



The horizontal motion of a rock (projectile) is given by the formula:

$$X = V_h T$$

Independently, the vertical motion is given by the formula

$$Y = H_0 + V_v T - \frac{1}{2} g T^2$$

The speed of the projectile has been described in terms of its vertical ( $V_v$ ) and horizontal ( $V_h$ ) speeds so that the total speed is given by the Pythagorean Theorem  $S = (V_h^2 + V_v^2)^{1/2}$ .

**Problem 1** – A rock is tossed horizontally from the top of the Eiffel Tower at a speed of 60 mph (40 feet/sec). The Eiffel Tower stands 1,063 feet above the street. How far from the centerline of the tower does the rock land? ( $g = 32 \text{ feet/sec}^2$ )

**Problem 2** – On Mars ( $g = 12 \text{ feet/sec}^2$ ) an astronaut throws a rock up in the air so that its vertical speed is 30 feet/sec and its horizontal speed is 10 feet/sec. The rock starts at a shoulder height of 5 feet. How high does the rock travel, and how far from the astronaut does it finally reach the ground?

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Answer:  $H_0 = 1063$  feet,  $V_h = 40$  feet/sec,  $V_v = 0.0$ ,  $g = 32$  feet/sec<sup>2</sup>. The vertical equation give us the time to reach the ground ( $y=0$ ):  $0 = 1063 - 16 T^2$ , so  $1063/16 = T^2$  and  $T = 8.1$  seconds. From the horizontal motion, it travels  $d = 40 \times 8.1 = \mathbf{324}$  feet.

**Problem 2** – On Mars ( $g = 12$  feet/sec<sup>2</sup>) an astronaut throws a rock up in the air so that its vertical speed is 30 feet/sec and its horizontal speed is 10 feet/sec. The rock starts at a shoulder height of 5 feet. How high does the rock travel, and how far from the astronaut does it finally reach the ground?

Answer: The two equations are  $X = 10 T$  and  $Y = 5.0 + 30T - 6T^2$

Write Y in terms of X:  $Y = 5.0 + 30(X/10) - 6 (X/10)^2$  so  $Y = \mathbf{5.0 + 3.0X - 0.06X^2}$

Solve for the roots of  $Y(X) = -0.06X^2 + 3.0X + 5.0$  with coefficients  $a = -0.06$ ,  $b=3.0$  and  $c = 5.0$ , to get the ground intercept points using the Quadratic Formula:

$$X = [-3.0 \pm (9-4(-0.06)(5.0))^{1/2}] / (2x-0.06) \text{ so}$$

$$X = (-3 + 3.19)/-0.12 = -1.6 \text{ feet, and the second root is}$$

$$X = (-3 - 3.19)/-0.12 = +51.6 \text{ feet. see graph below.}$$

The peak of the parabola is  $\frac{1}{2}$  way between the x-intercepts at  $x = (51.6-1.6)/2 = +25.0$  feet  
 And since  $X = 10T$ , we have  $25 = 10T$  so  $T = 2.5$  seconds. From  $Y(T)$ , the altitude of the peak is  $Y = 5.0 + 30(2.5) - 6(2.5)^2 = \mathbf{42.5}$  feet. From the x-intercept, it reaches a distance of  $\mathbf{51.6}$  feet from the astronaut.

