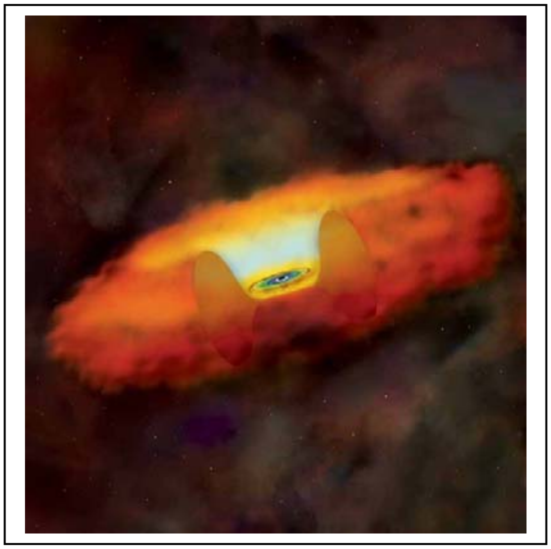
Black Hole Power...X



Black holes are sometimes surrounded by a disk of orbiting matter. This disk is very hot. As matter finally falls into the black hole from the inner edge of that disk, it releases about 7% of its rest-mass energy in the form of light. Some of this energy was already lost as the matter passed through, and heated up, the gases in the surrounding disk. But the over-all energy from the infalling matter is about 7% of its rest-mass in all forms (heat+ light).

The power produced by a black hole is phenomenal, with far more energy per gram being created than by ordinary nuclear fusion, which powers the sun.

Illustration of black hole accretion disk cut-away, showing the central black hole. Courtesy NASA/Chandra/ M.Weiss (CXC)

Problem 1 - The Event Horizon of a black hole has a radius of 2.93 M kilometers, where M is the mass of the black hole in multiples of the sun's mass. Assume the Event Horizon is a spherical surface, so its surface area is $S = 4 \pi R^2$. What is the surface area of A) a stellar black hole with a mass of 10 solar masses? B) a supermassive black hole with a mass of 100 million suns?

Problem 2 - What is the volume of a spherical shell with the surface area of the black holes in Problem 1, with a thickness of one centimeter?

Problem 3 - If the density of gas near the horizon is 10^{10} atoms/cc of hydrogen, how much matter is in each black hole shell, if the mass of a hydrogen atom is 1.6×10^{-24} grams?

Problem 4 - If $E = m c^2$ is the rest mass energy, E, in ergs, for a particle with a mass of m in grams, what is the rest mass energy equal to the masses in Problem 3 if $c = 3 \times 10^{10}$ cm/sec is the speed of light and only 7% of the mass produced energy?

Problem 5 - Suppose the material was traveling at 1/2 the speed of light as it crossed the horizon, how much time does it take to travel one centimeter if $c = 3 \times 10^{10}$ cm/sec is the speed of light?

Problem 6 - The power produced is equal to the energy in Problem 4, divided by the time in Problem 5. What is the percentage of power produced by each black hole compared to the sun's power of 3.8×10^{33} ergs/sec?

Answer Key:

Problem 1 - The Event Horizon of a black hole has a radius of 2.93 M kilometers, where M is the mass of the black hole in multiples of the sun's mass. Assume the Event Horizon is a spherical surface, so its surface area is $S = 4 \pi R^2$. What is the surface area of A0 a stellar black hole with a mass of 10 solar masses? B) a supermassive black hole with a mass of 100 million suns?

Answer; A) The radius is 2.93 x 10 = 29.3 kilometers. The surface area S = 4 x 3.14 x $(2.93 \times 10^6)^2 = 1.1 \times 10^{14}$ cm^2, B) 2.93×10^{13} cm S = 4 x 3.14 x $(2.93 \times 10^{13})^2 = 1.1 \times 10^{28}$ cm^2,

Problem 2 - What is the volume of a spherical shell with the surface area of the black holes in Problem 1, with a thickness of one centimeter?
Answer: Stellar black hole, V = Surface area x 1 cm = 1.1×10^{14} cm^3; Supermassive bklack hole, V = 1.1×10^{28} cm^3.

Problem 3 - If the density of gas near the horizon is 10^{10} atoms/cc of hydrogen, how much matter is in each black hole shell, if the mass of a hydrogen atom is 1.6 x 10-24 grams?

Answer - Stellar: M = $(1.1 \times 10^{14}$ cm^3) x $(1.0 \times 10^{10}$ atoms/cm^3) x $(1.6 \times 10^{-24}$ grams/atom = 1.76 grams. Supermassive: M = $(1.1 \times 10^{28}$ cm^3) x $(1.0 \times 10^{10}$ atoms/cm^3) x $(1.6 \times 10^{-24}$ grams/atom = 1.76×10^{14} grams.

Problem 4 - If $E = m c^2$ is the rest mass energy, E, in ergs, for a particle with a mass of m in grams, what is the rest mass energy equal to the masses in Problem 3 if c = 3×10^{10} cm/sec is the speed of light and only 7% of the mass produced energy?
Answer: Stellar: E = 0.07 x (1.76 grams x $(3 \times 10^{10})^2$) = 1.1×10^{20} ergs.;
Supermassive E = 0.07 x $(1.76 \times 10^{14}$ grams x $(3 \times 10^{10})^2$) = 1.1×10^{34} ergs

Problem 5 - Suppose the material was traveling at 1/2 the speed of light as it crossed the horizon, how much time does it take to travel one centimeter if c = 3×10^{10} cm/sec is the speed of light?
Answer; 1 cm / 3×10^{10} cm/sec = 3.3×10^{-11} seconds.

Problem 6 - The power produced is equal to the energy in Problem 4, divided by the time in Problem 5. What is the percentage of power produced by each black hole compared to the sun's power of 3.8×10^{33} ergs/sec?
Answer Stellar: 1.1×10^{20} ergs / 3.3×10^{-11} seconds = 3.3×10^{30} ergs/sec
Percent = 100% x $(3.3 \times 10^{30} / 3.8 \times 10^{33})$ = 0.08 %

Supermassive: 1.1×10^{34} ergs / 3.3×10^{-11} seconds = 3.3×10^{44} ergs/sec
= $(3.3 \times 10^{44} / 3.8 \times 10^{33})$ = 86 billion times the sun's power!