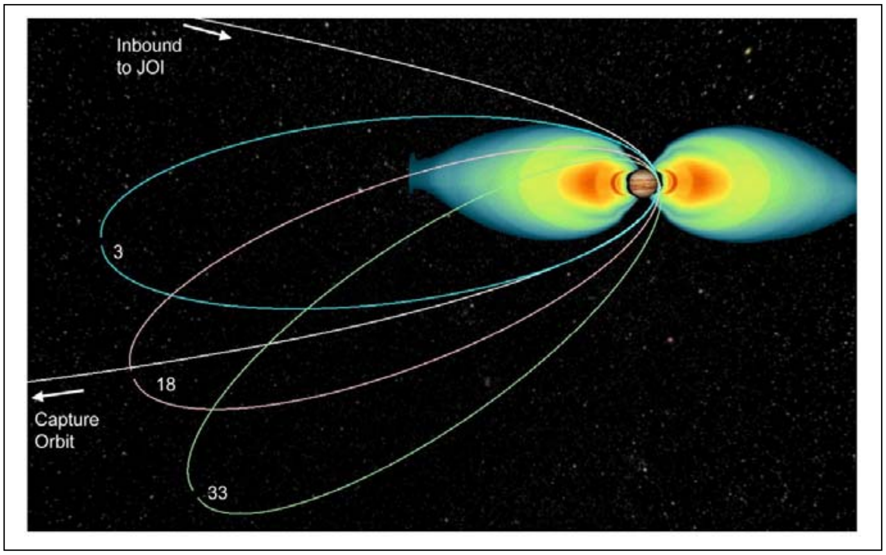
Juno Math: Exploring Jupiter's Gravitational Field



All objects in the universe produce a gravitational force, and in our solar system, the second largest source of gravity after the sun is the magnificent planet Jupiter!

Since Sir Isaac Newton first described the mathematics of gravity through his Law of Universal Gravitation and his detailed mathematical description of motion, astronomers have been able to use this theory to make some seemingly miraculous discoveries. For example, they can follow the minute changes in the way a satellite is accelerated as it orbits a planet to deduce how matter is distributed inside the planet. They can also determine the mass of the entire planet itself! This set of activities will show you how these marvels of astrophysics are performed using Jupiter as an example!

The Mass of a Planet.

In 1600, Johannes Kepler noticed from very careful studies of the planets, that they obeyed Three Golden Rules. First, they all orbited along paths that were similar to ellipses. Second, as they traveled along the orbit, they swept out equal areas in equal times. But the most exciting discovery was that there was a relationship between the square of the orbit period and the cube of the orbit distance; called Kepler's Third Law of Planetary Motion. It can be written as $T^2 = C \times A^3$ where T is the period in seconds, A is the distance (actually the semi-major axis of the elliptical orbit) in meters, and C is a constant.

Problem 1 - For Jupiter, the orbit period of Juno is 13.97 days and its semi-major axis is 1.39 million km. For the moon of Jupiter called Europa, its period is 3.6 days and A is 671,000 km. Show that the constants are the same to two significant figures.

Problem 2 - For Earth, the orbit period of the International Space Station is 92 minutes and its semi-major axis is 6,780 km. For the moon, its period is 27.5 days and A is 384,500 km. Show that the constants are the same to two significant figures.

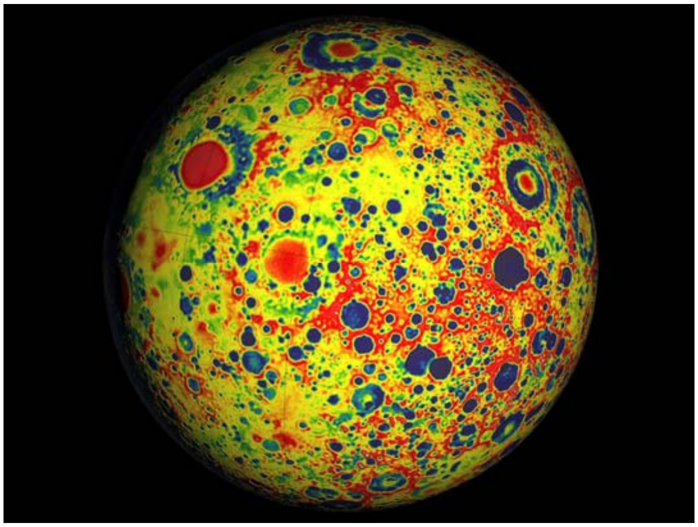
Problem 3 - For Earth, the orbit period is 365.24 days and its semi-major axis is 149 million km. For Pluto, its period is 90,560 days and A is 5.88 billion km. Show that the constants are the same to two significant figures.

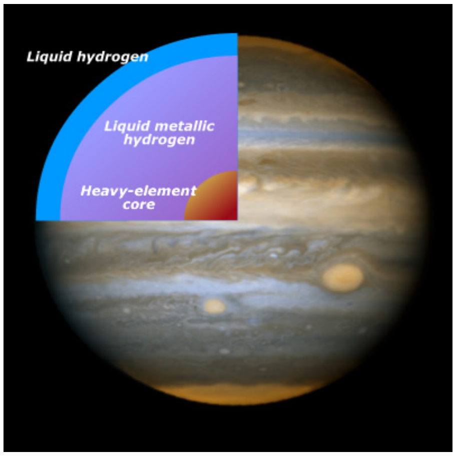
From the equations provided by Sir Isaac Newton, we can deduce that the constant term, C, discovered by Kepler can be derived from the mass of the object that the satellite orbits according to $M = 4\pi^2 C/G$, where G is the Universal Constant of Gravity given by $G = 6.67 \times 10^{-11}$ Newton meter^2/kg^2, and where the answer for M is in kilograms.

Problem 4 – From the values for C you determined for Jupiter, Earth and Sun, what are the respective masses of these objects in kilograms?

Modeling a Planet’s Mass Distribution

Because gravity is an attractive force, each of its pieces will pull on the orbit of a satellite causing its speed and altitude to change continuously. These slight changes can be measured, and from them we can figure out how matter is distributed inside a planet. For example, the twin Grail satellites have been tracked in their orbit around the moon for many years, and from the minute changes in satellite speed, altitude, and separation, a map of the irregularities in the lunar crust and interior has been created as shown here.





Although astronomers have accurate gravity maps for our Earth, Mars and Mercury, the best model of the interior of Jupiter today looks like this, and it is a very smooth-looking shape because it's only a first-guess based on very simple mathematical models.

What scientists really want to know is if there are any 'bumps and wiggles' in the interior shape and mass distribution. For example, if the Great Red Spot is from an impacted asteroid, can we detect the gravity of this asteroid mass under the clouds of Jupiter?

If a spacecraft went into a perfectly circular orbit around Jupiter at a fixed distance from the center of Jupiter, we can calculate how much gravity will be accelerating it downwards, and by how many kilometers it will appear to be falling as it orbited the planet. Because the planet has a perfectly uniform interior, this acceleration and falling will be exactly the same at each point along the spacecraft's orbit.

From Newton's Law of Gravity, the acceleration that a body will feel is given by $a = GM/R^2$ and the distance it will fall is $d = 1/2aT^2$ where d and R are measured in meters, M is the planet's mass in kilograms, T is the time in seconds, and G is the Universal Constant of Gravity $G = 6.67 \times 10^{-11}$ Newtons m^2/kg^2

Problem 5 – For Jupiter, $M = 1.9 \times 10^{27}$ kg, and R = 100,000 km, what will be the acceleration that the spacecraft feels in this orbit, and how far will the spacecraft fall every second as it orbits Jupiter?

Problem 6 – Suppose that near the core of Jupiter there was a mass irregularity of 1% the mass of Jupiter. If its distance from the spacecraft in Problem 5 were 90,000 km, what would be the gravitational acceleration that the spacecraft would feel from this mass irregularity alone?

Problem 7 – The effects of gravitational fields are additive. At the orbit of the spacecraft, what is a) the total acceleration felt by the spacecraft from the combination of the gravity of Jupiter and the gravity of the irregularity in Problem 6? and b) what is the total distance the spacecraft will fall in one second under the combined gravity?

By carefully measuring the orbit of the Juno spacecraft, irregularities in the way that mass is distributed inside Jupiter can be detected and mapped. Although 95% of the mass of Jupiter is in its massive hydrogen and helium atmosphere, computer models predict that it has a rocky core that contains 5% of the total mass of Jupiter. The Juno spacecraft should be able to detect irregularities in the mass of this rocky core from the slight changes in the spacecraft orbit and speed.

Answer Key

Problem 1 - Answer:
Juno: $C = (1{,}670{,}000{,}000)^3 / (13.97 \times 24 \times 3600)^2 = \mathbf{3.2 \times 10^{15}}$
Europa: $C = (671{,}000{,}000)^3 / (3.6 * 24 \times 3600)^2 = \mathbf{3.2 \times 10^{15}}$

Problem 2 - Answer:
ISS: $C = (6{,}780{,}000)^3 / (92 \times 60)^2 = \mathbf{1.0 \times 10^{13}}$
Moon: $C = (384{,}500{,}000)^3 / (27.5 \times 24 \times 3600)^2 = \mathbf{1.0 \times 10^{13}}$

Problem 3 - Answer:
Earth: $C = (149{,}000{,}000{,}000)^3 / (365.24 \times 24 \times 3600)^2 = \mathbf{3.3 \times 10^{18}}$
Pluto: $C = (5{,}880{,}000{,}000{,}000)^3 / (90560 \times 24 \times 3600)^2 = \mathbf{3.3 \times 10^{18}}$

Problem 4 – Answer: $M = 4\pi^2\, C/G$ and $G = 6.67 \times 10^{-11}$ so

Jupiter: $4 \times (3.14)^2\, (3.2 \times 10^{15})/(6.67 \times 10^{-11}) = \mathbf{1.9 \times 10^{27}\ kg}$.

Earth: $4 \times (3.14)^2\, (1.0 \times 10^{13})/(6.67 \times 10^{-11}) = \mathbf{5.9 \times 10^{24}\ kg}$.

Sun: $4 \times (3.14)^2\, (3.3 \times 10^{18})/(6.67 \times 10^{-11}) = \mathbf{1.9 \times 10^{30}\ kg}$.

Problem 5 – Answer:
Acceleration $= GM/R^2 = (6.67 \times 10^{-11})(1.9 \times 10^{27})/(100{,}000{,}000)^2 =$ **12.7 meters/sec^2**
Distance = ½ (12.7) (1 sec)^2 = **6.35 meters**.

Problem 6 – Answer: Acceleration $= GM/R^2 = (6.67 \times 10^{-11})(0.01 \times 1.9 \times 10^{27})/(90{,}000{,}000)^2 =$
0.15 meters/sec^2

Problem 7 –Answer: A) acceleration = 12.7 meters/sec^2 + 0.15 meters/sec^2 = **12.85 meters/sec^2**.
B) distance = $1/2aT^2$ = ½(12.85) (1.0)^2 = **6.43 meters.**

So, as the spacecraft travels over the irregularity, its orbit will appear to shift downwards by about (6.35-6.43) = - 0.08 meters from its normal orbit distance. This can be detected by carefully measuring the orbit of the Juno spacecraft.