



During 2012, NASA's twin Grail satellites orbited the moon at altitudes of only 30 km. As they traveled, minute changes in their speeds tracked from Earth revealed changes in the gravitational field of the moon. These changes could be mapped, and revealed density changes in the lunar surface below them. In this way, scientists could look hundreds of kilometers beneath the lunar surface and explore how the surface was formed billions of years ago! On Earth, the acceleration of gravity is $9,807 \text{ cm/sec}^2$. The normal acceleration of gravity on the average lunar surface is 1620 cm/sec^2 , but in the blue regions of the map this is as low as 1520 cm/sec^2 , and in the red regions it is as high as 1920 cm/sec^2 . A pendulum clock has a swinging period, T in seconds, given by the formula $T = 2\pi\sqrt{\frac{L}{g}}$ where L is the length of the pendulum in centimeters, and g is the acceleration of gravity in cm/sec^2 .

Problem 1 - A lunar colony in a lunar 'blue' area has a Blue Clock with a pendulum length $L = 100 \text{ cm}$. What is the swing period? (use $\pi = 3.141$)

Problem 2 - A lunar colony in a lunar 'red' area has an identical Red Clock. What is the swing period? (use $\pi = 3.141$)

Problem 3 - After how many swings on the Blue Clock will the clocks differ in time by 1 hour?

Problem 4 - If both clocks were synchronized to 1:00:00 am local time, what will the time on the Blue Clock and the Red Clock be when the two colony clocks are off by 1 hour relative to each other?

Problem 1 - A lunar colony in a lunar 'blue' area has a Blue Clock with a pendulum length $L = 100$ cm. What is the swing period?

Answer: $T = 2 (3.141) (100/1520)^{1/2} = \mathbf{1.61 \text{ seconds}}$.

Problem 2 - A lunar colony in a lunar 'red' area has an identical Red Clock. What is the swing period?

Answer; $T = 2 (3.141) (100/1920)^{1/2} = \mathbf{1.43 \text{ seconds}}$.

Problem 3 - After how many swings on the Blue Clock will the clocks differ in time by 1 hour?

Answer: Each swing on the slower Blue Clock pendulum is behind the faster Red Clock by $1.61 - 1.43 = 0.18$ seconds. We want this difference to be 3600 seconds in 1 hour, which will take $N = 3600/0.18 = \mathbf{20,000 \text{ swings}}$ on the Blue Clock.

Problem 4 - If both clocks were synchronized to 1:00:00 am local time, what will the time on the Blue Clock and the Red Clock be when the two colony clocks are off by 1 hour relative to each other?

Answer: On the Blue Clock, 20,000 swings have to pass, each taking 1.61 seconds for a total time of 32,200 seconds or 8 hours, 56 minutes, 40 seconds. So the time on the Blue Clock will read **09:56:40 am local time**.

On the Red Clock, because after 20,000 swings it is exactly 1 hour behind the Blue Clock, its time will read 08:56:40 am local time. Another 'long way' to see this is that we still need 20,000 swings to add up to a 1 hour time difference, but on the Red Clock each swing is only 1.43 seconds long and so this takes 28,600 seconds or 7 hours, 56 minutes, 40 seconds. The time on the Red Clock will be **08:56:40 am local time**.

This is why colonists will NOT be using pendulum clocks on the moon!!

Note: Devices that act like pendulum clocks were once used by prospectors on Earth to search for oil and other valuable materials below ground before the advent of more accurate magnetometer-based technology. Minute changes in the pendulum period indicate changes in the density of rock below ground and these can be used to identify high-gravity, density regions (like iron ore) or low-gravity regions (like caverns). Another way to measure minute gravity changes is by the shape of a satellite orbit, or by the subtle changes in speed between two satellites on the same orbit. Lunar scientists used this orbit method with the two Grail spacecraft only 200 kilometers apart.