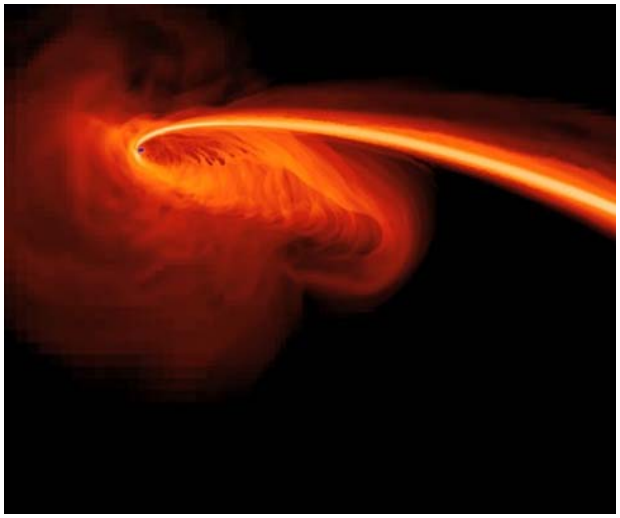
Simulation of black hole and tidal shredding of a star. (Courtesy Gezari and Guillochon: NASA)

$$r = 2.44R\left(\frac{D}{d}\right)^{\frac{1}{3}} \qquad r = R\left(\frac{M}{m}\right)^{\frac{1}{3}}$$

The gravitational force between two objects varies as the inverse-square of the distance between them. On the surface of Earth, we do not notice that the gravitational force of Earth pulling on our feet is slightly larger than the force pulling on our head. For astronomical bodies, however, the difference in gravity can be so great that it pulls the body apart! This is called the tidal gravitational force.

Around every body, there is a distance called the tidal radius within which an object will be gravitationally torn apart if the body is being held together by its own gravitational forces. This distance can be calculated using the formulae to the left. Let's explore what this distance is for some common astronomical bodies.

Problem 1 – The Moon has a density of d = 3400 kg/m^3. The Earth has an average density of D = 5500 kg/m^3. If the radius, R, of Earth is 6378 km, how close to Earth would the moon have to get in order to be tidally disrupted? If the Moon is moving away from Earth at a speed of 3 cm/year, and its present distance is about 340,000 km, will it ever be tidally disrupted?

Problem 2 - Saturn has a density of D=687 kg/m^3, and the average density of its innermost satellite Pan is about d=400 kg/m^3. Saturn has a radius of R=120,000 km. If Pan's distance from Saturn is about 134,000 km, is it in any danger of being tidally disrupted? Can a satellite gravitationally assemble itself from a collection of gas and dust if the orbit of this material is inside the tidal limit?

Problem 3 - A red, supergiant star with a mass of m=20 times that of our sun, and a radius equal to the orbit of Earth (R=150 million km). If the mass of the black hole is M=10 times the mass of our sun, and its radius is 60 kilometers, what is the closest distance, r, that this star can come to the black hole before it is disrupted? Suppose, instead, that the black hole were like the one in the center of the Milky Way with a mass of M=3 million suns. What would be the tidal distance, r?

Problem 1 – The Moon has a density of 3400 kg/m^3. The Earth has an average density of 5500 kg/m^3. If the radius, R, of Earth is 6378 km, how close to Earth would the moon have to get in order to be tidally disrupted? If the Moon is moving away from Earth at a speed of 3 cm/year, and its present distance is about 340,000 km, will it ever be tidally disrupted?

Answer: From Equation 1, $r = 2.44\ (6378)(5500/3400)^{1/3}$ = **18,300 km**. Because the Moon is moving away from Earth and is already at a distance of 340,000 km, it will never be gravitationally disrupted by Earth to form a ring like Saturn's.

Problem 2 - Saturn has a density of 687 kg/m^3, and the average density of its innermost satellite Pan is about 400 kg/m^3. Saturn has a radius of 120,000 km. If Pan's distance from Saturn is about 134,000 km, is it in any danger of being tidally disrupted? Can a satellite assemble itself from a collection of gas and dust if the orbit of this material is inside the tidal limit?

Answer: From Equation 1, $r = 2.44 \times (120{,}000)\ (687/400)^{1/3}$ = **351,000 km**. This is larger than Pan's distance so Pan could be tidally disrupted, however, unlike our moon, it is not a body held together by its own gravity but instead is held together by the stronger forces between atoms and molecules, called tensile forces. A body cannot be gravitationally assembled from local materials if its formation is occurring inside the tidal radius of a nearby body.

Problem 3 - A red, supergiant star with a mass of 20 times that of our sun, and a radius equal to the orbit of Earth (150 million km). If the mass of the black hole is 10 times the mass of our sun, and its radius is 60 kilometers, what is the closest distance that this star can come to the black hole before it is disrupted? Suppose, instead, that the black hole were like the one in the center of the Milky Way with a mass of 3 million suns. What would be the tidal distance?

Answer: From Equation 2, we have m = 20 and M = 10 and R = 150 million km, so
$r = 150 \text{ million } (10/20)^{1/3}$
= **119 million kilometers**.

For a M=3 million solar mass black hole, the tidal distance would be
$R = 150 \text{ million } (3 \text{ million}/20)^{1/3}$
= **7.9 billion kilometers**, or just beyond the orbit of Pluto!