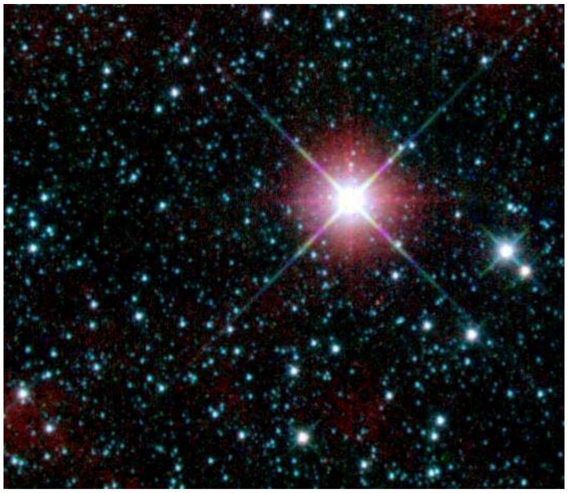


There are many situations in astrophysics when two distinct functions are multiplied together to form a new function.

If there are N light bulbs, each with a brightness of W watts, then the total brightness, T of all these bulbs is just N x W. For N=3 bulbs and W = 100 watts we have T = 300 watts.

Suppose N(m) tells us the number of stars in an area of the sky with a brightness of m. Let a second function, S(m), represent the number of watts per square meter at the Earth that a star with a brightness of m produces. Then N(m)S(m) will be the total number of watts/meter^2 produced by the stars in the sample that have a brightness of m.

NASA's, Wide-Field Infrared Survey Explorer (WISE) satellite is surveying the sky to catalog stars visible at a wavelength of 3.5 microns in the infrared spectrum. If the differential star count function $N(m)=0.000005\ m^{+7.0}$ stars, and the star brightness function is defined by $S(m)= 350\ 10^{-0.4m}$ Janskys. Use this information to answer the following problems:

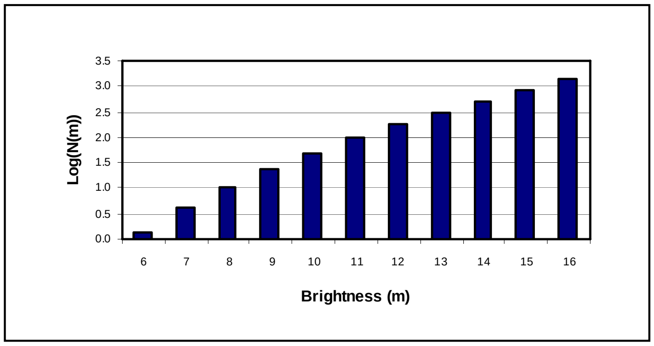
Problem 1 - Graph the functions Log(N(m)) and Log(S(m)) as individual histograms over the domain m:[+6, +16] for integer values of m.

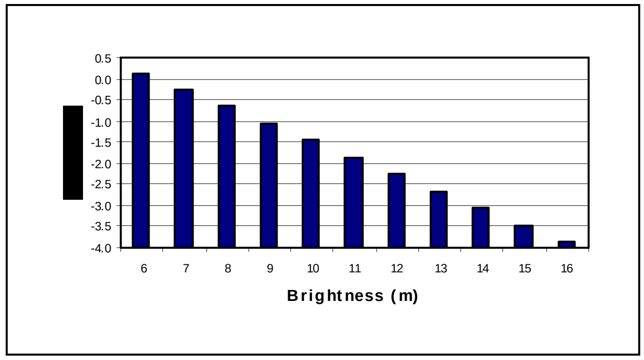
Problem 2 - Graph the product of these functions N(m)S(m) over the domain m:[+6, +16] for integer values of m.

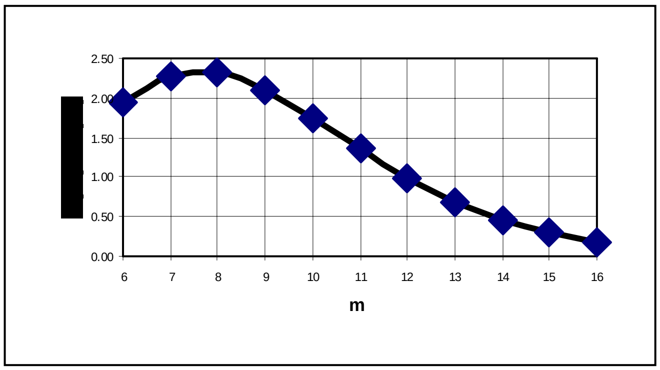
Problem 3 - What is the sum, T, of N(m)S(m) for each integer value of m in the domain m:[+6, +16], and how does this sum relate to the area under the curve for N(m)S(m)?

Problem 4 - What is the integral of N(m)S(m) from m=+6 to m= +16? You do not need to evaluate it!

Problem 1 and 2 - Answer: **See below.**







Problem 3 - Answer: The sum is T = N(6)S(6) + N(7)S(7) + … N(16)S(16)
T = 1.95 + 2.28 + 2.32 + 2.10 + 1.75 + 1.36 + 0.99 + 0.69 + 0.46 + 0.30 + 0.19
T = 14.39 Janskys. **This sum represents the approximate area under the curve N(m)S(m) vs m using vertical rectangles with a width of m= +1.0 and a height of N(m)S(m).**

Problem 4 - Answer:

$$T = \int_{6}^{16} (350)10^{-0.4m}(5x10^{-6})m^{+7}dm \quad \text{becomes}$$

$$T = 0.00175\int_{6}^{16} m^{7}e^{-(2.3)0.4m}dm$$

Changing variables to y=0.92x so dy = 0.92 dx we have

$$T = 0.0034\int_{5.5}^{14.7} y^{7}e^{-y}dy$$

This integral can be evaluated by approximation, as we did in Problem 3 using large rectangles with a base size of 1.0. Improved approximations can be created with base sizes of 1/2m, 1/4m, 1/8m…etc until a limit is reached for a desired degree of accuracy. Note: This integral can actually 'looked up' by advanced students, and its solution will be found to involve a recursive integral of $x^{m}e^{ax}$ where m=7 and a = -1. With computers, it is actually faster to evaluate it by successive approximation!