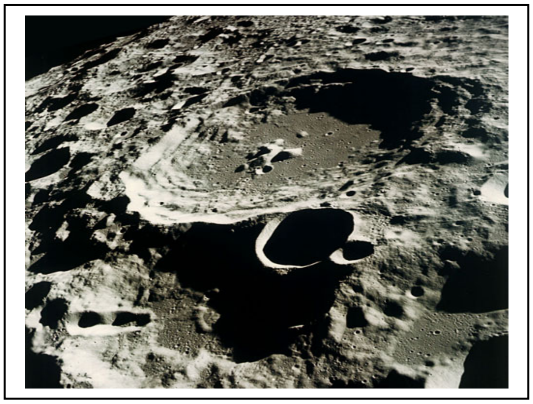
# Areas and Probabilities



There are many situations in astronomy where probability and area go hand in hand! The problems below can be modeled by using graph paper shaded to represent the cratered areas.

The moon's surface is heavily cratered, as the Apollo 11 photo to the left shows. The total area covered by them is more than 70% of the lunar surface!

1 – A 40km x 40km area of the Moon has 5 non-overlapping craters, each about 5km in radius. A) What fraction of this area is covered by craters? B) What is the percentage of the cratered area to the full area? C) Draw a square representing the surveyed region and shade the fraction covered by craters.

2 - During an 8-day period, 2 days were randomly taken off for vacation. A) What fraction of days are vacation days? B) What is the probability that Day-5 was a vacation day? C) Draw a square whose shaded area represents the fraction of vacation days.

3 – An asteroid capable of making a circular crater 40-km across impacts this same 40km x 40km area dead-center. About what is the probability that it will strike a crater that already exists in this region?

4 – During an 8-day period, 2 days were randomly taken of for vacation, however, during each 8-day period there were 4 consecutive days of rain that also happened randomly during this period of time. What is the probability that at least one of the rain days was a vacation day? (Hint: list all of the possible 8-day outcomes.)

**Inquiry** – How can you use your strategy in Problem 4 to answer the following question: An asteroid capable of making a circular crater 20-km across impacts this same 40km x 40km area dead-center. What is the probability that it will strike a crater that already exists in this region?

# Answer Key

1 – Answer: A) The area of a crater with a circular shape, is A = $\pi$ $(5km)^2$ = 78.5 $km^2$, so 5 non-overlapping craters have a total area of 393 $km^2$. The lunar area is 40 km x 40km = 1600 $km^2$, so the fraction of cratered area is 393/1600 **= 0.25.**

B) The percentage cratered is 0.25 x 100% = 25%.

C) Students will shade-in 25% of the squares.

2 - Answer: A) 2 days/8 days = 0.25

B) .25 x 100% = 25%.

C) The square should have 25% of area shaded.

3 –Answer: The area of the impact would be $\pi$ $(20km)^2$ = 1240 $km^2$ . The area of the full region is 1600 $km^2$. The difference in area is the amount of lunar surface not impacted and equals 360 $km^2$. Because the cratered area is 393 $km^2$ and is larger then the unimpacted area, the **probability is 100%** that at least some of the cratered area will be affected by the new crater.

4 – R = Rain days

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Vacation: There are 8 x 7/2 = **28 possibilities**

Rain 'area' = 50%. For each of the 5 possibilities for a rain period, there are 28 possibilities for a vacation series, which makes 5 x 28 = 140 combinations of rain and vacation. For a single rain pattern out of the 5, there are 28 vacation patterns, and of these, 50% will include at least one vacation day in the rain, because the other half of the days avoid the rain days entirely. So, out of the 140 combinations, 70 will include rain days and 70 will not, again reflecting the fact that the 'area' of the rain days is 50% of the total days.

**Inquiry:** The already cratered area is ¼ of the total. The area of the new impact is $\pi$ $(10km)^2$ = 314 $km^2$ which is 314/1600 = 1/5 of the total area. Draw a series of S=20 cells (like the 8-day pattern). The new crater represents 20x1/5 = 4 consecutive cells shaded. Work out all of the possible ways that 20 x 1/4 = 5 previously cratered areas can be distributed over the 20 days so that one of them falls within the 4 consecutive cells of the new crater. Example: X P X P X N N N N X X X X P X P X X P X
X = not cratered area, P = previously cratered area, N = new crater impact.

We are looking for: Probability = (The number of trials where a P is inside the 'N' region) / M , where M = the total number of possible trials. **Because there are 17 possibilities for the Ns, and 20!/ (15! 5!) = (20 x 19) / 2 = 190 possibilities for where the 5 Ps can go, m = 17 x 190 = 3230 . These do not all have to be worked out by hand to find the number of trials where a P is inside an N region in the series. You can also reduce the value of S so long as you keep the relative** areas between N, V and P the same.