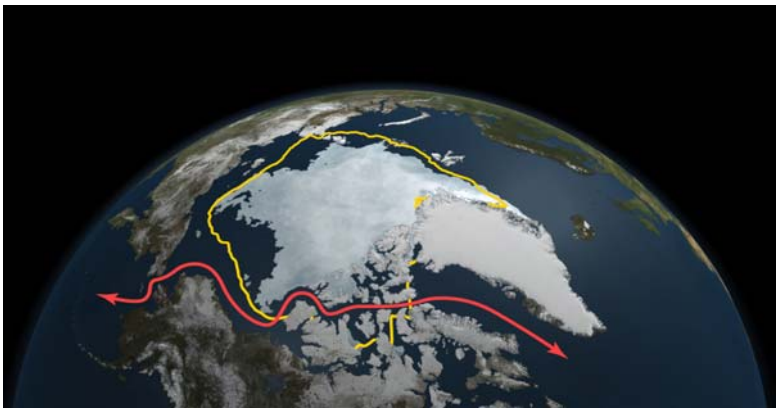
Problem 2 - Use a linear regression to model the data between 1979-2011 and compute the R^2 value of the fit.

Problem 3 - Use a quadratic regression to model the data between 1979-2011 and compute the R^2 value of the fit.

Problem 4 - To the nearest 100,000 km^2, what would you predict as the area of Arctic sea ice in the year 2030 using each regression model?

Problem 5 - Which of the two forecasts would you consider more statistically reliable? Which would you consider more believable?

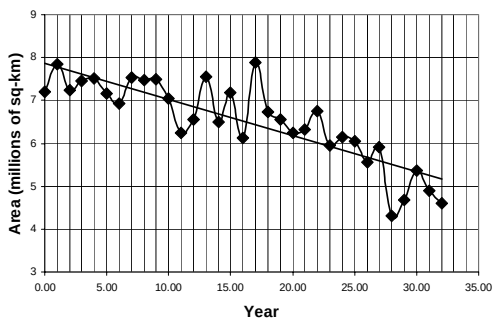
NASA satellite data reveals how this year's minimum sea ice extent, reached on Sept. 9, 2011 as depicted here, declined to a level far smaller than the 30-year average (in yellow) and opened up Northwest Passage shipping lanes (in red). ***(Credit: NASA Goddard's Scientific Visualization Studio)***

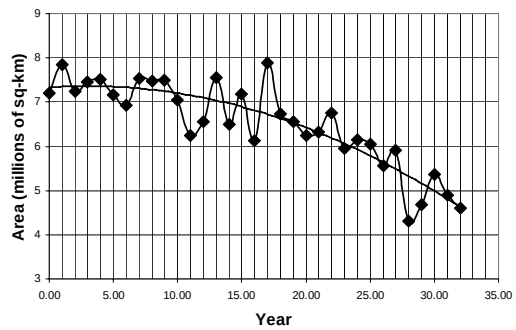


The tabular data was obtained from the NOAA, National Snow and Ice Data Center archive at
ftp://sidads.colorado.edu/DATASETS/NOAA/G02135/Sep/
The archive of sea ice data can be assessed at
http://nsidc.org/data/seaice_index/archives/index.html

NASA Press Release:
Arctic Sea Ice Continues Decline, Hits 2nd-Lowest Level
October 4, 2011
URL = http://www.nasa.gov/topics/earth/features/2011-ice-min.html

Problem 1 - Graph the tabular data using convenient scaling of the horizontal (year since 1979) and vertical (Area) axes.





Note: For regression purposes, because the year numbers are so large, convert X axis to X = 'Years since 1979'.

Problem 2 - With Excel Spreadsheet, a linear regression (top left) gives **A = 7.9561 -0.0847 (X)** with $R^2 = 0.71$.

Problem 3 - With Excel Spreadsheet, a quadratic regression (top right) gives **$A = -0.0033 (X)^2 + 0.0264(X) + 7.308$** with $R^2 = 0.79$

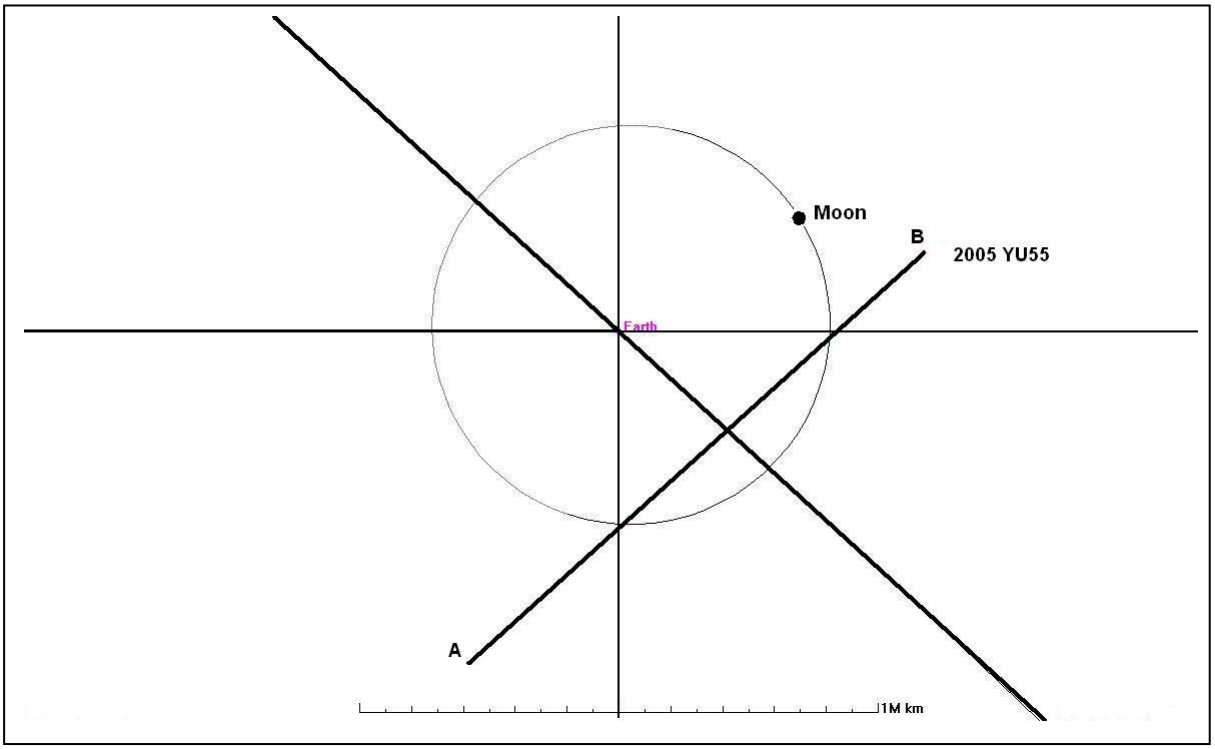
Problem 4 – To the nearest 100,000 km^2, what would you predict as the area of Arctic sea ice in the year 2030 using each regression model?

X = (2030-1979) = 51, then

Linear: A = 7.9561 – 0.0847 (51) so **A = 3.6 million km^2**.
Quadratic: $A = -0.0033 (X)^2 + 0.0264(X) + 7.308$ so **A = 0.1 million km^2.**

Problem 5 - Which of the two forecasts would you consider more statistically reliable? Which would you consider more believable?

Answer: The quadratic regression has the better regression coefficient, but if we had removed the last three points (2009, 2010 and 2011) the regression coefficient would be similar to the one for the linear regression, so the quadratic regression is not as 'believable'. More data beyond 2011 is needed.



The Asteroid 2005 YU55 is predicted to pass inside the orbit of our moon sometime between November 8 and November 9, 2011. The diagram shows the lunar orbit as a circle center on Earth. The diagonal line is the orbit of Earth around the sun. The line segment AB is a portion of the orbit of the asteroid. The horizontal line at the bottom of the page is 1 million kilometers long at the scale of the figure. Point A is the location of the asteroid on November 8.438. Point B is its location one day later on November 9.438, where we have used digital days to indicate a precise hour and minute within each endpoint date in terms of Universal Time.

Problem 1 - To the nearest minute, what is the Universal hour and minute for each of the endpoint dates?

Problem 2 - Using a millimeter ruler, what is the scale of this diagram in kilometers per millimeter?

Problem 3 - To the nearest 100 km, how far will the asteroid travel between the endpoint times?

Problem 4 - How fast will the asteroid be traveling in kilometers per hour?

Problem 5 - On what date and Universal Time will the asteroid be closest to Earth, and what is this distance?

The line segment AB is a portion of the orbit of the asteroid. The horizontal line at the bottom of the page is 1 million kilometers long at the scale of the figure. Point A is the location of the asteroid on November 8.438. Point B is its location one day later on November 9.438, where we have used digital days to indicate a precise hour and minute within each endpoint date in terms of Universal Time.

Problem 1 - What is the Universal hour and minute for each of the endpoint dates?

Answer; 0.438 days = 0.438 x 24hrs = 10.512h and (10.512-10)x60m = 31 m, so the time is 10:31 UT, then Point A is **November 8 at 10:31 UT** and Point B is **November 9 at 10:31 UT**.

Problem 2 - Using a millimeter ruler, what is the scale of this diagram in thousands of kilometers per millimeter?
Answer: 1 million km / 81 millimeters = **12346 km/mm**.

Problem 3 - How far will the asteroid travel between the endpoint times?
Answer; The distance is 81 mm or **1 million km**.

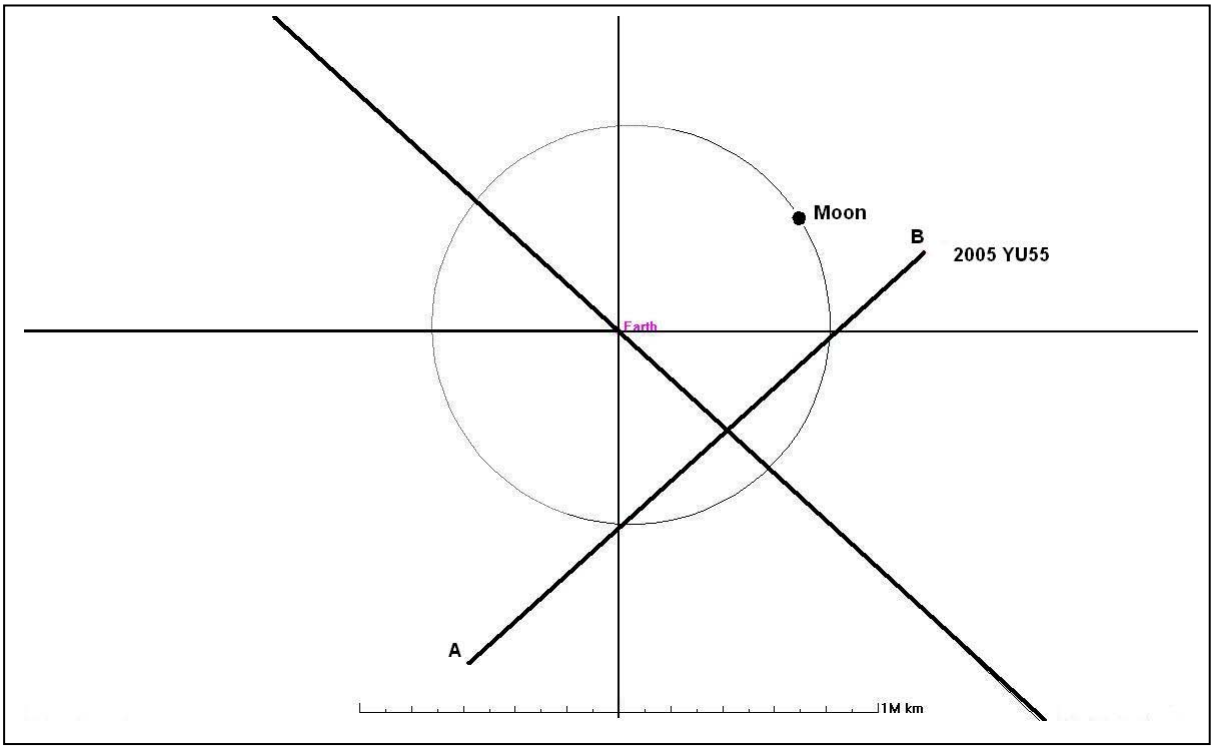
Problem 4 - How fast will the asteroid be traveling in kilometers per hour?
Answer; 1 million km / (9.438-8.438) = 1 million km/day or **41,667 km/hr**.

Problem 5 - On what date and Universal Time will the asteroid be closest to Earth?

Answer; This happens 46 mm from Point A.
D = 46 x 12346 km = 567,916 km.

The speed is 41,667 km/hr so T = 567,916 / 41667 = 13.63 hours from Point A.
This equals 0.568 days.

Then 8.438 + 0.568 = 9.006 which is **00:09 UT on November 9**.



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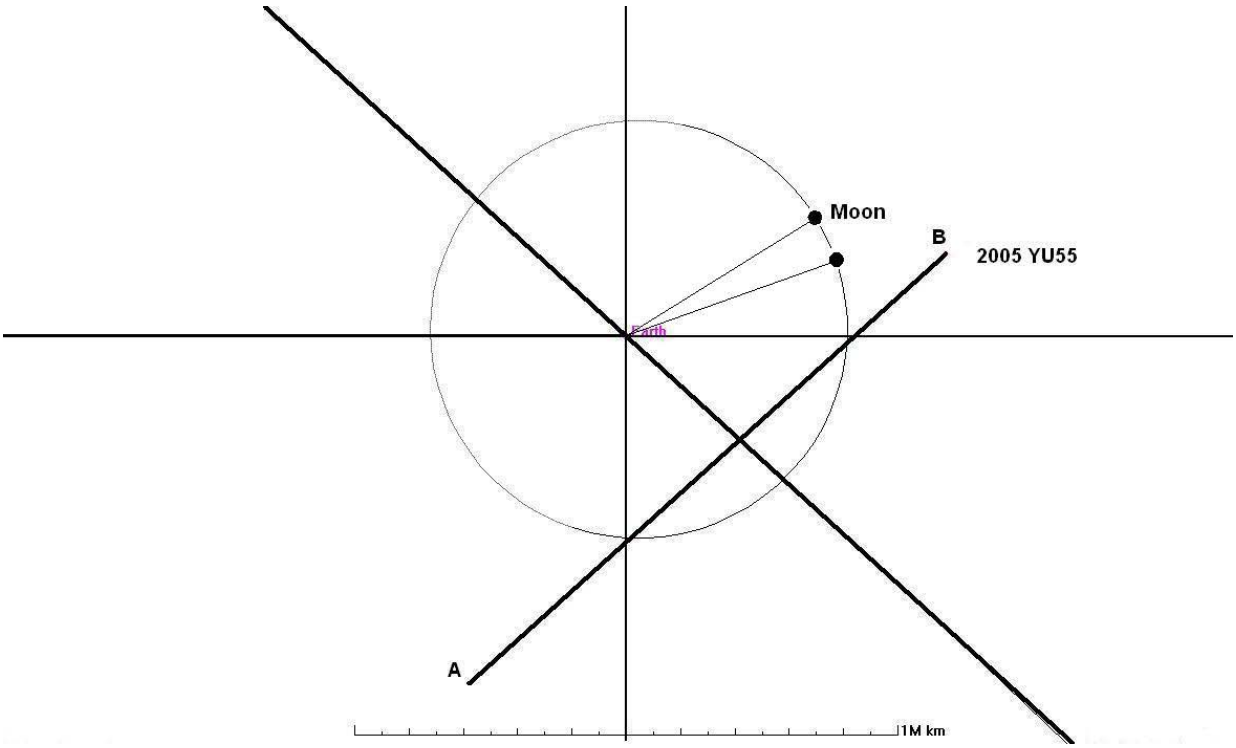
Problem 1 - The figure above shows the location of the Moon on November 9 at 10:13 Universal Time (9.438days) in its counter-clockwise journey around earth in a circular orbit. The period of the orbit is 27.3 days. On what date and time was the moon located when the asteroid was at Point A?

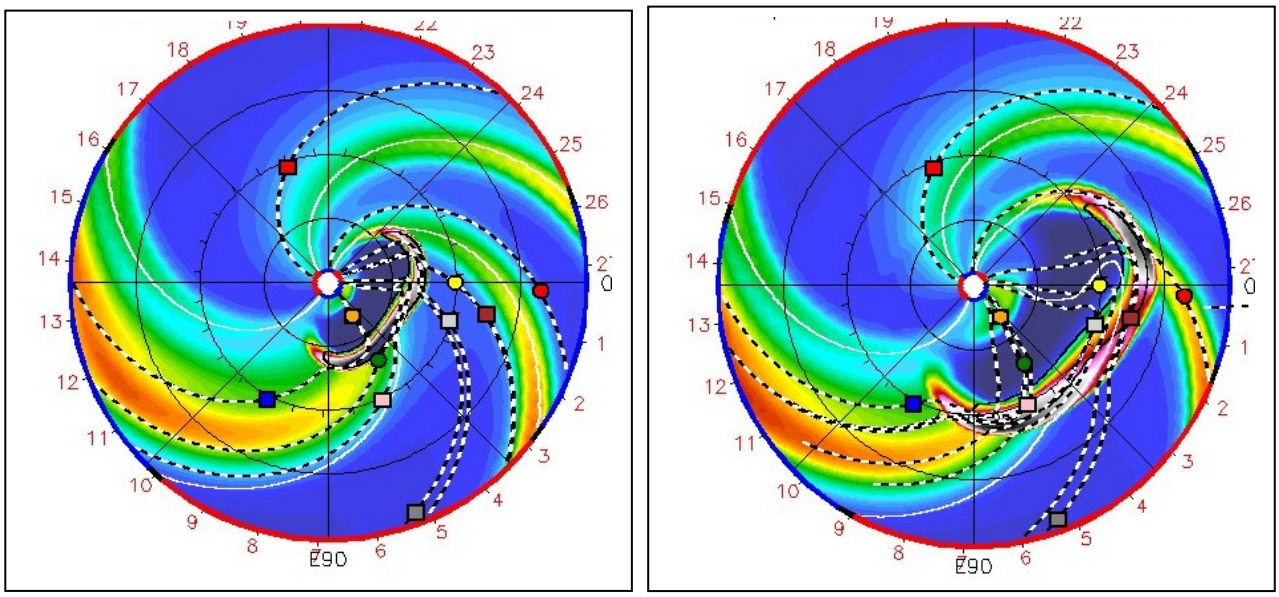
The line segment AB is a portion of the orbit of the asteroid. The horizontal line at the bottom of the page is 1 million kilometers long at the scale of the figure. Point A is the location of the asteroid on November 8.438. Point B is its location one day later on November 9.438, where we have used digital days to indicate a precise hour and minute within each endpoint date in terms of Universal Time.

Problem 1 - The figure above shows the location of the Moon on November 9 at 10:13 Universal Time (9.438days) in its counter-clockwise journey around earth in a circular orbit. The period of the orbit is 27.3 days. On what date and time was the moon located when the asteroid was at Point A?

Answer: The time between Point A and Point B is exactly 1.0 days, so we need to determine the position of the moon 1 day earlier that its location in the diagram.

The moon travels 360 degrees I n27.3 days for a speed of 13.2 degrees per day, so we need to find the position of the moon 13.2 degrees clockwise of its position in the diagram. Using a protractor, the figure below shows this position.





This pair of computed images shows the spiral pattern of the solar wind inside the orbit of Mars. It was created by the NASA Goddard, Coordinated Community Modeling Center to show the condition of the solar wind just after the March 6, 2012 solar storm. The images correspond to March 8 (00:00) and March 9 (00:00).

The planets are indicated at their correct positions by circles (yellow=earth; red=mars; orange=mercury and green=venus). Also shown are the positions of the STEREO A and B spacecraft (red and blue squares) and the Spitzer Space Telescope (pink square). The black concentric circles are drawn at intervals of 75 million kilometers.

The pinwheel pattern is formed from the high-speed gas streams leaving the sun through coronal holes. The crescent shaped cloud is the coronal mass ejection (CME) from the sun, which caused brilliant aurora on Earth.

Problem 1 - About what was the speed, in km/h, of the CME when it reached Earth?

Problem 2 - The dark cavity behind the CME represents a very low density region of space. How do you think this was created? Where did the gas go that once filled the cavity?

Problem 3 - At the orbit of Earth, about how fast do the high-speed gas streams sweep past the Earth in kilometers/hour?

Problem 4 - Assuming that the CME does not slow down, on what date will it arrive at Neptune, which is located 4.5 billion kilometers from the sun?

Problem 1 - About what was the speed, in km/h, of the CME when it reached Earth?

Answer: Students can estimate from the scaled figures that between the two days the crescent-shaped CMRE moves about one division or 75 million kilometers. This took 1 day or 24 hours, so the speed was 75 million km/24 h = **about 3 million km/h**.

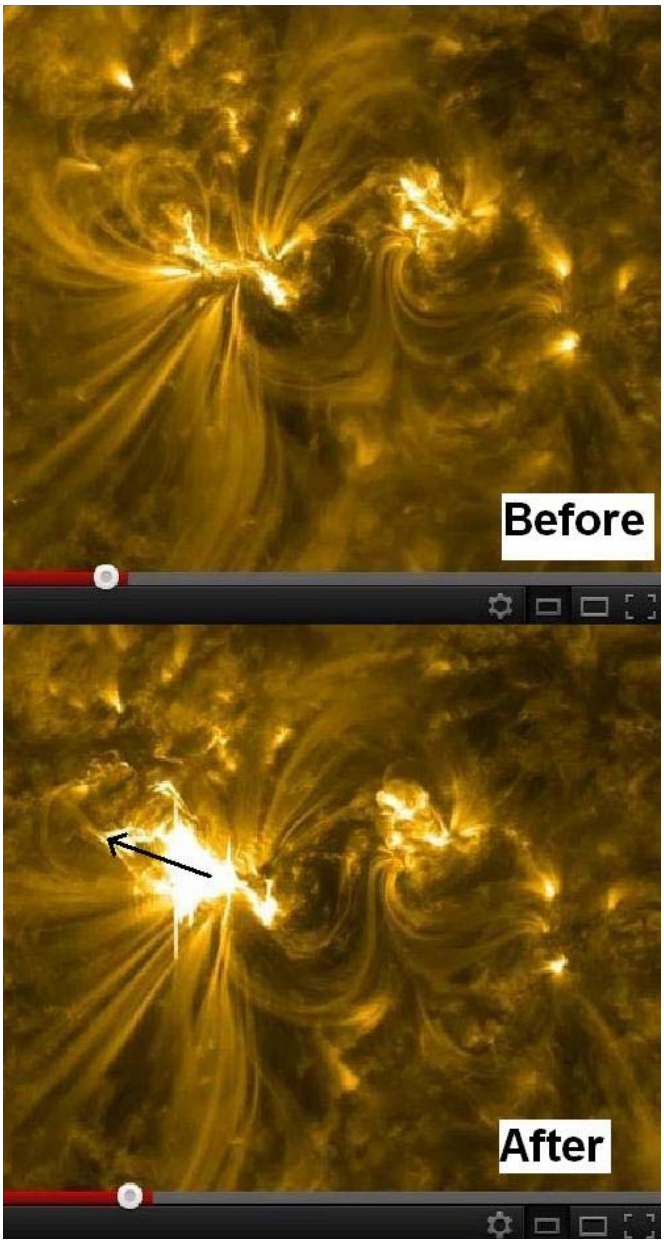
Problem 2 - The dark cavity behind the CME represents a very low density region of space. How do you think this was created? Where did the gas go that once filled the cavity?
Answer: **The CME traveled through the gas and swept it up like a snow-plow in front of the CME.**

Problem 3 - At the orbit of Earth, about how fast do the high-speed gas streams sweep past the Earth in kilometers/hour?

Answer: It will be a challenge to find a feature in the spiral gas streams that can be tracked between the two days, but a reasonable answer would be that the pattern rotated by about 25 million km in 1 day, so the speed is about **1 million km/h**.

Problem 4 - Assuming that the CME does not slow down, on what date will it arrive at Neptune, which is located 4.5 billion kilometers from the sun?

Answer: The CME travels about 3 million km/h so it traveled about 4500 million km during the transit time, so T = 4500/3 = 1500 hours which equals 62.5 days. When this is added to the launch date of March 6 in the first frame, we get a date of about **May 7.**



On March 6, 2012 Active Region 1429 produced a spectacular X5.4 solar flare shown in the pair of images taken by the NASA Solar Dynamics Observatory. The time between the two images is 2 seconds, and the width of each image is 318,000 kilometers.

Problem 1 - The arrowed line indicates how far the million-degree gas produced by the flare traveled in the time interval between the images. What was the speed of the gas, called a plasma, in kilometers/sec?

Problem 2 - By carefully looking at the two images, what other features can you find that changed their position in the time between the images?

Problem 3 - What would you estimate the speeds to be for the features that you identified in Problem 2?

The explosion can be seen in the movie located on *YouTube* at
http://www.youtube.com/watch?v=4xKRBkBBEP0

The above 'high definition' images of the spectacular March 6 solar flare were taken by the NASA Solar Dynamics Observatory (SDO) located in geosynchronous orbit around Earth. Within a few minutes, this flare produced more energy than a thousand hydrogen bombs going off all at once. This energy caused gasses nearby to be heated to millions of degrees, and produced a blast-wave that traveled across the face of the sun at over 4 million km/hour. In the movie above, watch for magnetic loops and filaments to be disturbed by the explosion as the blast wave passes by them.

Problem 1 - The arrowed line indicates how far the million-degree gas produced by the flare traveled in the time interval between the images. What was the speed of the gas, called a plasma, in kilometers/sec?

Answer: Students should compare the length of the arrowed line to the width of the image and solve the proportion to get the physical distance that the plasma traveled. Example, for standard printing onto an 81/2 x 11 page, the line width is about 14 millimeters long, and the width of the image is about 84 millimeters so the proportion for the true length of the line in kilometers is just

$$\frac{14}{84} = \frac{X}{318,000km}$$ so X = 53,000 km

The speed of the flare is simply the distance traveled (53,000 km) divided by the time between the two images (2 seconds) so V = 53,000/2 = **26,500 km/sec**.

The actual speed of the plasma will be much less than this because some of the brightening you see in the second image is because gases trapped in the magnetic field loops were heated in place and brightened rapidly without much movement. As one region dims and another brightens we have the appearance of something moving between the two locations when in fact there was little of no actual movement.

	CY	J	F	M	A	M	J	J	A	S	O	N	D
1996	1							1					
1997	2											3	
1998	3				2	2			5			5	
1999	4								2		1	1	
2000	5		1	3			4	3		1		5	
2001	6			1	8		1		1	1	4	2	3
2002	7				1	1		5	4		1		
2003	8			2		3	4				7	4	
2004	9							6	2		1	2	
2005	10	6						1		10			
2006	11												2

X-Class solar flares are among the most powerful, explosive events on the solar surface. They can cause short-wave radio interference, satellite malfunctions and can even cause the premature re-entry of satellites into the atmosphere.

The table above lists the number of X-class flares detected on the sun during the last sunspot cycle which lasted from 1996 to about 2008. The second column also gives the year from the start of the 11-year sunspot cycle in 1996. The counts are listed by year (rows) and by month (columns). Study this table and answer the following questions to learn more about how common these flares are.

Problem 1 – For the years and months considered, is the distribution of months with flares a uniform distribution? Explain.

Problem 2 - The sunspot cycle can be grouped into pre-maximum (1997,1998, 1999), maximum (2000,2001,2002) and post-maximum (2003,2004,2005). For each group, calculate A) The percentage of months with no flares; and B) The average number of weeks between flares.

Problem 3 – For each group, what is the median number of flares that occurs in the months that have flares?

Problem 4 – Taken as a whole, what is the average number of flares per month during the entire 11-year sunspot cycle?

Problem 5 – We are currently in Year-3 of the current sunspot cycle, which began in 2007. About how many X-class flares would you predict for this year using the tabulated flares from the previous sunspot cycle as a guide, and what is the average number of weeks between these flares for this year?

Problem 1 – For the years and months considered, is the distribution of months with flares a uniform distribution? Explain. Answer; If you shaded in all the months with flares you would see that most occur between 1999-2002 so the distribution is not random, and is not uniform.

Problem 2 - The sunspot cycle can be grouped into pre-maximum (1997,1998, 1999), maximum (2000,2001,2002) and post-maximum (2003,2004,2005). For each group, calculate A) the percentage of months with no flares, and B) The average number of weeks between flares. Answer: A) Pre-Maximum, N = 28 months so P = 100%x28/36 = 78% . Maximum; N = 17 months so P = 100% x 17/36 = 47%; post-maximum N= 24 months so P = 100% x 24/36 = 67%. B) pre-maximum N = 22 flares so T = 36 months/22 flares = 1.6 months. Maximum: N = 50 flares so T = 36 mo/50 = 0.7 months; post-maximum: N = 48 flares so T = 36 mo/48 = 0.8 months.

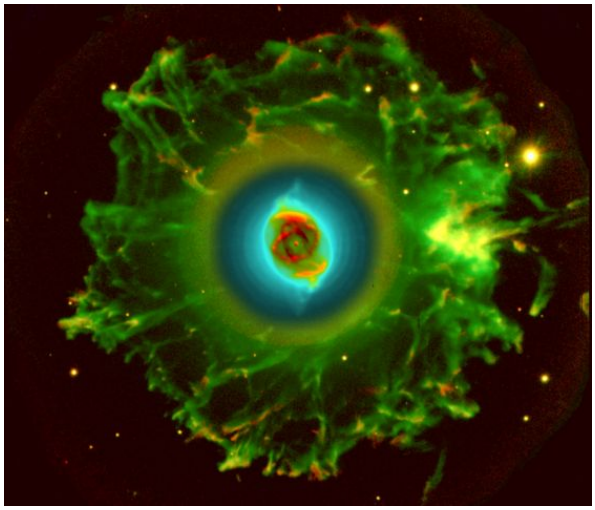
Problem 3 – For each group, what is the median number of flares that occurs in the months that have flares? Answer: Pre-maximum: 1,1,2,2,2,3,5,5 median = 3. Maximum: 1,1,1,1,1,1,1,1,1,2,3,3,3,4,4,4,5,8 median = 4; post-maximum 1,1,2,2,2,3,4,6,6,7 median = 6

Problem 4 – Taken as a whole, what is the average number of flares per month during the entire 11-year sunspot cycle? Answer: There were 122 flares detected during the 132 months of the sunspot cycle, so the average is 122 flares/132 months = 0.9 , which can be rounded to **1 flare/month**.

	CY	J	F	M	A	M	J	J	A	S	O	N	D	
1996	1							1						1
1997	2											3		3
1998	3				2	2			5			5		14
1999	4								2		1	1		4
2000	5		1	3			4	3		1		5		17
2001	6			1	8		1		1	1	4	2	3	21
2002	7				1	1		5	4		1			12
2003	8			2		3	4				7	4		20
2004	9							6	2		1	2		11
2005	10	6						1		10				17
2006	11												2	2
		6	1	6	11	6	9	16	14	12	14	22	5	

Problem 5 – We are in Year-3 of the current sunspot cycle, which began in 2007. About how many X-class flares would you predict for this year using the tabulated flares from the previous sunspot cycle as a guide, and what is the average number of weeks between these flares for this year? Answer: From the table we find for Year 3 that there were 14, X-class flares. Since there are 12 months in a year, this means that the average time between flares is about 14/12 = **1.2 months**.

The Spectacular Cats Eye Planetary Nebula



Thousands of years ago, a star reached the end of its life and began to eject its outer layers forming one of the most complex 'planetary nebulae' in the sky. Known as NGC 6543, it is located 3,300 light years from Earth in the constellation Draco. Its stunning appearance has been known to astronomers for a century as the Cat's Eye Nebula. Detailed telescopic studies of this nebula by astronomers using the Hubble Space Telescope (top left) and recently the Nordic Optical Telescope (top right) have uncovered a wealth of detail caused by the ejected gas and dust from the star (white spot at the center of the Hubble image). Careful measurements allow us to estimate how this nebula has evolved over the centuries.

Problem 1 – The gases that make up the bright central nebula are traveling at 16.4 km/sec. If the radius of the nebula is about 0.25 light years. If one light year = 9.3×10^{12} km, about how long did it take, in years, for the nebular shell to form?

Problem 2 – The eleven equally-spaced outer dust shells that have been detected in the Hubble image extend to a maximum radius of 2.5 light years. If the expansion speeds of the shells is 16.4 km/sec, how long ago was the outermost shell ejected? About how often did the central star eject these shells?

Problem 3 – The dying star was ejecting about 20 trillion tons of matter every second (2×10^{16} kg). If the estimated mass of the star was 5.0 times our sun, about how many percent of the stars mass was ejected to make the planetary nebula with the age calculated in Problem 2? (1 solar mass = 3.0×10^{30} kg).

http://www.nasa.gov/mission_pages/hubble/science/dark-matter-survey.html
Ambitious Hubble Survey Obtaining New Dark Matter Census
Oct 13, 2011

http://apod.nasa.gov/apod/ap020904.html

HST ACS image http://w3.iaa.es/xpn/pn_files/NGC6543/optical.html

Problem 1 – The gases that make up the bright central nebula are traveling at 16.4 km/sec. If the radius of the nebula is about 0.25 light years. If one light year = 9.3×10^{12} km, about how long did it take, in years, for the nebular shell to form?

Answer: Time = Distance/speed, so
Time = $0.25 \times 9.3 \times 10^{12}$ km/16.4 and
Time = 1.4×10^{11} seconds. x (1 year / 3.1×10^{7} sec)
Time = 4,600 years.

Problem 2 – The eleven equally-spaced outer dust shells that have been detected in the Hubble image extend to a maximum radius of 2.5 light years. If the expansion speeds of the shells is 16.4 km/sec, how long ago was the outermost shell ejected? About how often did the central star eject these shells?

Answer:

Time = Distance/speed
= $2.5 \times 9.3 \times 10^{12}$ km / (16.4 km/s)
= **46,000 years**.

The shell spacing is about 2.5 light years/(11 shells) so the ejection interval is 46,000 years/11 shells= **4,200 years per shell ejection event**.

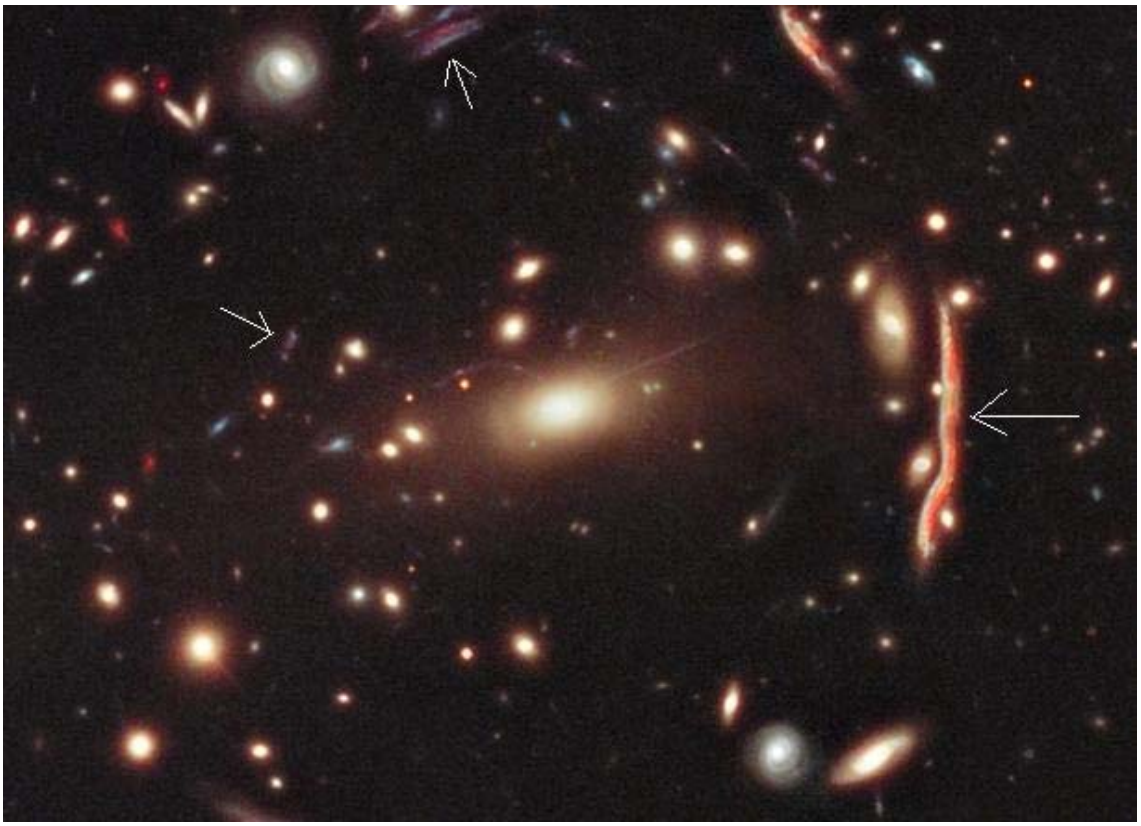
The dying star ejected its first detectable shell about 46,000 years ago, and every 4,000 years would eject a new shell until about 4,500 years ago when it ejected a massive cloud of gas and dust in a giant explosion.

Problem 3 – The dying star was ejecting about 20 trillion tons of matter every second (2×10^{16} kg/sec). If the estimated mass of the star was 5.0 times our sun, about how many percent of the stars mass was ejected to make the planetary nebula with the age calculated in Problem 2? (1 solar mass = 3.0×10^{30} kg).

Answer:

Mass = (2×10^{16} kg/sec) x 46000 years x (3.1 x 107 sec / 1 year)
= 2.8 x 1028 kg

Fraction = 2.8 x 1028 kg / (5.0 x 3.0 x 1030 kg) = 0.0018
Percentage = 100% x 0.0018 = **0.18 %.**



Decades after its discovery, astronomers still do not know the nature of dark matter; a form of 'matter' unlike anything familiar in every-day life to humans. It is commonly found mixed with ordinary matter inside great clusters of galaxies, however, there is often ten times more of it than luminous matter in stars. A detailed study of galaxy clusters such as MACS J1206.2 shown in the Hubble Telescope image above now shows how much dark matter can exist, and how it is spread through out such clusters. This cluster is located 4.5 billion light years from our Milky Way. It contains over 100 individual galaxies.

When light passes through a strong gravitational field it is bent. This causes the image of the source to become distorted. Astronomers have counted in the cluster image a total of 47 'ghost' images from 12 background galaxies behind this massive cluster. A few of these images is shown by the arrows above. A careful study of the number and shapes of these images lets astronomers estimate the total mass of the cluster inside spheres of different radii centered on the massive galaxy at the center of the cluster. They found that inside a radius of 515,000 light years the total mass is 1.3×10^{14} Msun; inside 312,000 light years the total mass is about 8.0×10^{13} Msun. Only about 10% of the total mass is in the form of stars and galaxies. (Note: The Milky Way has a mass of about 5.0×10^{11} Msun)

Problem 1 – From the mass data within the two radii, and comparing the density of matter found within the two concentric volumes, is the dark matter uniformly mixed within each cubic light year of the cluster's volume?

http://www.nasa.gov/mission_pages/hubble/science/dark-matter-survey.html
Ambitious Hubble Survey Obtaining New Dark Matter Census
Oct 13, 2011

Problem 1 – From the mass data within the two radii, is the dark matter uniformly mixed within each cubic light year of the cluster's volume?

Answer: To determine whether dark matter is uniformly distributed in space, we can compare the average density of the total mass inside the first volume, with the total mass found within the shell of the second sphere.

Inner sphere:

Volume = 4/3 π (312,000 ly)^3 = 1.3x10^{17} ly^3.

Mass = 1.3 x 10^{14} Msun

Density = Mass/Volume
= (8.0 x 10^{13} Msun) / (1.3x10^{17} ly^3)
= **0.00062 Msun/ ly^3**

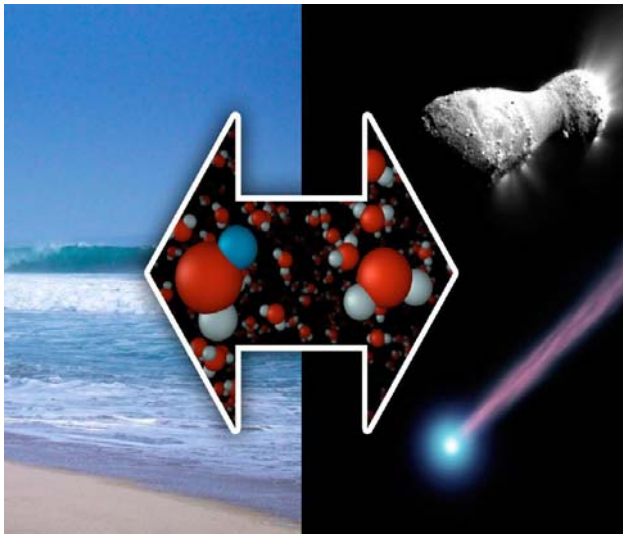
Outer shell between 515,000 and 312,000 light years

V = 4/3 π (515,000)^3 – 4/3 π (312,000)^3 = 4.4 x 10^{17} ly^3

The difference in mass residing in this shell volume is
M = 13 x 10^{13} Msun – 8.0 x 10^{13} Msun = 5 x 10^{13} Msun.

So the density of mass in this outer shell is just
D = 5.0 x 10^{13} Msun / 4.4 x 10^{17} Ly^3
= **0.00011 Msun/Ly^3**.

From this we see that there is about 6 times more mass in the core volume than in the shell volume, so dark matter is not evenly spread through out every volume of space within the cluster.



The abundance of heavy-water in Earth's oceans is about 0.015%. The abundance of heavy-water in Hartley-2 is about 0.016%, so comets like Hartley-2 could have impacted Earth and deposited over time Earth's ocean water. (Image courtesy NASA/JPL-Caltech)

New measurements from the Herschel Space Observatory show that comet Hartley 2, which comes from the distant Kuiper Belt, contains water with the same chemical signature as Earth's oceans. This remote region of the solar system, some 30 to 50 times as far away as the distance between Earth and the sun, is home to icy, rocky bodies including Pluto, other dwarf planets and innumerable comets.

Herschel detected the signature of vaporized water in this coma and, to the surprise of the scientists, Hartley 2 possessed half as much "heavy water" as other comets analyzed to date. In heavy water, one of the two normal hydrogen atoms has been replaced by the heavy hydrogen isotope known as deuterium. The amount of deuterium is similar to the abundance of this isotope in Earth's ocean water.

The deposition of Earth's oceans probably occurred between 4.2 and 3.8 billion years ago. Suppose that the comet nuclei consisted of three major types, each spherical in shape and made of pure water-ice: Type 1 consisting of 2km in diameter bodies arriving once every 6 months, Type-2 consisting of 20 km diameter bodies arriving once every 600 years and Type-3 consisting of 200 km diameter bodies arriving every one million years.

Problem 1 - What are the volumes of the three types of comet nuclei in km^3?

Problem 2 - The volume of Earth's liquid water oceans is 1.33×10^9 cubic kilometers. If solid ice has 6 times the volume of liquid water, what is the volume of cometary ice that must be delivered to Earth's surface every year to create Earth's oceans between 4.2 and 3.8 billion years ago?

Problem 3 - What is the annual ice deposition rate for each of the three types of cometary bodies?

Problem 4 - How many years would it take to form the oceans at the rate that the three types of cometary bodies are delivering ice to Earth's surface?

Space Observatory Provides Clues to Creation of Earth's Oceans

http://www.nasa.gov/mission_pages/herschel/news/herschel20111005.html

Problem 1 - What are the volumes of the three types of comet nuclei in km3?

Answer: $V = 4/3\ \pi R^3$ so

Type 1 Volume = **4.2 km^3**

Type 2 Volume = **4,200 km^3**

Type 3 Volume = **4.2×10^6 km^3**

Problem 2 - The volume of Earth's liquid water oceans is 1.33×10^9 cubic kilometers. If solid ice has 6 times the volume of liquid water, what is the volume of cometary ice that must be delivered to Earth's surface every year to create Earth's oceans between 4.2 and 3.8 billion years ago?

Answer: 1.33×10^9 km^3 of water requires
$6 \times (1.33 \times 10^9\ km^3) = 8.0 \times 10^9\ km^3$ of ice.

The average delivery rate would be about
R = 8.0×10^9 km^3 of ice / 400 million years
= **20 km^3 of ice per year**.

Problem 3 - What is the annual ice deposition rate for each of the three types of cometary bodies?

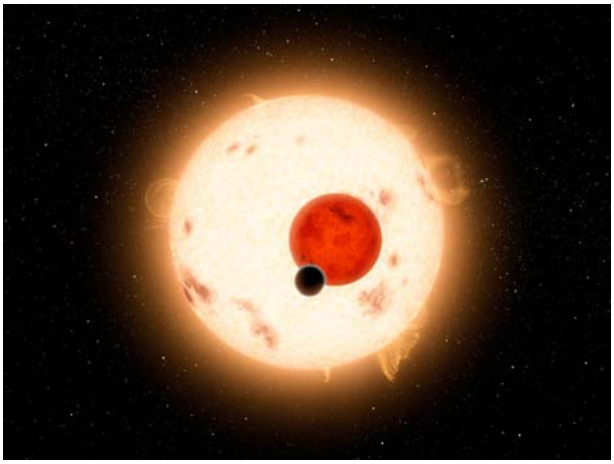
Type 1: R1 = 4.2 km3/0.5 yrs = **8.4 km^3/yr**

Type 2: R2 = 4200 km3/600 yrs = **7.0 km^3/yr**

Type 3: R3 = 4.2×10^6 km3 / 1000000 yrs = **4.2 km^3/yr**

Problem 4 - How many years would it take to form the oceans at the rate that the three types of cometary bodies are delivering ice to Earth's surface?

Answer: The total deposition rate is R1+R2+R3 = 20 km^3/yr, so it would take
T = 8.0×10^9 km^3 of ice / (20 km^3/yr) = **400 million years**.



NASA's Kepler mission has discovered a world where two suns set over the horizon instead of just one. The planet, called Kepler-16b, is the most "Tatooine-like" planet yet found in our galaxy. Tatooine is the name of Luke Skywalker's home world in the science fiction movie *Star Wars*. In this case, the planet it not thought to be habitable. It is a cold world, with a gaseous surface, but like Tatooine, it circles two stars.

The planet orbits two stars, Kepler 16A and Kepler-16B located 200 light years from Earth. (Image credit: NASA/JPL-Caltech/R. Hurt)

The Saturn-sized planet Kepler-16b, orbits Kepler-16A at a distance of 0.7 Astronomical Units or 105 million kilometers. Its orbit is nearly a perfect circle. The smaller star Kepler-16B orbits its more massive companion at a distance of 0.2 Astronomical Units or 30 million kilometers. The diameters of the two stars are 890,000 kilometers and 300,000 kilometers respectively.

Problem 1 – Suppose you were standing on the surface of ‘Tatooine’ and watching (safely!) these two stars in the sky, perhaps much like Luke Skywalker was watching his twin suns near sunset. In terms of angular degrees, and by using the definition of the tangent of an angle, what would be the greatest separation between these two stars as they orbited one another to the nearest degree?

Problem 2 - At the time the stars are farthest apart in the sky, what would be the angular diameters of each star as viewed from Tatooine?

Problem 1 – Suppose you were standing on the surface of 'Tatooine' and watching (safely!) these two stars in the sky, perhaps much like Luke Skywalker was watching his twin suns near sunset. In terms of angular degrees, and by using the definition of the tangent of an angle, what would be the greatest separation between these two stars as they orbited one another to the nearest degree?

Answer: The geometry of this viewing triangle is that the adjacent side is the distance of Tatooine from Kepler-16A, which is 105 million kilometers, and the opposite side is the distance between Kepler-16A and 16B, which is 30 million kilometers.

The maximum separation angle is then

$\text{Tan}(\theta) = 30/105$
$= 0.286$, so arctangent(0.286) = **16 degrees**.

Problem 2 - At the time the stars are farthest apart in the sky, what would be the angular diameters of each star as viewed from Tatooine?

Answer: The distance to **Kepler-16A** is just 105 million kilometers. Its diameter is 890,000 km, then

$\text{Tan}(\theta) = 0.89/105$
$= 0.00848$ and arcTan(0.00848) = **0.49 degrees**.

For **Kepler-16B**, the problem is a bit tricky because the distance from Tatooine to Kepler-16B is now the hypothenuse of the triangle in Problem 1, which is $d^2 = (105)^2 + (30)^2$ so d = 109 million km. Then the angular diameter of Kepler-16B is just

$\text{Tan}(\theta) = 0.3/109$
$= 0.00275$ so arcTan(0.00275) = **0.16 degrees**.

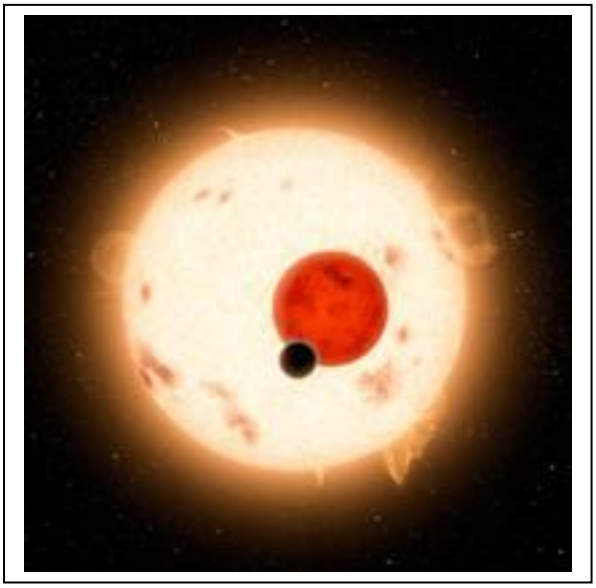
As seen from Earth, our sun (and moon) have angular diameters of 0.5 degrees by comparison.

Note, as a preliminary activity, have students sketch the two stars in the afternoon sky at what they think might be their sizes as seen from Tatooine. After completing these two problems, students may sketch the same scene in which the two stars are located in the sky at their proper angular scales, and compare with their first guesses.

For more details read the Press Release on September 15, 2011:

NASA's Kepler Mission Discovers a World Orbiting Two Stars

http://www.nasa.gov/mission_pages/kepler/news/kepler-16b.html



In 2011, the Kepler observatory detected a Saturn-sized planet orbiting the binary stars Kepler 16A and Kepler 16B. Nicknamed 'Tatooine', the view from this planet of its twin suns would be spectacular. The smaller star, Kepler-16B orbits once every 41 days at a distance of 30 million km from the larger star Kepler 16A, and the planet is in a circular orbit 108 million km from Kepler 16A which takes 229 days to complete. As seen from 'Tatooine', the small orange star Kepler-16B never gets more than 18 degrees from the much larger, yellow star Kepler 16A.

Predicting when Kepler 16B will pass across the face of Kepler 16A, called a transit, is made a bit more difficult because Tatooine is also moving along its orbit while Kepler 16B is in motion around the main star. To see a transit, Kepler 16B and Tatooine must be located in their orbits so that a line through their centers passes through the center of Kepler 16A at the center of the orbits. The circumferences of the two orbits are different, and they take different amounts of time to complete a full orbit. Given that you have just observed one transit, the time you must wait in order to see the next one depends on how long it takes for Kepler 16B and Tatooine to return to their 'straight line' configuration.

Problem 1 - What is the speed of the star Kepler 16B in its orbit in terms of degrees per day?

Problem 2 - What is the speed of Tatooine in its orbit in terms of degrees per day?

Problem 3 - What is the difference in angular speed between fast-moving Kepler 16B and slower-moving Tatooine?

Problem 4 - How many days will it take Kepler 16B to overtake Tatooine?

Problem 5 - Can you show that your answer to Problem 4 is just the time between transits?

Problem 1 - What is the speed of Kepler 16B in its orbit in terms of degrees per day?

Answer: SK = 360 degrees /41days = **8.78 degrees/day.**

Problem 2 - What is the speed of Tatooine in its orbit in terms of degrees per day?

Answer: ST = 360 degrees / 229 days = **1.57 degrees/day.**

Problem 3 - What is the difference in angular speed between fast-moving Kepler 16B and slower-moving Tatooine?

Answer: 8.78 - 1.57 = **7.21 degrees/day**

Problem 4 - How many days will it take Kepler 16B to overtake Tatooine?

Answer: T = 360 degrees / 7.21 = **50 days**.

Problem 6 - Can you show that your answer to Problem 4 is just the time between transits?

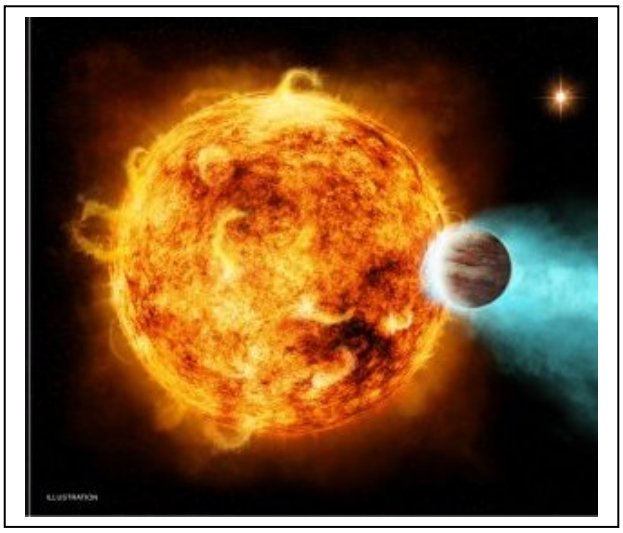
Answer: Students can sketch two concentric circles with Kepler 16B on the inner circle and Tatooine on the outer circle. Draw a line from Kepler 16A at the center of the circle to the location of Tatooine. Place Kepler 16B in its orbit on the line you drew to 'start' the clock. In 41 days, Kepler 16B will go once around its circle and return to this spot, while Tatooine will move 41 days further along its orbit. After 50 days, Kepler 16B and Tatooine will once again beon the same line to Kepler 16A.

Note: In astronomy, the synodic period, P, is related to the orbit periods of a fast-moving planet, t, and a slow-moving planet, T, by the formula

$$\frac{360}{t} - \frac{360}{T} = \frac{360}{P}$$

In our problem t = 41 days and T = 229 days so P = 50 days. The synodic period is a measure of the time taken for two planets to return to the same spatial configuration with respect to a central star. The synodic period of the moon is 29 days and this is the time between corresponding phases of the moon (full moon to full moon), when Earth, sun and moon are in the same orientation to produce the moons illumination.

Chandra 'Sees' a Distant Planet Evaporating



NASA's Chandra Observatory has discovered that the star CoRot-2a is a powerful X-ray source. This is unfortunate because it is also known that a planet orbits this star at a distance of only 5 million kilometers. Called CoRoT-2b, the planet is three times the mass of Jupiter, and probably has an atmosphere similar to that of Jupiter given its size. Every second, the planet loses about 5 million tons of matter due to its star's radiation.

The star CoRoT-2a is located 880 light years from Earth. The planet orbits its star once every 1.7 days.

The star, CoRoT-2a is similar to our own sun while the planet is about 1.4 times the diameter of Jupiter. The heating of its upper atmosphere to temperatures of 1,500 K have caused the planet to 'puff out' in size due to the added heat energy.

A simple model of this planet's interior suggests that its atmosphere might account for as much as 50% the mass of the planet. By comparison, the mass of Jupiter is about 1.9×10^{27} kilograms or 315 times the mass of Earth.

Problem 1 - About what is the mass of the atmosphere of CoRoT-2b in kilograms?

Problem 2 - Based on the rate at which the planet is being evaporated, about how many years might the planet survive before losing all of its atmosphere if the rate is constant the whole time? (1 ton = 1000 kg)

Problem 1 - About what is the mass of the atmosphere of CoRoT-2b in kilograms?

Answer: Mass = 0.50 x 3.0 Jupiters x (1.9 x 10^{27} kilograms / 1 Jupiter)
= **2.9 x 10^{27} kilograms**

Problem 2 - Based on the rate at which the planet is being evaporated, about how many years might the planet survive before losing all of its atmosphere if the rate is constant the whole time? (1 ton = 1000 kg)

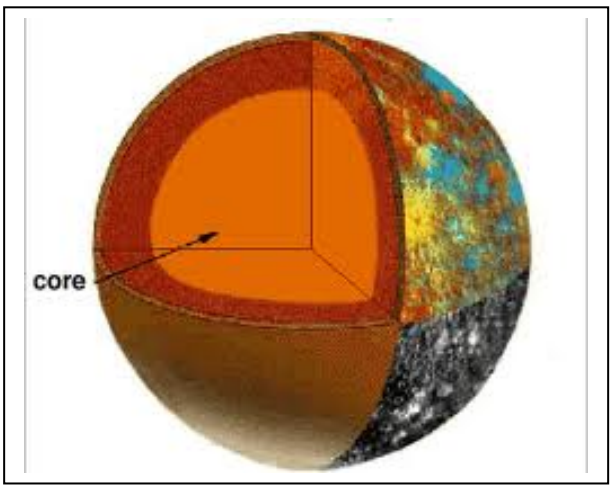
Answer: The mass loss rate is stated as 5 million tons per second .In terms of its loss per year R = 5 x 10^{6} tons x (1000 kg/1 ton) x (3.1 x 10^{7} seconds/ 1 year)
= 1.6 x 10^{17} kilograms/year

The mass of the planets atmosphere is M = 2.9 x 10^{27} kilograms, so
Time = Amount / Rate and so
Time = 2.9 x 10^{27} kilograms / (1.6 x 10^{17} kilograms/year)
= **18 billion years**.

Note: Mass loss rates for planets can sound HUGE, but with few exceptions, planets can usually survive for a period of time longer than the lifetime of their star (10 - 20 billion years) even with such very high rates of mass loss!

See the NASA press release at:

Star Fries Nearby Planet
http://www.nasa.gov/mission_pages/chandra/multimedia/corot2a_photo.html
Sep 13, 2011CoRoT-2a



This MESSENGER spacecraft is in orbit around the planet Mercury, with an elliptical orbit period of 8 hours at an average distance of 7715 kilometers. At its closest distance it is only 320 km above the surface of Mercury, which allows the spacecraft to obtain high-resolution images of the mysterious surface of Mercury.

A detailed study of the spacecrafts speed and altitude also lets astronomers study the gravity field of Mercury and deduce something about its interior.

We can create a simple model of the interior of Mercury by dividing it into a spherical core region, and an overlying shell of matter that reaches to the observed surface of the planet. Here is how we do this!

Problem 1 - The mass of Mercury is 3.31×10^{23} kilograms. If the planet is a perfect sphere with a radius of 2,425 kilometers, what is the average density of the planet Mercury in kg/meter^3 defined as density = mass/volume?

Problem 2 - Astronomers believe that the crust of the planet has an average density of 3,000 kg/m^3 and the iron-rich core has a density of 7,800 kg/m^3.

A) What is the formula that gives the total mass of the core, if the core has a radius of Rc in meters?

B) What is the formula for the outer shell of Mercury if the density equals the density of the crust, and its inner radius is Rc and its outer radius is the actual radius of the planet of 2,425 kilometers?

Problem 3 - If the sum of the core and shell masses must equal the mass of the planet, what is the value for Rc, the radius of the core in kilometers, that leads to a solution for this simple model?

Problem 1 - The mass of Mercury is 3.31 x 10^{23} kilograms. If the planet is a perfect sphere with a radius of 2,425 kilometers, what is the average density of the planet Mercury in kg/meter^3 defined as density = mass/volume?

Answer: Volume = $\frac{4}{3}\pi R^3$

so Volume = $\frac{4}{3}(3.141)(2,425,000)^3$ = 5.97 x 10^{19} meters^3

Density = 3.31 x 10^{23} kilograms / 5.97 x 10^{19} meters^3 = **5542 kg/m^3**.

Problem 2 - Astronomers believe that the crust of the planet has an average density of 3,000 kg/m^3 and the iron-rich core has a density of 7,800 kg/m^3. A) What is the formula that gives the total mass of the core, if the core has a radius of Rc in kilometers? B) What is the formula for the outer shell of Mercury if the density equals the density of the crust, and its inner radius is Rc and its outer radius is the actual radius of the planet of 2,425 kilometers?

Answer A) M(core) = $\frac{4}{3}\pi(1000Rc)^3(7800)$ = 3.26x10^{13}Rc^3 kilograms

B) Volume of the spherical shell = $\frac{4}{3}\pi(2,425,000)^3 - \frac{4}{3}\pi(1000Rc)^3$ cubic meters

Then M(shell) = 3,000 x V(shell)

$= 2,200\left(\frac{4}{3}\pi(2,425,000)^3 - \frac{4}{3}\pi(1000Rc)^3\right)$ kilograms

$=1.3x10^{23} - 9.21x10^{12}Rc^3$ kilograms

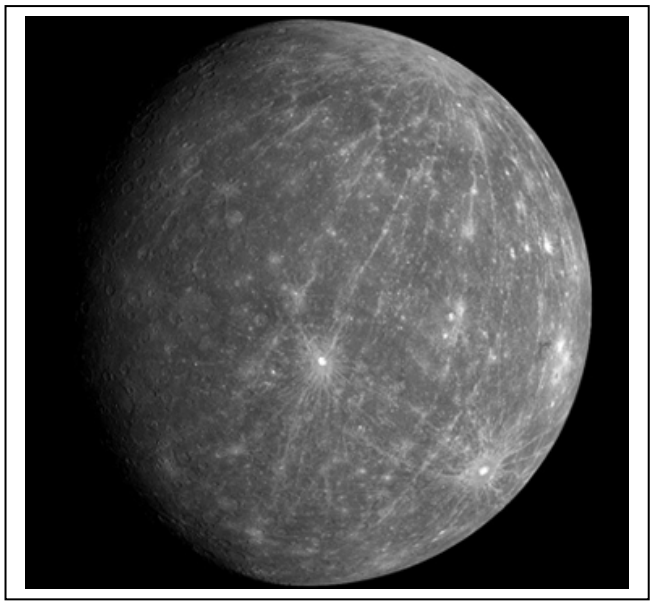
Problem 3 - If the sum of the core and shell masses must equal the mass of the planet, what is the value for Rc, the radius of the core in kilometers, that leads to a solution for this simple model?

Answer: 3.31 x 10^{23} = 3.26x10^{13}Rc^3 + 1.3x10^{23} - 9.21x10^{12} Rc^3

So 2.01x10^{23} = 2.34x10^{13} Rc^3

And so Rc = **2,046 kilometers!**

So the dense iron core of Mercury occupies 100%x(2046/2425) = 84% of the radius of Mercury! By comparison, Earth's core is only 30% of its radius.



According to Kepler's Third Law, the time it takes a satellite to go once around its planet is given by the formula

$$\frac{R^3}{T^2} = 1.69x10^{-12} M$$

where M is the mass of the planet in kilograms, T is the orbit period in seconds, and R is the radius of the orbit in meters.

For example, the International Space Station orbits Earth with R = 6738 km, T= 92.5 minutes so the mass of Earth is just M = 5.9 x10^{24} kg.

The MESSENGER spacecraft orbits the planet Mercury, but has changed its orbit several times since it arrived in April 2011. The following problems explore how these orbit changes affect the estimate for the mass of Mercury using the Kepler formula.

Problem 1 - On April 25, 2011 the orbit period of MESSENGER was 12 hours and 2 minutes, and its distance was 10,124 km from the center of Mercury. To three significant figures, what was the estimated mass of Mercury?

Problem 2 - On September 14, 2011 the orbit was changed to a distance of 10,085 kilometers and a period of 11hours 58 minutes. To three significant figures, what was the mass of Mercury?

Problem 3 - On May 25, 2012 the final orbit had a period of 8.0 hours and a distance of 7,715 kilometers. To three significant figures, what was the mass of Mercury?

Problem 4 - Explain what the formula is telling us about the properties of the orbit of a satellite and the mass of the body?

Problem 1 - On April 25, 2011 the orbit period of MESSENGER was 12 hours 2 minutes and its distance was 10,124 km from the center of Mercury. To three significant figures, what was the estimated mass of Mercury?

Answer: We first need to convert R and T to meters and seconds so that R = 10,124,000 meters and T = 12h x (3600 s/1hr) +120 sec = 43,320 seconds. Then from the formula:

$$M = 5.92x10^{11}\frac{(10124000)^3}{(43320)^2} = \mathbf{3.27 \times 10^{23}\ kg}$$

Problem 2 - On September 14, 2011 the orbit was changed to a distance of 10,085 kilometers and a period of 11 hours 58 minutes. To three significant figures, what was the mass of Mercury?

Answer: R = 10,085,000 meters and T = 43,080 seconds and so

$$M = 5.92x10^{11}\frac{(10085000)^3}{(43080)^2} = \mathbf{3.27 \times 10^{23}\ kg}$$

Problem 3 - On May 25, 2012 the final orbit had a period of 8.0 hours and a distance of 7715 kilometers. To three significant figures, what was the mass of Mercury?

Answer: R = 7715000 meters and T = 28,800 seconds and so

$$M = 5.92x10^{11}\frac{(7715000)^3}{(28800)^2} = \mathbf{3.27 \times 10^{23}\ kg}.$$

Problem 4 - Explain what the formula is telling us about the properties of the orbit of a satellite and the mass of the body?

Answer: Although the orbit properties change, the mass of Mercury remains the same because the spacecraft is orbiting the same body and the values for R ,T and M must lead to consistent solutions.

Note: From more careful orbital studies, astronomers use the adopted mass of Mercury of 3.31×10^{23} kg.

The SpaceX Falcon 9 rocket soared into space from Space Launch Complex-40 on Cape Canaveral Air Force Station in Florida, carrying the Dragon capsule (left) to orbit on May 22, 2012.

During the flight, there were a series of check-out procedures to test and prove Dragon's systems, including rendezvous and berthing with the International Space Station. If the capsule performed as planned, the cargo and experiments it was carrying would be transferred to the station.

Problem 1 – The Dragon Capsule has the shape shown in the photo above (Courtesy NASA). The diameter of the base just above the curved head shield at its bottom is 3.2 meters. The diameter of the top is 2.2 meters. The capsule is 2.3 meters tall, and it would be 5.2 meters tall to its apex at the top, if it were the shape of an upside-down ice cream cone.

The volume of an ice cream cone with a base radius of R and a height of h is given by the formula

$$V = \frac{1}{3}\pi R^2 h$$

From the information provided, what is the volume of the Dragon Capsule in cubic meters?

Problem 2 - Suppose you had a room in your house with an 8-foot (2.7 meter) ceiling. If the floor area were a perfect square, what would be the dimensions of the floor so that the volume of this room were the same as the volume of the Dragon Capsule A) in meters? B) in feet?

Problem 1 – The Dragon Capsule has the shape shown in the photo above (Courtesy NASA). The diameter of the base just above the curved head shield at its bottom is 3.2 meters. The diameter of the top is 2.2 meters. The capsule is 2.3 meters tall, and it would be 5.2 meters tall to its apex at the top, if it were the shape of an upside-down ice cream cone. From the information provided, what is the volume of the Dragon Capsule in cubic meters?

Answer: Students should first compute the volume of the full 'ice cream cone' with a base radius of R = (3.2/2)=1.6 meters and a height h = 5.2 meters, then subtract the cone with a height of h = (5.2-2.3) = 2.9 meters and a base radius of R = (2.2/2)=1.1 meters. The difference is the volume of the capsule.

$$V = \frac{1}{3}\pi(1.6)^2(5.4) \; - \; \frac{1}{3}\pi(1.1)^2(2.9)$$

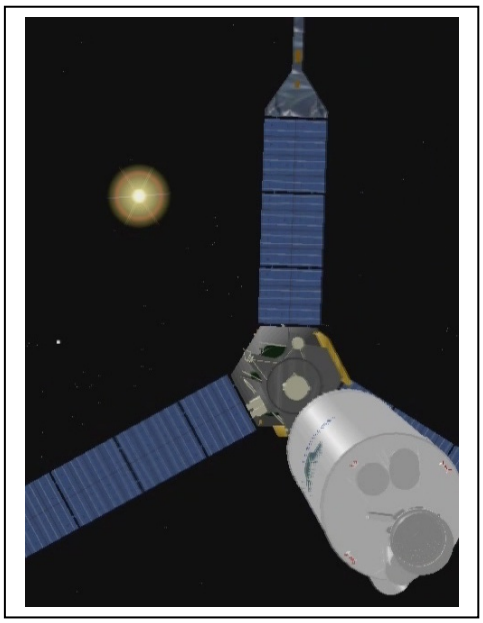
$$= \quad 14.5 \; - \; 3.7$$

= **10.8 cubic meters**

Problem 2 - Suppose you had a room in your house with an 8-foot (2.7 meter) ceiling. If the floor area were a perfect square, what would be the dimensions of the floor so that the volume of this room were the same as the volume of the Dragon Capsule A) in meters? B) in feet?

Answer: A) The volume of the room would be V = 2.7 x Area, and since Area = L2 for a square floor, the length would be given by 10.8 = 2.7 x L2, so **L = 2 meters**.
B) 1 meter = 3 feet, so the length would be **L = 6 feet**. The dimensions of the room would be 2m x 2m x 2.7m or 6 feet x 6 feet x 8 feet.

Note: Have the students try to imagine 5 astronauts lying on couches in this volume with computer equipment and spacesuits too!



As the Juno spacecraft travels to Jupiter, it gets farther from the sun every day. Because the spacecraft generates its electrical power using solar cells, as the sun gets farther away, the amount of power constantly diminishes. At Earth, the solar panels generate about 12,700 watts. Because the spacecraft's trajectory is a portion of an ellipse, the formula for its distance, r, from the sun, located at one of the ellipse's foci, is given by the formula:

$$r(\theta) = \frac{a(1-e^2)}{1-e\cos\theta}$$

where r is in Astronomical Units, a is the semi-major axis length and e is the orbit eccentricity.

The specific equation for the Juno spacecraft can be approximately represented by a = 3.0 AU and e = 2/3, where all distances are given in units of the Astronomical Unit. The Astronomical Unit is a measure of the distance between Earth and the sun; a physical distance of 150 million km.

Problem 1 – What is the simplified form for R given the initial parameters, a and e, for the Juno spacecraft?

Problem 2 – If the amount of solar energy falling on the Juno solar panels is determined by the inverse-square law, and the amount of solar energy generated by the solar panels at r = 1.0 AU is exactly 12,690 watts, what is the formula for the solar panel power at any distance defined by the function P(r)?

Problem 3 – For what angle, θ, will the spacecraft be able to generate only ¼ of the electrical power it did when it left Earth orbit?

Problem 4 – What will the spacecraft power be when it reaches Jupiter at its maximum distance from the sun?

Problem 1 – What is the simplified form for R given the initial parameters, a and e, for the Juno spacecraft?

$$r(\theta) = \frac{5}{3 - 2\cos\theta}$$

Problem 2 – If the amount of solar energy falling on the Juno solar panels is determined by the inverse-square Law, and the amount of solar energy generated by the solar panels at r = 1.0 AU is 12,000 watts, what is the formula for the solar panel power at any distance defined by the function P(r)?

$$P(r) = 480(3 - 2\cos\theta)^2$$

Problem 3 – For what angle, θ in degrees, will the spacecraft be able to generate only ¼ of the electrical power it did when it left Earth orbit?

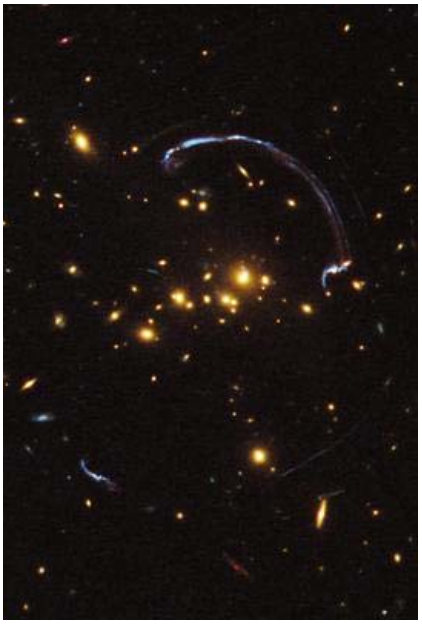
P = 12000/4 = 3000 watts

$3000 = 480\,(3-2\cos\theta)^2$ so $2.5 = 3 - 2\cos\theta$ and $\cos\theta = 0.25$ so **θ= 76 degrees**

Problem 4 – What will the spacecraft power be when it reaches Jupiter at its maximum distance from the sun?

Answer: The maximum value for r occurs at $\theta = 0$, so **P = 480 watts**.

Using a Gravity Lens to 'Weigh' a Cluster of Galaxies



The Hubble Space Telescope recently photographed a distant galaxy being 'lensed' by the gravity field of a foreground cluster of galaxies called RCS2-032727-132623 located 5.4 billion light years away in the southern constellation Cetus the Whale. The cluster lies directly between the more distant galaxy and Earth. Its gravity causes the image of the distant galaxy to be distorted into several arcs (blue).

Thanks to the magnification provided by this 'gravity lens' astronomers can study the blue galaxy located 9.7 billion light years from Earth – details that would have been impossible to see otherwise.

Astronomers can also use the geometry of the arc and its radius to determine the total mass of the cluster of galaxies, including any dark matter that it might contain!

The diameter of a gravity lens ring is related to the mass of the object producing the gravitational field by the simple formula:

$$M = \frac{\theta^2 c^2 (dD)}{4G(d-D)}$$

where θ is the angular diameter in radians of the ring, D is the distance in meters from Earth to the cluster of galaxies, d is the distance in meters between Earth and the galaxy whose image is being lensed, c is the speed of light (3×10^8 m/s) and G is the constant of gravity (6.67×10^{-11} m^3 kg^{-1} sec^{-2})

Problem 1 - For convenience, astronomers often re-write equations in the common units of astronomy where angles, θ, are measured in arcseconds (1 radian = 206265 arcseconds), distances (d and D) are measured in light years and masses (M) are given in multiples of the sun's mass). If 1 light year = 9.5×10^{15} meters and Msun = 2.0×10^{30} kg, what is the value of the constant, C, in the new equation

$$M = C\left[\frac{\theta^2 (dD)}{d-D}\right]$$

Problem 2 - The diameter of the gravitational lens ring in RCS2-032727-132623 is 38 arcseconds. What is the mass of the galaxy cluster?

Hubble Zooms in on a Magnified Galaxy
http://www.nasa.gov/mission_pages/hubble/science/magnified-galaxy.html
February 2, 2012

Problem 1 - For convenience, astronomers often re-write equations in the common units of astronomy where distances (d and D) are measured in light years and masses (M) are given in multiples of the sun's mass). If 1 light year = 9.5 x 10^{15} meters and Msun = 2.0 x 10^{30} kg and 1 radian = 2.0x10^{5} arcseconds, constant, C, in the new equation.

Answer: First insert the indicated units into the formula to get the mass, M, in kilograms. Note 1 arcsecond = 1/2.0x10^{5} = 5x10^{-6} radians so

$$M = \frac{(5.0x10^{-6})^2(3.0x10^8)^2(9.5x10^{15})^2}{4(6.67x10^{-11})(9.5x10^{15})} \quad = \quad 7.5 \text{ x } 10^{31}$$

Then convert M to solar mass units by dividing by 2.0x10^{30} to get C = 37.7

Problem 2 - The diameter of the gravitational lens ring in RCS2-032727-132623 is 38 arcseconds. What is the mass of the galaxy cluster?

Answer; From the text, d = 9.7 billion light years, D = 5.4 billion light years and θ = 38 arcseconds, so

M = (37.7) $(38)^2$ (9.7 billion)(5.4 billion) / (9.7-5.4) billion

M = **6.5 x 10^{14} solar masses**!

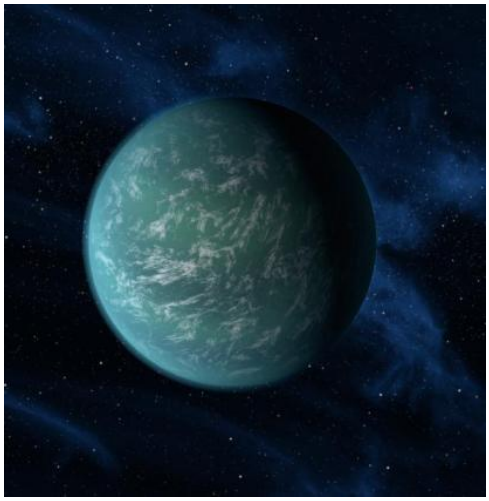
Note: Another independent way to measure the mass of the cluster is by using the formula for the speed of an object in a circular orbit:

$$V^2 = \frac{GM}{R}$$

The average speed of the galaxies in the cluster is V = 988 km/sec and the average radius, R, of the cluster is 9.5 million light years

So M = (9.5 x 10^{15})(9.5 x 10^{6})$(988{,}000)^2$ / (6.67x10^{-11})
M = 1.3 x 10^{45} kg

Or M = 6.6 x 10^{14} solar masses



An artists illustration of Kepler-22b (Credit NASA/Ames/JPL-Caltech)

NASA's Kepler mission has confirmed its first planet in the "habitable zone," the region where liquid water could exist on a planet's surface.

The newly confirmed planet, Kepler-22b, is the smallest yet found to orbit in the middle of the habitable zone of a star similar to our sun.

The planet is about 2.4 times the radius of Earth. Scientists don't yet know if Kepler-22b has a rocky, gaseous or liquid composition, but its discovery is a step closer to finding Earth-like planets.

Problem 1 - Suppose Kepler-22b is a spherical, rocky planet like Earth with an average density similar to Earth (about 5,500 kg/meter^3). If the radius of Kepler-22b is 15,000 km, what is the mass of Kepler-22b in A) kilograms? B) multiples of Earth's mass (5.97x1024 kg)?

Problem 2 - The acceleration of gravity on a planetary surface is given by the formula

$$a = \frac{GM}{R^2}$$

where M is in kilograms, R is in meters and G is the Newtonian Constant of Gravity with a value of 6.67 x 10^{-11} m^3 kg^{-1} sec^{-2} What is the surface acceleration of Kepler-22b A) In meters/sec^2? B) In multiples of Earth's surface gravity 9.8 meters/sec^2?

Problem 3 - The relationship between surface acceleration and your weight is a direct proportion. The surface acceleration of Earth is 9.8 meters/sec^2. If you weigh 150 pounds on the surface of Earth, how much will you weigh on the surface of Kepler-22b?

Problem 4 - The dimensions of a typical baseball park are determined by the farthest distance that an average batter can bat a home-run. This in turn depends on the acceleration of gravity, which is the force that pulls the ball back to the ground to shorten its travel distance. For a standard baseball field, the distance to the back-field fence from Home Plate may not be less than 325 feet, and the baseball diamond must be exactly 90 feet on a side.

A) If the maximum travel distance of the baseball scales linearly with the acceleration of gravity, what is the minimum distance to the back-field fence from Home Plate along one of the two foul lines?

B) What are the dimensions of the baseball diamond?

Problem 1 - Suppose Kepler-22b is a spherical, rocky planet like Earth with an average density similar to Earth (about 5,500 kg/meter^3). If the radius of Kepler-22b is 15,000 km, what is the mass of Kepler-22b in A) kilograms? B) multiples of Earth's mass (5.97×10^{24} kg)?

Answer: A) First find the volume of the spherical planet in cubic meters, then multiply by the density of the planet to get the total mass.

$R = 15{,}000 \text{ km} \times (1000 \text{ m}/1 \text{ km}) = 1.5 \times 10^7$ meters.

$V = 4/3\ \pi R^3$
$= 1.33 \times 3.14 \times (1.5 \times 10^7 \text{ meters})^3 = 1.41 \times 10^{22} \text{ meters}^3$

Then M = density x volume
$= 5{,}500 \text{ kg/m}^3 \times (1.41 \times 10^{22})$
$\mathbf{= 7.75 \times 10^{25}}$ **kg**

B) $M = 7.75 \times 10^{25}$ kg / 5.97×10^{24} kg = **12.9 Earths**.

Problem 2 - A) In meters/sec^2? B) In multiples of Earth's surface gravity 9.8 meters/sec^2?

Answer: A) $a = 6.67 \times 10^{-11} (7.75 \times 10^{25}) / (1.5 \times 10^7)^2$ = **23.0 meters/sec^2**
B) 23.0 / 9.8 = **2.3 times earth's surface gravity**

Note: From the formula for M and a, we see that the acceleration varies directly with the radius change, which is a factor of 2.4 times Earth, so a = 2.4xa(earth)

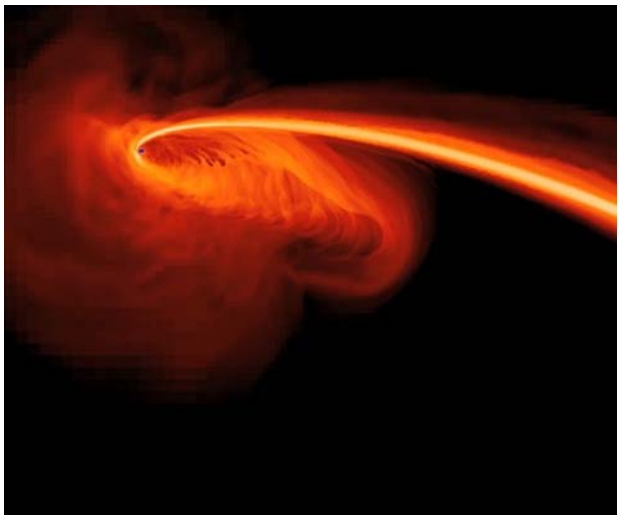
Problem 3 – The relationship between surface acceleration and your weight is a direct proportion. The surface acceleration of Earth is 9.8 meters/sec^2. If you weigh 150 pounds on the surface of Earth, how much will you weigh on the surface of Kepler-22b?

Answer: By a simple proportion: X/150 = 2.3/1.0 so x = 2.3x150 = **345 pounds**.

Problem 4 - For a standard baseball field, the distance to the back-field fence from Home Plate may not be less than 325 feet, and the baseball diamond must be exactly 90 feet on a side.

A) If the maximum travel distance of the baseball scales linearly with the acceleration of gravity, what is the distance to the back-field fence from Home Plate along one of the two foul lines? Answer: 325 / 2.3 = **141 feet**.

B) What are the dimensions of the baseball diamond? Answer: 90/2.3 = **39 feet** on a side.



Simulation of black hole and tidal shredding of a star. (Courtesy Gezari and Guillochon: NASA)

$$r = 2.44R\left(\frac{D}{d}\right)^{\frac{1}{3}} \qquad r = R\left(\frac{M}{m}\right)^{\frac{1}{3}}$$

The gravitational force between two objects varies as the inverse-square of the distance between them. On the surface of Earth, we do not notice that the gravitational force of Earth pulling on our feet is slightly larger than the force pulling on our head. For astronomical bodies, however, the difference in gravity can be so great that it pulls the body apart! This is called the tidal gravitational force.

Around every body, there is a distance called the tidal radius within which an object will be gravitationally torn apart if the body is being held together by its own gravitational forces. This distance can be calculated using the formulae to the right. Let's explore what this distance is for some common astronomical bodies.

Problem 1 – The Moon has a density of d = 3400 kg/m^3. The Earth has an average density of D = 5500 kg/m^3. If the radius, R, of Earth is 6378 km, how close to Earth would the moon have to get in order to be tidally disrupted? If the Moon is moving away from Earth at a speed of 3 cm/year, and its present distance is about 340,000 km, will it ever be tidally disrupted?

Problem 2 - Saturn has a density of D=687 kg/m^3, and the average density of its innermost satellite Pan is about d=400 kg/m^3. Saturn has a radius of R=120,000 km. If Pan's distance from Saturn is about 134,000 km, is it in any danger of being tidally disrupted? Can a satellite gravitationally assemble itself from a collection of gas and dust if the orbit of this material is inside the tidal limit?

Problem 3 - A red, supergiant star with a mass of m=20 times that of our sun, and a radius equal to the orbit of Earth (R=150 million km). If the mass of the black hole is M=10 times the mass of our sun, and its radius is 60 kilometers, what is the closest distance, r, that this star can come to the black hole before it is disrupted? Suppose, instead, that the black hole were like the one in the center of the Milky Way with a mass of M=3 million suns. What would be the tidal distance, r?

Problem 1 – The Moon has a density of 3400 kg/m^3. The Earth has an average density of 5500 kg/m^3. If the radius, R, of Earth is 6378 km, how close to Earth would the moon have to get in order to be tidally disrupted? If the Moon is moving away from Earth at a speed of 3 cm/year, and its present distance is about 340,000 km, will it ever be tidally disrupted?

Answer: From Equation 1, $r = 2.44\ (6378)(5500/3400)^{1/3}$ = **18,300 km**. Because the Moon is moving away from Earth and is already at a distance of 340,000 km, it will never be gravitationally disrupted by Earth to form a ring like Saturn's.

Problem 2 - Saturn has a density of 687 kg/m^3, and the average density of its innermost satellite Pan is about 400 kg/m^3. Saturn has a radius of 120,000 km. If Pan's distance from Saturn is about 134,000 km, is it in any danger of being tidally disrupted? Can a satellite assemble itself from a collection of gas and dust if the orbit of this material is inside the tidal limit?

Answer: From Equation 1, $r = 2.44 \times (120{,}000)\ (687/400)^{1/3}$ = **351,000 km**. This is larger than Pan's distance so Pan could be tidally disrupted, however, unlike our moon, it is not a body held together by its own gravity but instead is held together by the stronger forces between atoms and molecules, called tensile forces. A body cannot be gravitationally assembled from local materials if its formation is occurring inside the tidal radius of a nearby body.

Problem 3 - A red, supergiant star with a mass of 20 times that of our sun, and a radius equal to the orbit of Earth (150 million km). If the mass of the black hole is 10 times the mass of our sun, and its radius is 60 kilometers, what is the closest distance that this star can come to the black hole before it is disrupted? Suppose, instead, that the black hole were like the one in the center of the Milky Way with a mass of 3 million suns. What would be the tidal distance?

Answer: From Equation 2, we have m = 20 and M = 10 and R = 150 million km, so

$r = 150 \text{ million } (10/20)^{1/3}$

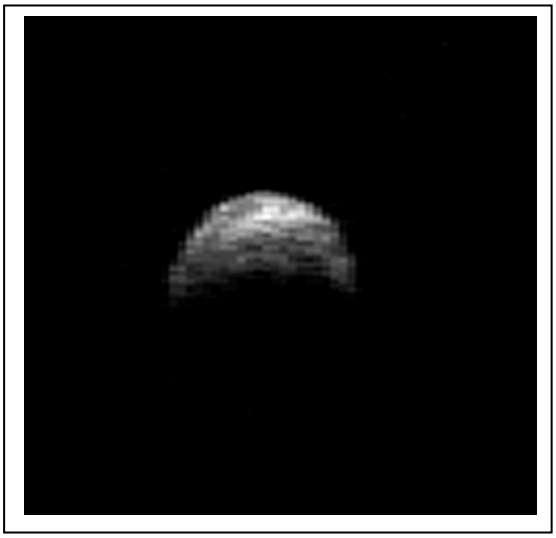
= **119 million kilometers**.

For a M=3 million solar mass black hole, the tidal distance would be

$R = 150 \text{ million } (3 \text{ million}/20)^{1/3}$

= **7.9 billion kilometers**, or just beyond the orbit of Pluto!

The Closest Approach of Asteroid 2005 YU55 - III



NASA scientists will be tracking asteroid 2005 YU55 with antennas of the agency's Deep Space Network at Goldstone, Calif., as the space rock safely flies past Earth slightly closer than the moon's orbit on Nov. 8.

Scientists are treating the flyby of the 1,300-foot-wide (400-meter) asteroid as a science target of opportunity – allowing instruments on "spacecraft Earth" to scan it during the close pass.

The image above was made at radio wavelengths by the Arecibo Radio Telescope in Puerto Rico in April, 2011. (Credit NASA/Cornell/Arecibo). The trajectory of asteroid 2005 YU55 is well understood.

On a standard Cartesian grid with Earth at the Origin, the Asteroid is predicted to be located at Point A (-247, -543) on November 8.438, and at Point B (+506, +1360 on November 9.438. The Moons orbit is represented by a circle with a radius of 346. All units are in thousands of kilometers so '346' is 346,000 kilometers.

Problem 1 - What is the slope-intercept form of the linear equation that represents the trajectory of the Asteroid between November 8 and November 9?

Problem 2 - What is the equation representing the orbit of the moon?

Problem 3 - What are the coordinates of the points that represent the intersection of the asteroid's trajectory and the lunar orbit?

Problem 4 - Over how many hours will the asteroid be inside the orbit of the moon if the asteroid was traveling at a speed of 41,700 km/hr?

Problem 5 - If the closest distance to Earth occurs when the asteroid is at the mid-point of its time inside the orbit of the Moon, A) about when does this happen and B) what is the distance to Earth at that time?

Problem 1 - Answer: Start with the two-point formula and simplify to the slop-intercept form.

$y - y_1 = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1)$ so $y + 543 = \frac{(136+543)}{(506+247)}(x+247)$ and so **y = 0.902x - 320**

Problem 2 - Answer: r = 346 so $x^2 + y^2 = (346)^2$

Problem 3 - Answer: Eliminate y in the equation for the lunar orbit by substituting the linear equation formula. Then solve the quadratic equation for the intersection points.

$x^2 + (0.902x - 320)^2 = (346)^2$ so $1.814x^2 - 577x - 17316 = 0$

use the quadratic formula with a = 1.814, b=-577 and c = -17316 to get:

$x = \frac{577 \pm \sqrt{(577^2 - 4(1.814)(-17316))}}{2(1.814)}$ so x = 159 +/- 187 x1 = +346 and x2 = -28

Find the corresponding y coordinates
Y1 = 0.902(346) - 320 = -8.0 so **First Point = (+346,-8.0)**
Y2 = 0.902(-28) - 320 = -345. so **Second Point = (-28, -345)**

Note, if you use the equation for the circle, you get y1 = 0.0 and y2 = -345. so **First Point (+346,0.0)** and **Second Point (-28, -345)**. The differences can be attributed to rounding error.

Problem 4 - Answer: Using the linear equation result, the distance between the First and Second Points is

$d = \sqrt{(-28-346)^2 + (-345+8)^2}$ so d = 503 units or 503,000 km. At a speed of 41,700 km/h, the asteroid travels this distance in about **12.1 hours**.

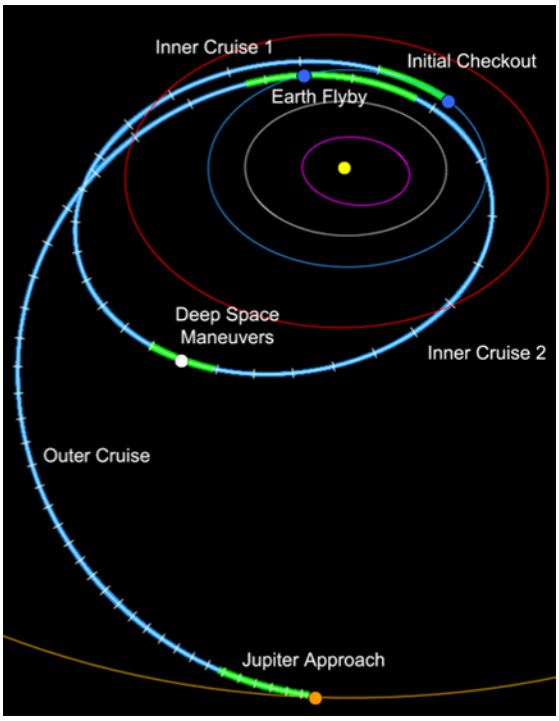
Problem 5 - If the closest distance to Earth occurs when the asteroid is at the mid-point of its time inside the orbit of the Moon, A) about when does this happen and B) what is the distance to Earth at that time?
Answer: T(mid) = (12.1)/2 = 6.05 hours.
Distance from Point A (-247, -543) to the Second Point at (-28, -345) is

$d = \sqrt{(-28+247)^2 + (-345+543)^2}$ so d = 295 or 295,000 km. At a speed of 41,700 km/h this time interval is just 7.07 hours. Adding the two times together we get 5.25+7.07 = 12.32 hours or 0.51 days after Point A. Since Point A occurs on November 8.438d we add 0.51d to this and get
A) **November 8.95 or November 8 at 22:48 Universal Time**.

B) The point that is half way between the
First point (+346,-8.0) and the Second point (-28, -345) is at
x=(346-28)/2= +159, and y = (-345 - 8)/2 = -177.
This point is located at a distance of $d = \sqrt{(159)^2 + (177)^2}$ = **238 units or 238,000 km from Earth**.

Note: The actual answer is 324,600 km when more accurate modeling is performed in 3-dimensions. The asteroid is off-set by an additional +221,000 km in the 'Z' direction.

Investigating Juno's Elliptical Transfer Orbit



The Juno spacecraft was initially placed in an elliptical orbit near Earth soon after its launch on August 5, 2011. The orbit was elliptical and designed so that a Deep Space Maneuver in August 2012 would send the spacecraft into a flyby of Earth in 2013. This encounter with Earth would boost the spacecraft's speed and place it into an elliptical transfer orbit that would intersect Jupiter's orbit in 2016.

This added speed would not require extra fuel by the spacecraft making it a free resource that keeps the cost of the mission small. These kinds of 'billiard shot' gravitational assists are commonly used by NASA to place spacecraft in trajectories to the outer solar system.

An approximate equation for the transfer orbit is given by the formula:
$5.15x^2 + 9.61y^2 = 49.49$. The units for x and y are given in terms of Astronomical Units where 1 AU = 150 million kilometers, which is the average orbit distance of Earth from the Sun.

Problem 1 - What is the equation of the orbit written in Standard Form for an ellipse?

Problem 2 – What is the semimajor axis length in AU?

Problem 3 – What is the semiminor axis length in AU?

Problem 4 – What is the distance between the focus of the ellipse and the center of the ellipse, defined by c?

Problem 5 - What is the eccentricity, e, of the orbit?

Problem 6 – What are the spacecraft's aphelion and perihelion distances?

Problem 7 – Kepler's Third Law states that the period, P, of a body in its orbit is given by $P = a^{3/2}$ where a is the semimajor axis distance in AU, and the period is given in years.If Juno spends ½ of its orbit to get to Jupiter after October, 2013 about when will it arrive at Jupiter?

Problem 1 - What is the equation of the orbit written in Standard Form for an ellipse?
Answer:
$5.15x^2 + 9.61y^2 = 49.49$ Divide both sides by 49.49 to get

$$\frac{x^2}{9.61} + \frac{y^2}{5.15} = 1$$

Problem 2 – What is the semimajor axis length in AU?
Answer: For an ellipse written in standard form: $x^2/a^2 + y^2/b^2 = 1$
Comparing with the equation from Problem 1 we get that the longest axis of the ellipse is along the x axis so the semimajor axis is $a^2 = 9.61$ so **a = 3.1 AU**

Problem 3 – What is the semiminor axis length in AU?
Answer: The semiminor axis is along the y axis so $b^2 = 85.15$ and **b = 2.3 AU**

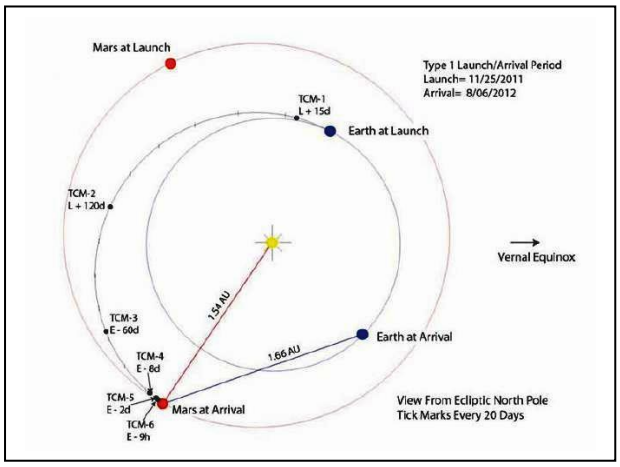
Problem 4 – What is the distance between the focus of the ellipse and the center of the ellipse, defined by c?
Answer: $c = (a^2 - b^2)^{1/2}$. With a= 3.1 and b=2.3 we have **c = 2.1**.

Problem 5 - What is the eccentricity, e, of the orbit?
Answer: The eccentricity e = c/a so e = 2.1/3.1 and so **e = 0.68**

Problem 6 – What are the spacecraft's aphelion and perihelion distances?
Answer: The closest distance to the focus along the orbit is given by a – c so the perihelion distance is 3.1 – 2.1 = **1.00 AU**. The farthest distance is a + c = 3.1 + 2.1 = **5.2 AU**. *Note the perihelion distance is at Earth's orbit and the aphelion distance is at Jupiter's orbit.*

Problem 7 – Kepler's Third Law states that the period, P, of a body in its orbit is given by $P = a^{3/2}$ where a is the semimajor axis distance in AU, and the period is given in years. If Juno spends ½ of its orbit to get to Jupiter after October, 2013 about when will it arrive at Jupiter?

Answer: Since a = 3.1 we have $P = 3.1^{3/2} = 5.5$ years for a full orbit. For Juno it spans 5.5/2 = 2.8 years in the elliptical transfer orbit, so it arrives at Jupiter in October 2013 + 2.8 years = October, 2013 + 2 years 8 months = **June, 2016**.



The Mars Science Laboratory was launched from Cape Canaveral on November 26, 2011 for a 251-day journey to Mars along an orbital path 567 million kilometers in length. The path is along a portion of an elliptical orbit called a Hohmann Transfer Orbit.

Hohmann Transfer Orbits require the least amount of fuel to transfer a spacecraft from Earth's orbit around the sun to the orbit of Mars.

A Hohmann Transfer Orbit to Mars has a perihelion distance of 1.0 Astronomical Units, and an aphelion distance equal to the distance of Mars from the sun at the time of interception, which for August 6, 2012 is 1.5 Astronomical Units. One Astronomical Units (1 AU) equals 149 million km. One focus of the transfer orbit is located on the sun, as are the elliptical orbits of all the other planets. The relationship between the aphelion and perihelion distances, A and P, and the semi-major axis, A, and eccentricity, e, of the corresponding ellipse is given by

$P = a - c \qquad A = a + c$

where a is the semi-major axis and also the semi-minor axis is $b = (a^2 - c^2)^{1/2}$

Problem 1 – For the desired transfer orbit, what is the equation of the required elliptical orbit in standard form?

Problem 2 – From the diagram above, where would Earth be in its orbit if the spacecraft could complete its original transfer orbit and attempt to return to Earth?

Problem 3 - To the proper number of significant figures, what is the average speed of the Mars Science Laboratory spacecraft in its journey to Mars A) in kilometers/hr? B) miles/hour?

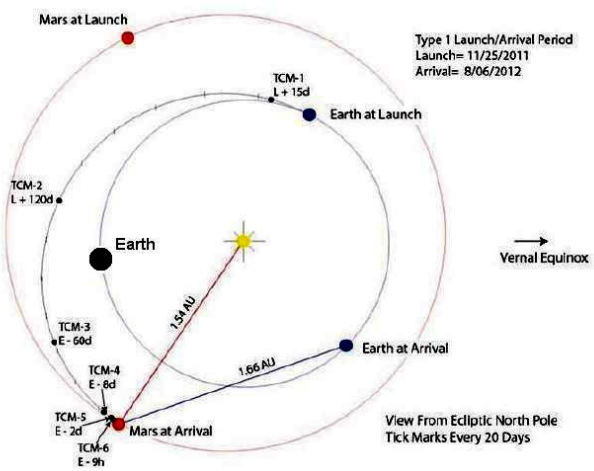
Problem 1 – For the desired transfer orbit, what is the equation of the required elliptical orbit in standard form?

Answer: The major axis has a total length of 1.0 + 1.0 + 0.5 = 2.5 AU, so a = 2.5/2 = 1.25 AU. Then from P = a – c and A = a + c we have 1.0 = 1.25 –c and 1.5 = 1.25 + c so that c = 0.25 AU. Then b = $(1.25^2 - 0.25^2)^{1/2}$ = 1.2 AU. Then from the standard form for an ellipse we get:

$$1 = \frac{x^2}{(1.25)^2} + \frac{y^2}{(1.2)^2}$$

Problem 2 – From the diagram above, where would Earth be in its orbit if the spacecraft could complete its original transfer orbit and attempt to return to Earth?

Answer: By symmetry, the dot on the orbit for Earth on August 6, 2012 would have to move an equal distance to its journey since November 25, 2011, which would place it near the indicated point in the diagram below:



Problem 3 - To the proper number of significant figures, what is the average speed of the Mars Science Laboratory spacecraft in its journey to Mars A) in kilometers/hr? B) miles/hour?

Answer: The 567 million km journey will take 251 days, so the average speed will be A) 567 million km/251 days = 2.26 million km/day or **94,200 km/hr** B) **58,400 miles/hr**. (since 1 km = 0.62 miles)

This spectacular night-time image taken by photographed Dominic Agostini shows the launch of the STEREO mission from Pad 17B at the Kennedy Space Center. The duration of the time-lapse image was 2.5 minutes. The distance to the horizon was 10 km, and the width of the image was 40 degrees. Assume that the trajectory is at the distance of the horizon, and in the plane of the photograph (the camera was exactly perpendicular to the plane of the launch trajectory).

Problem 1 – By using trigonometry formula, what is the horizontal scale of this image in meters/mm at the distance of the horizon?

Problem 2 - What was the altitude, in kilometers, of the rocket when it crossed the point directly in front of the camera's field of view at a position 2.4 km from the launchpad?

Problem 3 – From three measured points along the trajectory, what is the formula for the simplest parabola that can be fitted to this portion of the rocket's trajectory with the form $H(x) = ax^2 + bx + c$? (use kilometers for all measurements)

Problem 4 – Because of projection effects, although the rocket is continuing to gain altitude beyond the vertex of the parabola shown in the photograph, it appears as though the rocket reaches a maximum altitude and then starts to lose altitude as it travels further from the launch pad. About what is the apparent maximum altitude, in kilometers, that the rocket attains from the vantage point of the photographer?

More of Dominic's excellent night launch images can be found at www.dominicphoto.com

Problem 1 – By using trigonometry formula, what is the horizontal scale of this image in meters/mm at the distance of the horizon?
Answer: Solve the right-triangle to determine the hypotenuse h as h = 10 km/cos(20) = 11 km, then the width of the field of view at the horizon is just 2(11sin(20)) = 2(3.8 km) = 7.6 km. This is the width of the photograph along the distant horizon. With a millimeter ruler, the width of the mage is 125 mm, so the scale of this image at the horizon is about 7600m/125mm = **61 meters/mm**.

Problem 2 - What was the altitude, in kilometers, of the rocket when it crossed the point directly in front of the camera's field of view at a position 2.4 km from the launchpad? Answer: Draw a vertical line through the center of the image. Measure the length of this line between the distant horizon and the point on the trajectory. Typical values should be about 40 mm. Then from the scale of the image of 61 meters/mm, we get an altitude of H = 40 mm x 61 m/mm = **2440 meters or 2.4 kilometers**.

Problem 3 – From three measured points along the trajectory, what is the formula for the simplest parabola that can be fitted to this trajectory with the form $H(x) = ax^2 + bx + c$? (use kilometers for all measurements with the launch pad as the origin of the coordinates)

Answer: Because the launch pad is the origin of the coordinates, one of the two x-intercepts must be at (0,0) so the value for c = 0. We already know from Problem 2 that a second point is (2.4,2.4) so that $2.4 = a(2.4)^2 + b(2.4)$ and so the first equation for the solution is just
1 = 2.4a + b. We only need one additional point to solve for a and b. If we select a point at (1.2,1.5) we have a second equation $1.5 = a(1.2)^2 + 1.2b$ so that 1.5 = 1.4a + 1.2b. Our two equations to solve are now just

$$2.4\,a + 1.0\,b = 1.0$$
$$1.4\,a + 1.2\,b = 1.5$$

which we can solve by elimination of b to get a = -0.2 and then b = 1.48
so $\mathbf{H(x) = -0.2\,x^2 + 1.48\,x}$

Problem 4 – Because of projection effects, although the rocket is continuing to gain altitude beyond the vertex of the parabola shown in the photograph, it appears as though the rocket reaches a maximum altitude and then starts to lose altitude as it travels further from the launch pad. About what is the apparent maximum altitude, in kilometers, that the rocket attains from the vantage point of the photographer?

Answer: Students can use a ruler to determine this as about **2.7 kilometers**. Alternatively they can use the formula they derived in Problem 3 to get the vertex coordinates at
x = -b/2a
= +3.7 km and so h = **2.7 km**.

Useful Internet Resources

Space Math @ NASA

http://spacemath.gsfc.nasa.gov

A Math Refresher

http://istp.gsfc.nasa.gov/stargaze/Smath.htm

Developers Guide to Excelets

http://academic.pgcc.edu/~ssinex/excelets/

Interactive Science Simulations

http://phet.colorado.edu/en/simulations/category/physics

My Physics Lab

http://www.myphysicslab.com/

NASA Press Releases

http://www.nasa.gov/news/index.html

NCTM - Principles and Standards for School Mathematics

http://www.nctm.org/standards/content.aspx?id=16909

Practical Uses of Math and Science (PUMAS)

http://pumas.gsfc.nasa.gov

Teach Space Science

http://www.teachspacescience.org

A note from the Author:

July, 2012

Hi again!

Here is another collection of 'fun' problems based on NASA space missions across the solar system and the universe

This year has been quite suspenseful and filled with many new discoveries. We began the year with the launches of Juno to Jupiter and the Mars Science Laboratory to Mars; the Arctic polar icecap and ozone layer were seen to be in decline due to global warming; and the MESSENGER spacecraft continued its exploration of Mercury taking its 100,000 pictures of this planet's bizarre moon-like surface. In recent months, the Higgs boson has been discovered some 40 years after it was predicted; Pluto has gained a fifth moon, and Space X has made an historic delivery of cargo to the International Space Station.

We implicitly understand that all of these events required mathematics to make them possible. Without math, there would have been no way to connect theory and design, with the reality of numbers, quantity and measurement. Mathematics lets us 'weigh' a distant galaxy that has been around for 12 billion years, or figure out the mass of a sub-atomic particle that has lived barely a trillionth of a trillionth of a second.

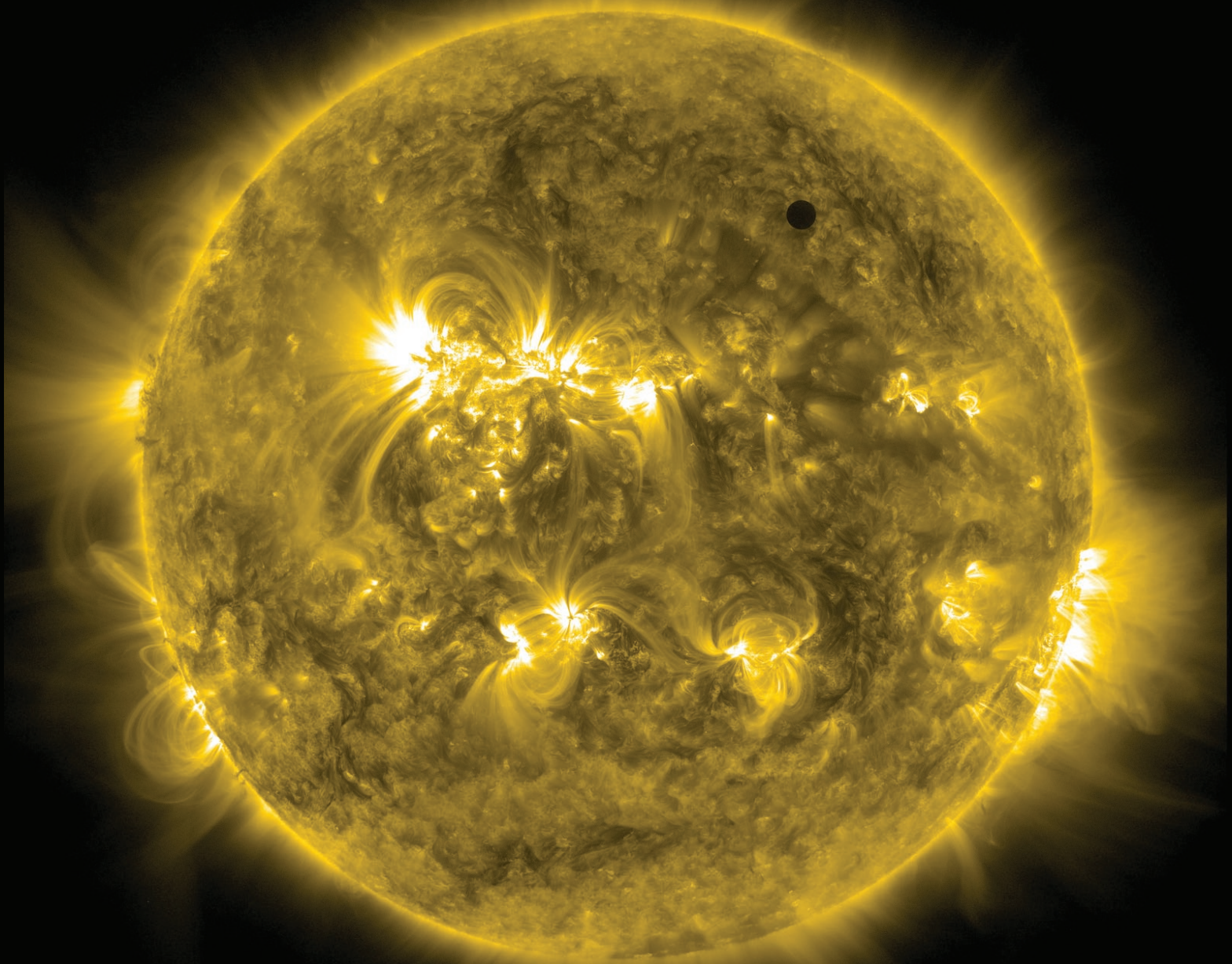
The 49 problems in this collection of SpaceMath@NASA problems from 2011-2012 span the gamut from fractions and percentages, to the challenges of algebraic manipulation. New challenges for young astronomers include working with fractions to figure out the sizes of moons and planets relative to each other. For more advanced students, algebraic challenges include calculating elliptical orbits, and weighing galaxies using gravity lenses and warped space.

As an astronomer, I remain amazed at how just a little bit of math can go such a long way in helping me understand something interesting about our world and beyond. A simple percentage tells me where the boundary is between objects that I know something about, and ones that have turned up in my survey for which I still do not understand.

The challenge, as always, is to make students see math as a partner to understanding the world, and not as an obfuscating adversary to be tolerated. The Rosetta Stone for understanding Nature's many mysteries and unknowns is as close as the right number set into the right equation. The equation becomes the artist's palate, and the paint brushes are but the various forms of mathematical tools (arithmetic, algebra, geometry, calculus) that we can employ in creating a scientific work of art. The more of these tools you have as an artist, the more subtle and spectacular will be your work of art, and the more insightful your understanding.

Sincerely,

Dr. Sten Odenwald
NASA Astronomer
sten.f.odenwald@nasa.gov



National Aeronautics and Space Administration

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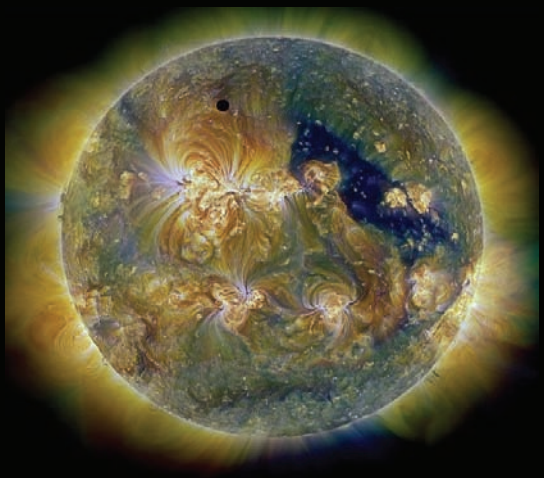
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