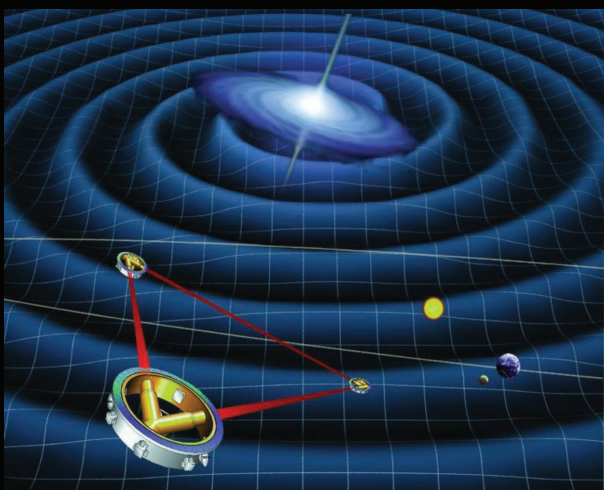
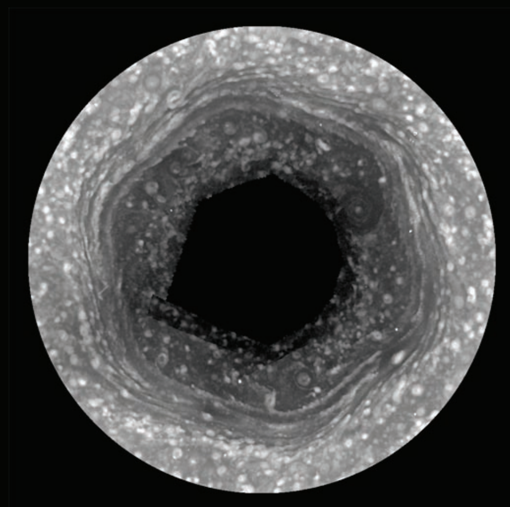
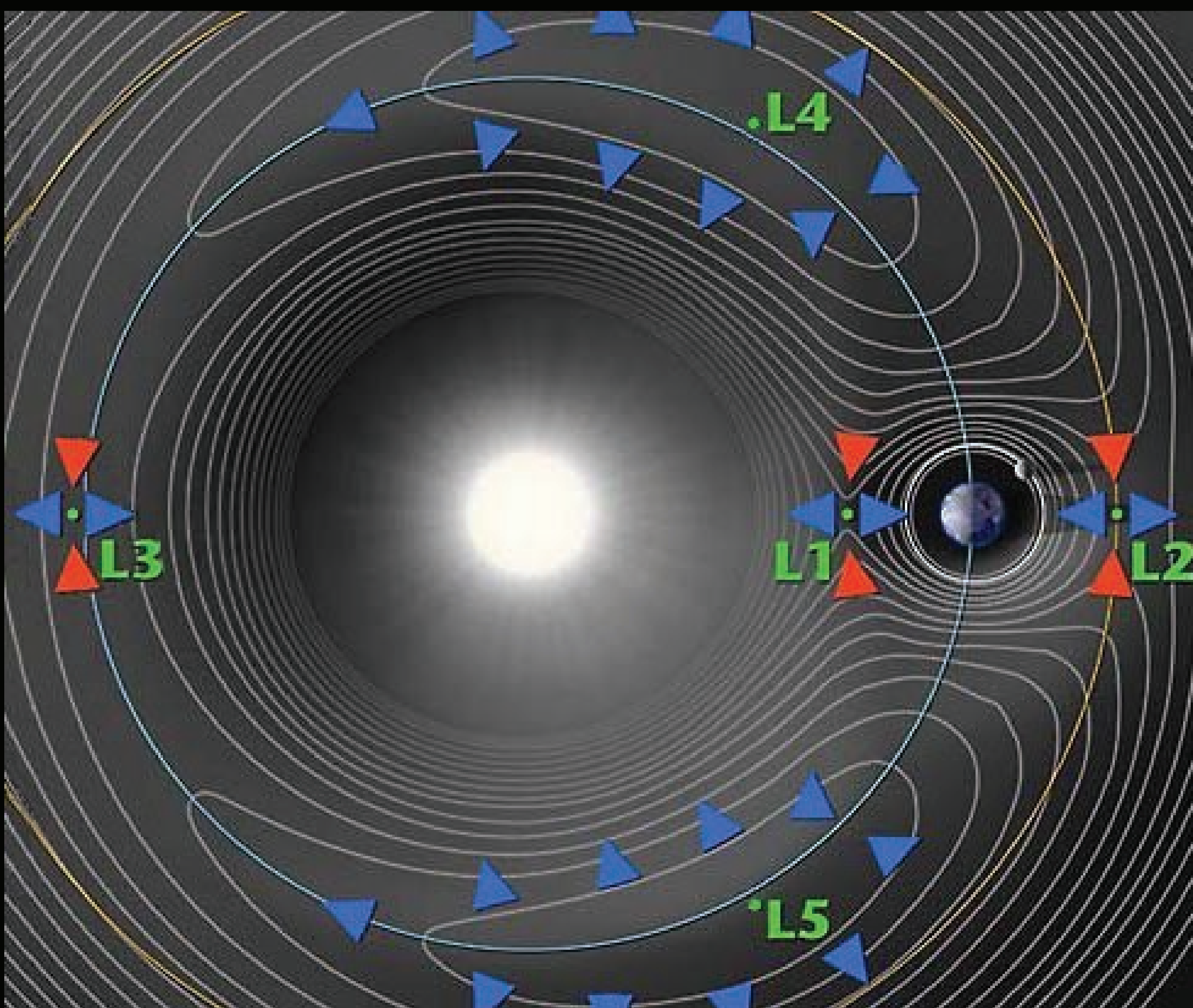
National Aeronautics and Space Administration

Space Math VI







This collection of activities is based on a weekly series of space science problems distributed to thousands of teachers during the 2009-2010 school year. They were intended for students looking for additional challenges in the math and physical science curriculum in grades 5 through 12. The problems were created to be authentic glimpses of modern science and engineering issues, often involving actual research data.

The problems were designed to be 'one-pagers' with a Teacher's Guide and Answer Key as a second page. This compact form was deemed very popular by participating teachers.

For more weekly classroom activities about astronomy and space visit the NASA website,

http://spacemath.gsfc.nasa.gov

Add your email address to our mailing list by contacting Dr. Sten Odenwald at

Sten.F.Odenwald@nasa.gov

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Mathematics Topic Matrix

Topic	Problem Numbers																													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
Inquiry																															
Technology, rulers						X	X					X	X			X	X				X			X	X						
Numbers, patterns, percentages	X	X	X	X	X			X		X					X			X		X		X		X			X				
Averages										X					X																
Time, distance,speed						X	X										X				X							X			
Areas and volumes															X									X							
Scale drawings						X	X					X	X			X	X	X			X				X	X					
Geometry																		X		X		X				X		X			
Scientific Notation						X										X					X										
Unit Conversions									X					X																	
Fractions																														X	
Graph or Table Analysis										X	X								X				X								
Solving for X																														X	X
Evaluating Fns																													X		
Modeling																			X								X	X		X	
Probability					X																	X									
Rates/Slopes																			X												
Logarithmic Fns																															X
Polynomials																															
Power Fns																															
Conics																															
Piecewise Fns																															
Trigonometry																										X					
Integration																															
Differentiation																															
Limits																															

Mathematics Topic Matrix (cont'd)

Topic	Problem Numbers																													
	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62
Inquiry																															
Technology, rulers			X								X																				
Numbers, patterns, percentages																															
Averages																															
Time, distance,speed																															
Areas and volumes				X	X			X			X		X											X		X	X	X			
Scale drawings			X								X																				
Geometry	X		X	X	X			X			X	X											X	X		X	X	X			X
Scientific Notation							X	X					X		X	X	X	X			X					X	X	X	X	X	
Unit Conversions				X	X		X						X	X	X	X	X									X	X	X			
Fractions																															
Graph or Table Analysis		X				X				X												X									
Solving for X	X						X												X	X	X	X			X	X	X	X	X	X	
Evaluating Fns						X			X									X		X	X	X			X	X	X	X	X	X	X
Modeling						X				X									X		X		X	X	X	X	X	X			
Probability																															
Rates/Slopes										X															X						
Logarithmic Fns																															
Polynomials																				X									X	X	X
Power Fns																					X										
Exponential Fns																															
Conics																							X								
Piecewise Fns																															
Trigonometry	X											X																			
Integration																										X	X	X			
Differentiation																															
Limits																															

Mathematics Topic Matrix (cont'd)

Topic	Problem Numbers																													
	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93
Inquiry																															
Technology, rulers								X	X	X	X	X		X		X	X	X	X		X	X	X								
Numbers, patterns, percentages													X		X																
Averages																															
Time, distance,speed														X			X				X						X	X	X		
Areas and volumes							X					X				X		X	X		X	X	X			X					
Scale drawings								X	X	X	X	X	X	X		X	X	X	X		X	X	X			X					
Geometry					X	X	X		X							X	X	X	X		X	X	X			X					
Scientific Notation	X														X				X		X		X	X	X	X					X
Unit Conversions								X	X	X	X	X				X	X	X	X	X	X	X	X	X	X					X	
Fractions																															
Graph or Table Analysis																										X					
Solving for X	X	X	X	X		X																		X			X	X	X	X	X
Evaluating Fns	X	X	X	X		X																		X			X	X	X	X	X
Modeling	X	X	X	X		X	X					X	X									X	X			X	X	X	X	X	X
Probability																															
Rates/Slopes																															
Logarithmic Fns		X																													
Polynomials	X																														
Power Fns			X																												
Exponential Fns																															
Conics																															
Piecewise Fns																															
Trigonometry					X																										
Integration			X		X																										
Differentiation				X	X	X																									
Limits				X																											

Mathematics Topic Matrix (cont'd)

Topic	Problem Numbers																														
	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110														
Inquiry																															
Technology, rulers						X			X								X														
Numbers, patterns, percentages										X																					
Averages																															
Time, distance,speed						X																									
Areas and volumes		X	X	X					X		X	X	X			X	X														
Scale drawings					X	X			X								X														
Geometry			X	X	X				X		X						X														
Scientific Notation	X	X	X	X		X	X		X	X	X	X		X		X															
Unit Conversions	X	X	X	X		X	X		X																						
Fractions																															
Graph or Table Analysis													X		X																
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Evaluating Fns	X					X	X	X				X			X																
Modeling	X	X	X	X		X	X	X							X	X	X														
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Integration													X	X		X															
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Limits																															

How to use this book

Teachers continue to look for ways to make math meaningful by providing students with problems and examples demonstrating its applications in everyday life. Space Math offers math applications through one of the strongest motivators-Space. Technology makes it possible for students to *experience* the value of math, instead of just reading about it. Technology is essential to mathematics and science for such purposes as "access to outer space and other remote locations, sample collection and treatment, measurement, data collection and storage, computation, and communication of information." 3A/M2 authentic assessment tools and examples. The NCTM standards include the statement that "Similarity also can be related to such real-world contexts as photographs, models, projections of pictures" which can be an excellent application for all of the Space Math applications.

This book is designed to be used as a supplement for teaching mathematical topics. The problems can be used to enhance understanding of the mathematical concept, or as a good assessment of student mastery.

An integrated classroom technique provides a challenge in math and science classrooms, through a more intricate method for using **Space Math VI**. Read the scenario that follows:

Ms. Green decided to pose a new activity using Space Math for her students. She challenged each student team with math problems from the Space Math VI book. She copied each problem for student teams to work on. She decided to have the students develop a factious space craft. Each team was to develop a set of criteria that included reasons for the research, timeline and budget. The student teams had to present their findings and compete for the necessary funding for their space craft. The students were to use the facts available in the Space Math VI book and images taken from the Space Weather Media Viewer, http://sunearth.gsfc.nasa.gov/spaceweather/FlexApp/bin-debug/index.html#

Space Math VI can be used as a classroom challenge activity, assessment tool, enrichment activity or in a more dynamic method as is explained in the above scenario. It is completely up to the teacher, their preference and allotted time. What it does provide, regardless of how it is used in the classroom, is the need to be proficient in math. It is needed especially in our world of advancing technology and physical science.

Alignment with Standards

AAAS: Project:2061 Benchmarks

(3-5) - Quantities and shapes can be used to describe objects and events in the world around us. 2C/E1 --- Mathematics is the study of quantity and shape and is useful for describing events and solving practical problems. 2A/E1 **(6-8)** Mathematicians often represent things with abstract ideas, such as numbers or perfectly straight lines, and then work with those ideas alone. The "things" from which they abstract can be ideas themselves; for example, a proposition about "all equal-sided triangles" or "all odd numbers". 2C/M1 **(9-12)** - Mathematical modeling aids in technological design by simulating how a proposed system might behave. 2B/H1 ---- Mathematics provides a precise language to describe objects and events and the relationships among them. In addition, mathematics provides tools for solving problems, analyzing data, and making logical arguments. 2B/H3 ----- Much of the work of mathematicians involves a modeling cycle, consisting of three steps: (1) using abstractions to represent things or ideas, (2) manipulating the abstractions according to some logical rules, and (3) checking how well the results match the original things or ideas. The actual thinking need not follow this order. 2C/H2

NCTM: Principles and Standards for School Mathematics

Grades 6–8 :

- work flexibly with fractions, decimals, and percents to solve problems;
- understand and use ratios and proportions to represent quantitative relationships;
- develop an understanding of large numbers and recognize and appropriately use exponential, scientific, and calculator notation; .
- understand the meaning and effects of arithmetic operations with fractions, decimals, and integers;
- develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios.
- represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules;
- model and solve contextualized problems using various representations, such as graphs, tables, and equations.
- use graphs to analyze the nature of changes in quantities in linear relationships.
- understand both metric and customary systems of measurement;
- understand relationships among units and convert from one unit to another within the same system.

Grades 9–12 :

- judge the reasonableness of numerical computations and their results.
- generalize patterns using explicitly defined and recursively defined functions;
- analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior;
- understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions;
- draw reasonable conclusions about a situation being modeled.

Teacher Comments

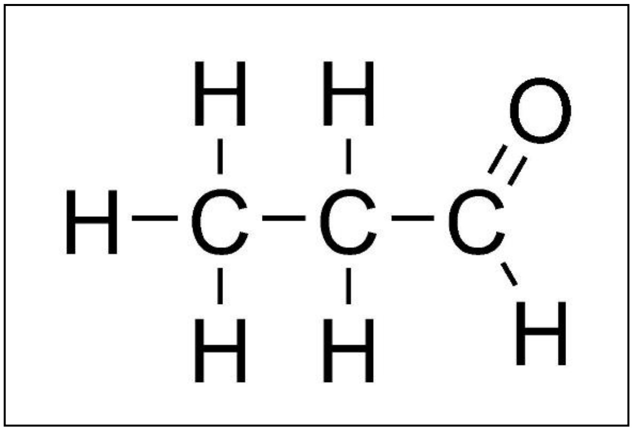
"Your problems are great fillers as well as sources of interesting questions. I have even given one or two of your problems on a test! You certainly have made the problems a valuable resource!" (Chugiak High School, Alaska)

"I love your problems, and thanks so much for offering them! I have used them for two years, and not only do I love the images, but the content and level of questioning is so appropriate for my high school students, they love it too. I have shared them with our math and science teachers, and they have told me that their students like how they apply what is being taught in their classes to real problems that professionals work on." (Wade Hampton High School ,SC)

"I recently found the Space Math problems website and I must tell you it is wonderful! I teach 8th grade science and this is a blessed resource for me. We do a lot of math and I love how you have taken real information and created reinforcing problems with them. I have shared the website with many of my middle and high school colleagues and we are all so excited. The skills summary allows any of us to skim the listing and know exactly what would work for our classes and what will not. I cannot thank you enough. I know that the science teachers I work with and I love the graphing and conversion questions. The "Are U Nuts" conversion worksheet was wonderful! One student told me that it took doing that activity (using the unusual units) for her to finally understand the conversion process completely. Thank you!" (Saint Mary's Hall MS, Texas)

"I know I'm not your usual clientele with the Space Math problems but I actually use them in a number of my physics classes. I get ideas for real-world problems from these in intro physics classes and in my astrophysics classes. I may take what you have and add calculus or whatever other complications happen, and then they see something other than "Consider a particle of mass 'm' and speed 'v' that…" (Associate Professor of Physics)

Counting Atoms in a Molecule



The complex molecule Propanal was discovered in a dense interstellar cloud called Sagittarius B2(North) located near the center of the Milky Way galaxy about 26,000 light years from Earth. Astronomers used the giant radio telescope in Greenbank, West Virginia to detect the faint signals from a massive cloud containing this molecule. It is one of the most complex molecules detected in the 35 years that astronomers have searched for molecules in space. Over 140 different chemicals are now known.

Problem 1 - How many atoms of hydrogen (H), carbon (C) and oxygen (O) are contained in this molecule?

Problem 2 - What percentage of all atoms are hydrogen? Carbon? Oxygen?

Problem 3 - What is the ratio of carbon atoms to hydrogen atoms in propanal?

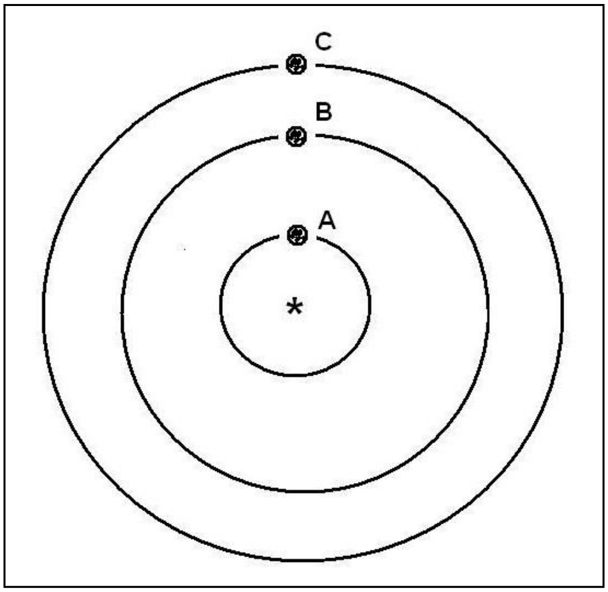
Problem 4 - If the mass of a hydrogen atom is defined as 1 AMU, and carbon and oxygen have masses of 12.0 and 16.0 AMUs, what is the total mass of a propanal molecule in AMUs?

Problem 5 - What is the complete chemical formula for propanal?

$$C_3\ H\_\_\ O\_\_$$

Problem 6 - If this molecule could be broken up, how many water molecules could it make if the formula for water is H_2O?

Planetary Conjunctions



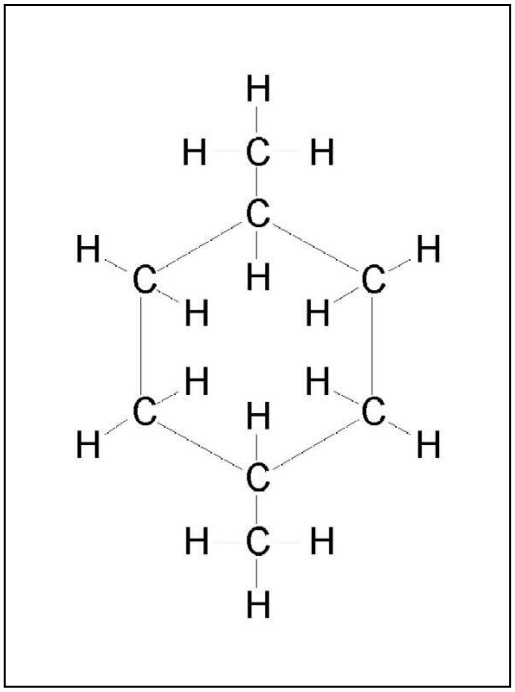
Since 1995, astronomers have detected over 350 planets orbiting distant stars. Our solar system has 8 planets, and for thousands of years astronomers have studied their motions. The most interesting events happen when planets are seen close together in the sky. These are called conjunctions, or less accurately, alignments.

The figure shows a simple 3-planet solar system with the planets starting out 'lined up' with their star. Each planet revolves around the star at a different pace, so it is a challenge to predict when they will all line up again.

Problem 1 - An astronomer detects three planets, A, B, C, that orbit their star once every 1, 2 and 4 years in a clockwise direction. Using the diagram above, draw a series of new diagrams that show where will the planets be in their orbits after: A) 1 year? B) 2 years? C) 3 years? D) 4 years?

Problem 2 - Suppose the three planets, A, B and C, orbit their star once every 2 years, 3 years and 12 years. A) How long would it take for all three planets to line up again? B) Where would the planets be after 6 years?

Parts Per Hundred (pph)



A common way of describing the various components of a population of objects is by the number of parts that they represent for every 100, 1000 or 1 million items that are sampled.

For instance, if there was a bag of 100 balls: 5 were red and 95 were white, you would say that the red balls represented 5 parts per hundred (5 pph) of the sample.

This also means that if you had a bag with 300 balls in the same proportions of red and white balls, the red balls would still be '5 parts per hundred' even though there are now 15 red balls in the sample (15/300 = 5/100 = 5 pph).

For each of the situations below, calculate the parts-per-hundred (pph) for each sample.

Problem 1 - Your current age compared to 1 century

Problem 2 - 10 cubic centimeter (10 cc) of food coloring blended into 1 liter (1000 cc) of water.

Problem 3 - The 4 brightest stars in the Pleiades star cluster, compared to the total population of the cluster consisting of 200 stars.

Problem 4 - One day compared to one month (30 days)

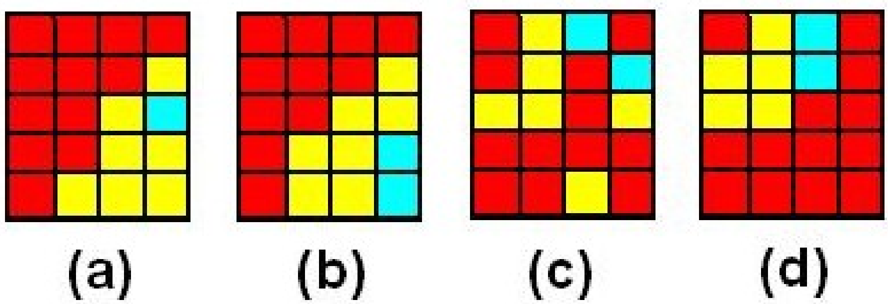
Problem 5 - Five percent of anything.

Problem 6 - The figure above shows the atoms of hydrogen (H) and carbon (C) in the molecule of dimethylcyclohexane. What is the pph of the carbon atoms in this molecule?

Star Cluster Math

Astronomers classify stars so that they can study their similarities and differences. A very common way to classify stars is by their temperature. This scale assigns a letter from the set [O, B, A, F, G, K, M] to represent stars with temperatures from 30,000 C (O-type) and 6,000 C (G-type), to 3,000 C (M-type).

Problem 1 – An astronomer studies a sample of stars in a cluster and identifies 6 as G-type like our Sun, 12 as M-type like Antares, and 2 stars as O-type like Rigel. Circle the pattern below, a, b, c or d, that graphically represents this information.



Problem 2 – What fraction of the stars in the sample are G-type?

A) 6/9 B) 20/6 C) 6/20 D) 6/8

Problem 3 – What fraction of the G and M-type stars in the cluster are G-type?

A) 12/18 B) 6/12 C) 12/6 D) 6/18

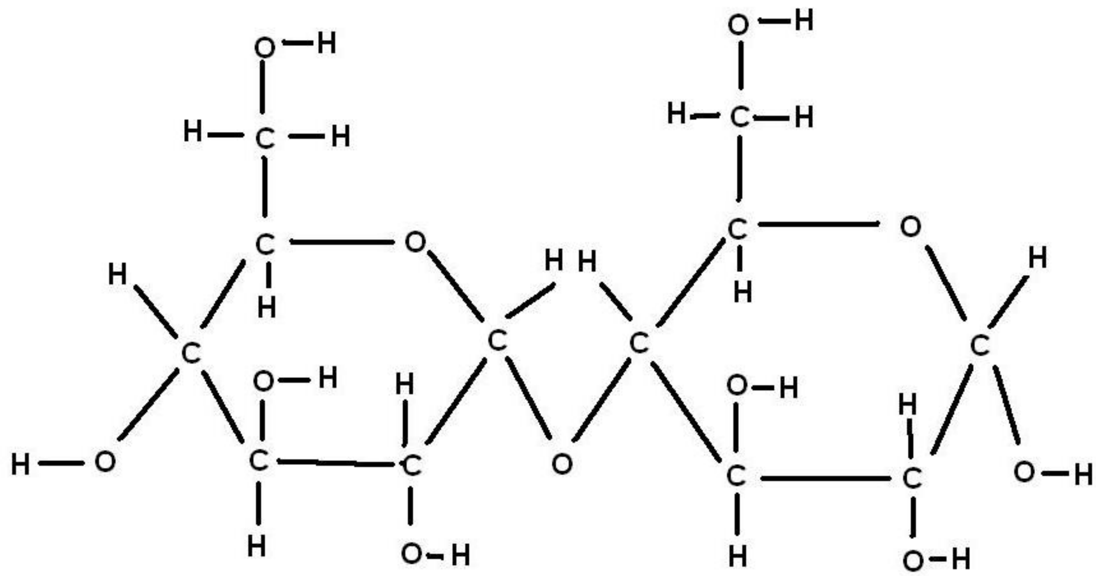
Problem 4 – If you selected 2 stars randomly from this cluster, which calculation would give the probability that these would both be O-type stars?

A) 1/20 x 1/20 B) 2/20 x 1/20 C) 1/20 x 1/19 D) 2/20 x 1/19

Problem 5 – A second star cluster has a total of 2,040 stars. If the proportion of O, G and M-types stars is the same as in the first cluster, how many G-type stars would be present?

A) 612 B) 340 C) 1428 D) 680

Atoms - How sweet they are!



Glucose is a very important sugar used by all plants and animals as a source of energy. Maltose is the next most complicated sugar, and is formed from two glucose molecules. The atomic ingredients of the maltose molecule is shown in the diagram above, which is called the structural formula for maltose. As an organic compound, it consists of three types of atoms: hydrogen (H), carbon (C), and oxygen (O).

Problem 1 - How many molecules does maltose contain of A) hydrogen? B) oxygen? C) carbon?

Problem 2 - What is the ratio of the number of hydrogen atoms to oxygen atoms?

Problem 3 - What fraction of all the maltose atoms are carbon?

Problem 4 - If the mass of 1 hydrogen atoms is 1 AMU, and 1 carbon atom is 12 AMU and 1 oxygen atom is 16 AMU, what is the total mass of one maltose molecule in AMUs?

Problem 5 - Write the chemical formula of maltose by filling in the missing blanks:

$C\_\_ \quad H\_\_ \quad O_{11}$

The Hand of Chandra!

A small, dense object only twelve miles in diameter is responsible for this beautiful X-ray nebula that spans 150 light years and resembles a human hand!

At the center of this image made by NASA's Chandra X-ray Observatory is a very young and powerful pulsar known as PSR B1509-58.

The pulsar is a rapidly spinning neutron star which is spewing energy out into the space around it to create complex and intriguing structures, including one that resembles a large cosmic hand.

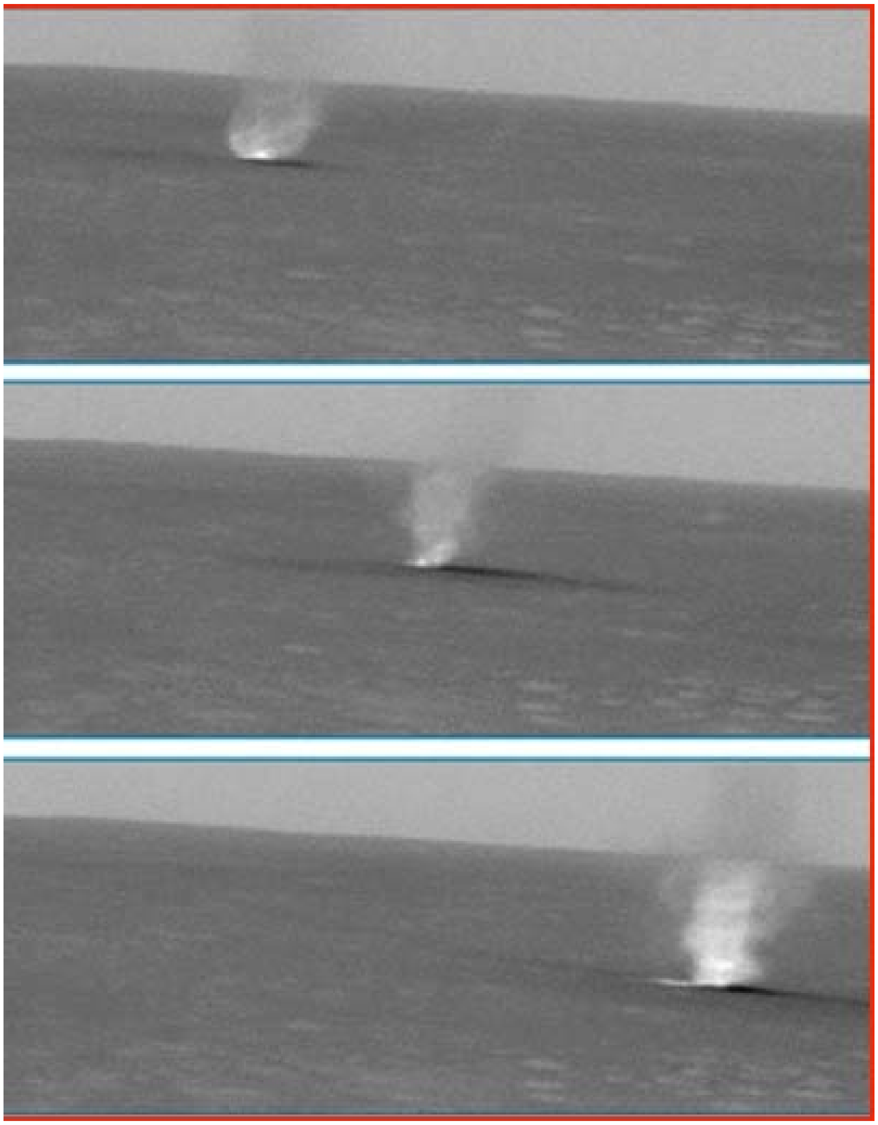
Astronomers think that the pulsar and its nebula is about 1,700 years old, and is located about 17,000 light years away. Finger-like structures extend to the upper right in the image, apparently energizing knots of material in a neighboring gas cloud known as RCW 89. The transfer of energy from the wind to these knots makes them glow brightly in X-rays (orange and red features to the upper right).

Problem 1 - This field of view is 19 arcminutes across, where one arcminute is exactly 1/60 of a degree. Using similar triangles and proportions, if 1 arcminute at a distance of 3,260 light years equals a length of 1 light year, how wide is the image in light years?

Problem 2 - Measure the width of this image with a millimeter ruler. What is the scale of this image in parsecs per millimeter? How far, in light years, is the bright spot in the 'palm' where the pulsar is located, from the center of the ring-like knots in RCW 89? (1 parsec = 3.26 light years). Round your answer to the nearest light year.

Problem 3 - If the speed of the interstellar gas is 10,000 km/sec, how many years did it take for the gas to reach RCW-89 if 1 light year = 9.5×10^{12} kilometers, and there are 3.1×10^{7} seconds in 1 year?

The Martian Dust Devils



A dust devil spins across the surface of Gusev Crater just before noon on Mars. NASA's Spirit rover took the series of images to the left with its navigation camera on March 15, 2005.

The images were taken at:

11:48:00 (T=top)
11:49:00 (M=middle)
11:49:40 (B=bottom)

based upon local Mars time.

The dust devil was about 1.0 kilometer from the rover at the start of the sequence of images on the slopes of the "Columbia Hills."

A simple application of the rate formula

$$speed = \frac{\text{distance}}{\text{time}}$$

lets us estimate how fast the dust devil was moving.

Problem 1 - At the distance of the dust devil, the scale of the image is 7.4 meters/millimeter. How far did the dust devil travel between the top (T) and bottom (B) frames?

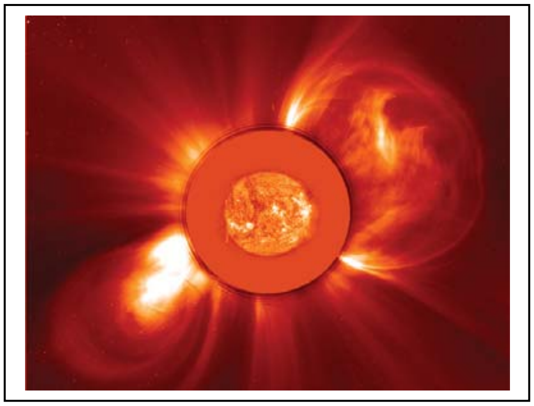
Problem 2 - What was the time difference, in seconds, between the images T-M, M-B and T-B?

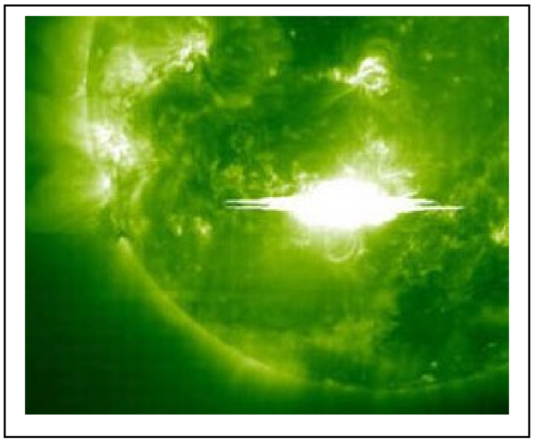
Problem 3 - What was the distance, in meters, traveled between the images T-M and M-B?

Problem 4 - What was the average speed, in meters/sec, of the dust devil between T-B. If an astronaut can briskly walk at a speed of 120 meters/minute, can she out-run a martian dust devil?

Problem 5 - What were the speeds during the interval from T-M, and the interval M-B?

Problem 6 - Was the dust devil accelerating or decelerating between the times represented by T-B?



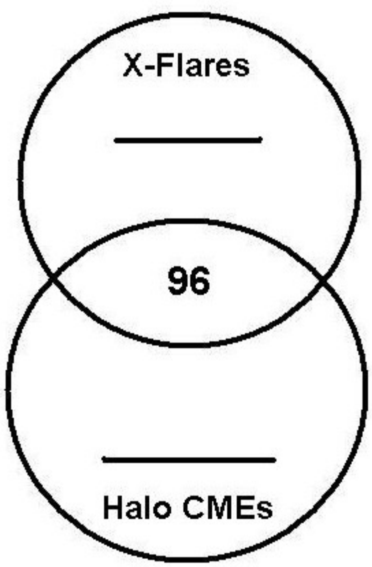


Solar storms come in two varieties:

Coronal Mass Ejections (CMEs) are clouds of gas ejected from the sun that can reach Earth and cause the Aurora Borealis (Northern Lights). These clouds can travel at over 2 million kilometers/hr, and carry billions of tons of matter in the form of charged particles (called a plasma). The picture to the left shows one of many CMEs witnessed by the SOHO satellite.

Solar Flares are intense bursts of X-ray energy that can cause short-wave radio interference on Earth. The picture to the left shows a powerful X-ray flare seen by the SOHO satellite on October 28, 2003.

Between 1996 and 2006, astronomers detected 11,031 coronal mass ejections (CMEs), and of these, 593 were directed towards Earth. These are called 'Halo CMEs' because the ejected gas surrounds the sun's disk on all sides and looks, like a halo around the sun. During these same years, astronomers also witnessed 122 solar flares that were extremely intense X-flares. Of these X-flares, 96 happened at the same time as the Halo CMEs.



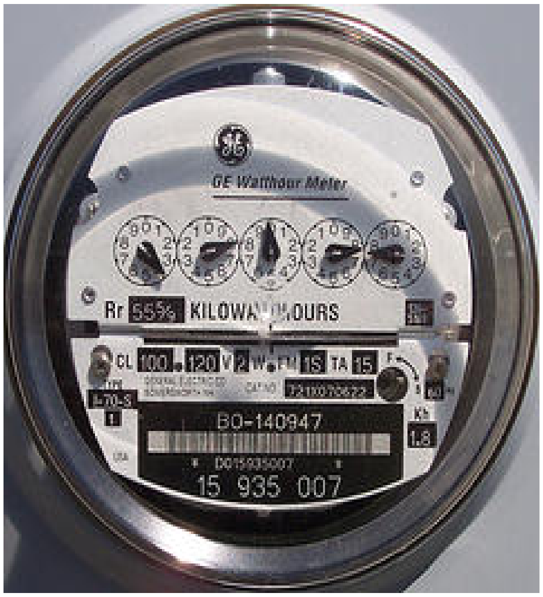
Problem 1 - From this statistical information, fill-in the missing numbers in the circular Venn Diagram to the left.

Problem 2 – What percentage of X-Flares also happened at the same time as a Halo CME?

Problem 3 – What percentage of Halo CMEs happened at the same time as an X-Flare?

Problem 4 – What percentage of all CMEs detected between 1996 and 2006 produced X-Flares?

Energy in the Home



Every month, we get the Bad News from our local electrical company. A bill comes in the mail saying that you used 900 Kilowatt Hours (kWh) of electricity last month, and that will cost you $100.00! *What is this all about?*

Definition: 1 kiloWatt hour is a unit of energy determined by multiplying the electrical power, in kilowatts, by the number of hours of use.

Example: A 100-watt lamp is left on all day. E = 0.1 kilowatts x 24-hours = 2.4 kWh. Note: At 11-cents per kWh, this costs you 2.4 x 11 = 26 cents!

Problem 1 – You and your sister fired-up your two computers at 3:00 PM, and finished your homework at 9:00 PM, but you forgot to turn them off before going to bed. At 7:00 AM, they were finally shut off after being on all night. If this happened each school day in the month (25 days):

A) How many kilowatt hours did it cost to run the computers this way for 25 days?

B) How many kilowatt hours were wasted?

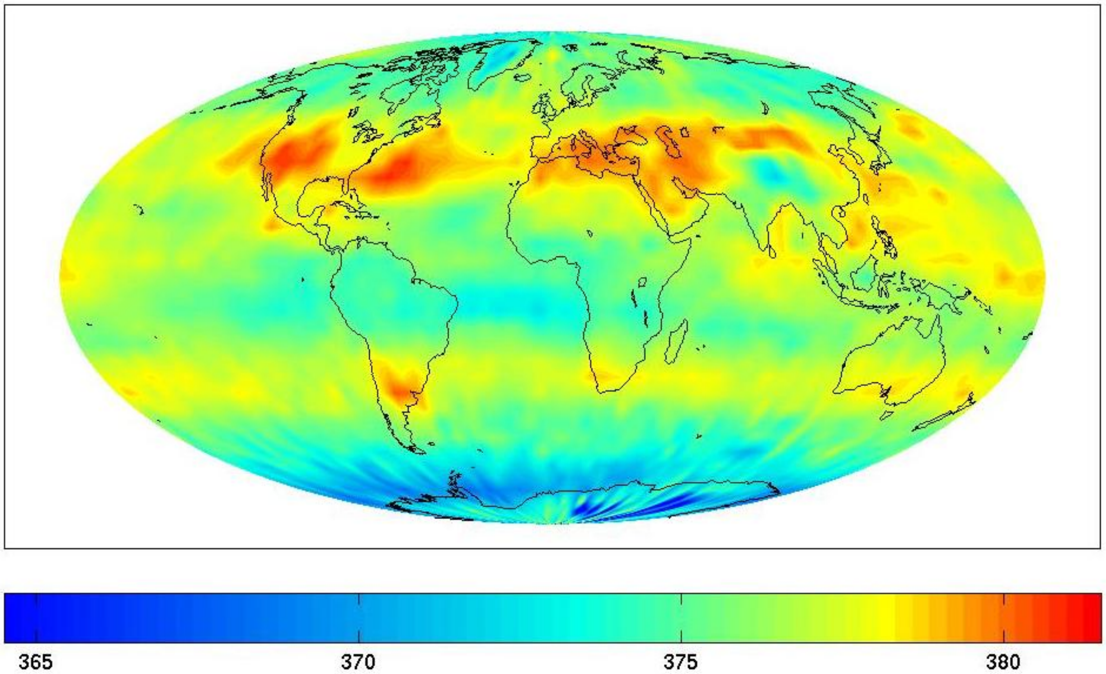
C) If each computer runs at 350 watts, and if electricity costs 11-cents per kilowatt hour, how much did this waste cost each month?

D) How many additional songs can you buy with iTunes for the wasted money each month?

Problem 2 – The Tevatron 'atom smasher' at Fermilab in Batavia, Illinois collides particles together at nearly the speed of light to explore the innermost structure of matter. When operating, the accelerator requires 70 megaWatts of electricity – about the same as the power consumption of the entire town of Batavia (population: 27,000). If an experiment, from start to finish, lasts 24 hours:

A) What is the Tevatron's electricity consumption in kilowatt hours?

B) At $0.11 per kilowatt hour, how much does one experiment cost to run?



The Atmospheric Infrared Sounder (AIRS) instrument on NASA's Aqua spacecraft has been used by scientists to observe atmospheric carbon dioxide. The above map shows the concentrations of atmospheric carbon dioxide in units of 'parts per million', and range from 363 ppm (dark blue) to 380 ppm (red). The data was obtained in July 2003, and the gas is at an altitude of 8 kilometers. The map shows that carbon dioxide is not evenly mixed in the atmosphere, but there are regional differences that change in time. For example, the red 'clouds' move in time and change size and shape.

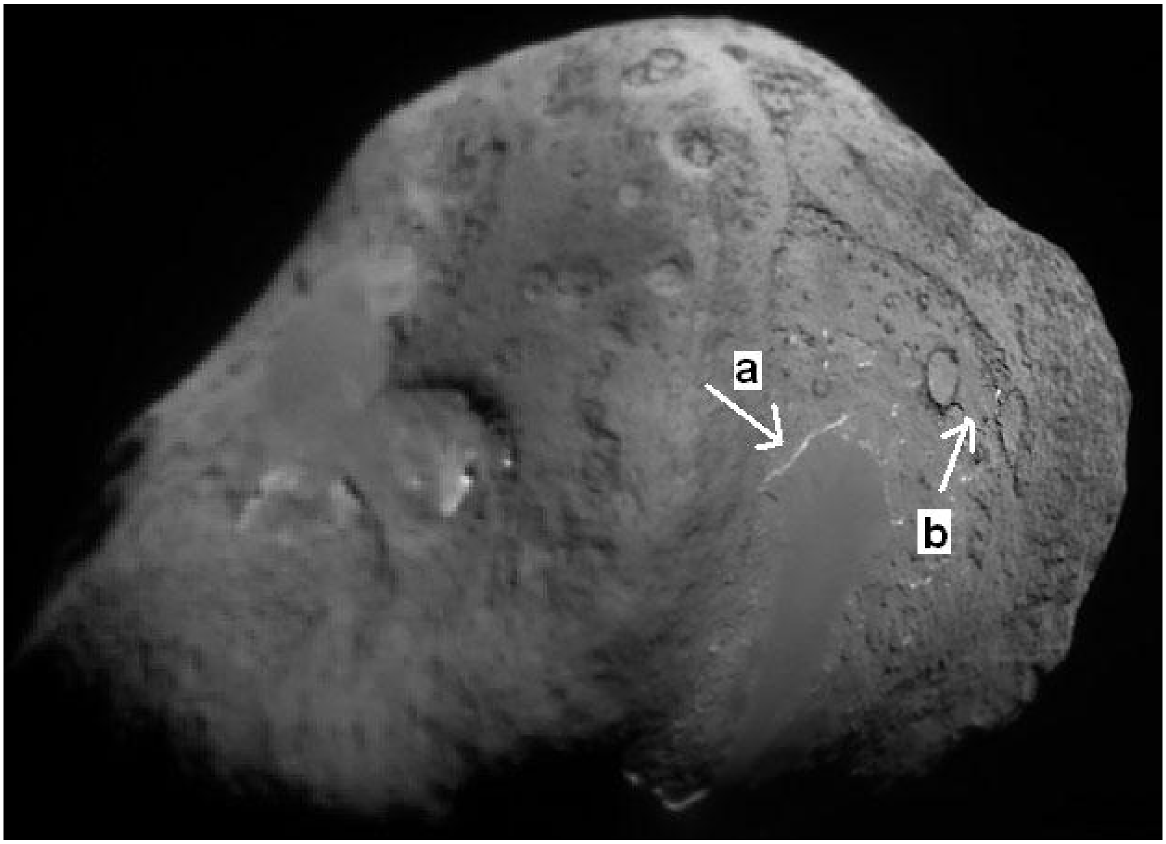
Problem 1 - From the color bar, about what is the average concentration of carbon dioxide across the globe, in ppm, not including the orange or red areas?

Problem 2 - What is the difference in ppm between your answer to Problem 1, and the highest levels of concentration?

Problem 3 - At these altitudes, atmospheric winds generally blow from west to east (left to right on the map). What geographic regions are nearest the highest concentrations of carbon dioxide in this map?

Problem 4 - The average mass of carbon dioxide in the atmosphere, at a concentration of 1 ppm equals 15 tons per square kilometer. How many tons/km^2 are represented by the: A) Red color? B) Yellow color? and C) The difference between red and yellow?

Tempel 1 - Close-up of a Comet!



On July 4, 2005, the Deep Impact spacecraft flew within 500 km of the nucleus of comet Tempel 1. This composite image of the surface of the nucleus was put together from images taken by the Impactor probe as it plummeted towards the comet before finally hitting it and excavating its own crater. The width of this picture is 8.0 kilometers.

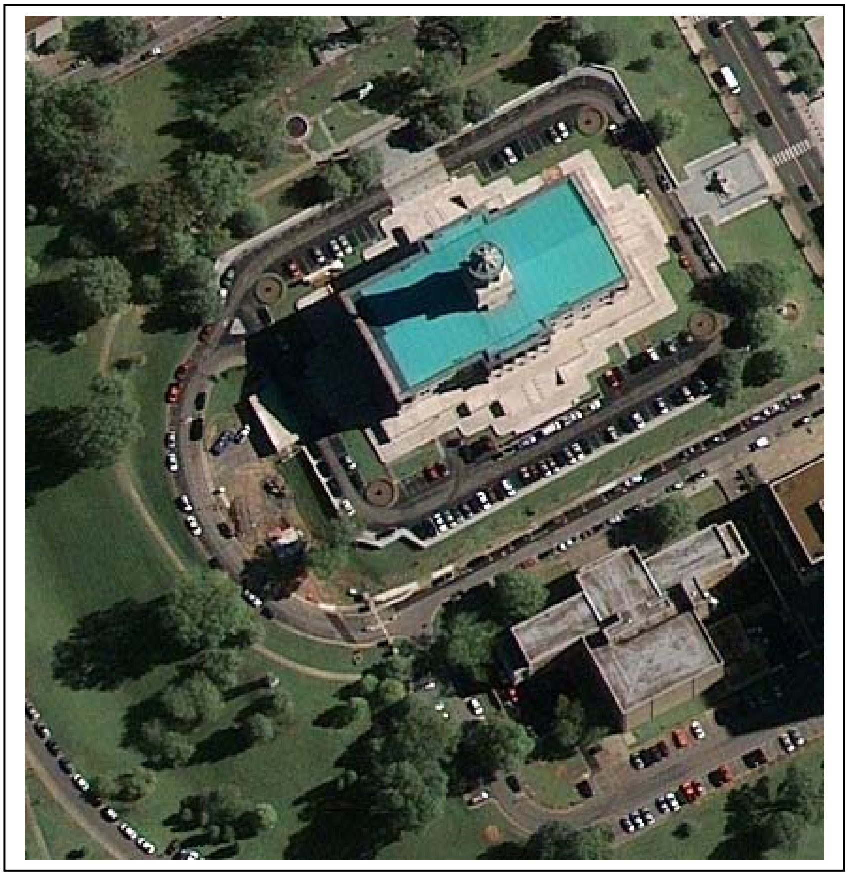
Problem 1 - By using a millimeter ruler: A) what is the scale of this image in meters per millimeter? B) What is the approximate size of the nucleus of this comet in kilometers? C) How big are the two craters near the right-hand edge of the nucleus by the arrow? D) What is the size of some of the smallest details you can see in the picture?

Problem 2 - The white streak identified by Arrow A is a cliff face. What is the height of the cliff in meters, (the width of the white line) and the length of the cliff wall in meters?

Problem 3 - The Deep Impact Impactor probe collided with the comet at the point marked by the tip of Arrow B. If there had been any uncertainty in the accuracy of the navigation, by how many meters might the probe have missed the nucleus altogether?

A High-Resolution Satellite Photo

The Lunar Reconnaissance Orbiter (LRO) will take photographs of the lunar surface at a resolution of 0.5 meters per pixel. The 425x425 pixel image below (Copyright © 2009 GeoEye) was taken of the Tennessee Court House from the GeoEye-1 satellite with a width of about 212 meters.



Problem 1 - What is the scale of the image in: A) meters per millimeter? B) meters per pixel?

Problem 2 - How does the resolution of the expected LRO images compare with the resolution of the above satellite photo?

Problem 3 - What are the smallest features you can easily identify in the above photo?

Problem 4 - From the length of the shadows, what would you estimate as the elevation of the sun above the horizon?

Story 1: On September 23, 1999 NASA lost the $125 million Mars Climate Orbiter spacecraft after a 286-day journey to Mars. Miscalculations due to the use of English units instead of metric units apparently sent the craft slowly off course -- 60 miles in all. Thrusters used to help point the spacecraft had, over the course of months, been fired incorrectly because data used to control the wheels were calculated in incorrect units. Lockheed Martin, which was performing the calculations, was sending thruster data in English units (pounds) to NASA, while NASA's navigation team was expecting metric units (Newtons).

Problem 1 - A solid rocket booster is ordered with the specification that it is to produce a total of 10 million pounds of thrust. If this number is mistaken for the thrust in Newtons, by how much, in pounds, will the thrust be in error? (1 pound = 4.5 Newtons)

Story 2: On January 26, 2004 at Tokyo Disneyland's Space Mountain, an axle broke on a roller coaster train mid-ride, causing it to derail. The cause was a part being the wrong size due to a conversion of the master plans in 1995 from English units to Metric units. In 2002, new axles were mistakenly ordered using the pre-1995 English specifications instead of the current Metric specifications.

Problem 2 - A bolt is ordered with a thread diameter of 1.25 inches. What is this diameter in millimeters? If the order was mistaken for 1.25 centimeters, by how many millimeters would the bolt be in error?

Story 3: On 23 July 1983, Air Canada Flight 143 ran completely out of fuel about halfway through its flight from Montreal to Edmonton. Fuel loading was miscalculated through misunderstanding of the recently adopted metric system. For the trip, the pilot calculated a fuel requirement of 22,300 kilograms. There were 7,682 liters already in the tanks.

Problem 3 - If a liter of jet fuel has a mass of 0.803 kilograms, how much fuel needed to be added for the trip?

Identifying Materials by their Reflectivity 15

50%	78%
3%	30%

Material	Reflectivity
Snow	80%
White Concrete	78%
Bare Aluminum	74%
Vegetation	50%
Bare Soil	30%
Wood Shingle	17%
Water	5%
Black Asphalt	3%

When light falls on a material, some of the light energy is absorbed while the rest is reflected. The absorbed energy usually contributed to heating the body. The reflected energy is what we use to actually see the material! Scientists measure reflectivity and absorption in terms of the percentage of energy that falls on the body, which is called its albedo. The combination must add up to 100%.

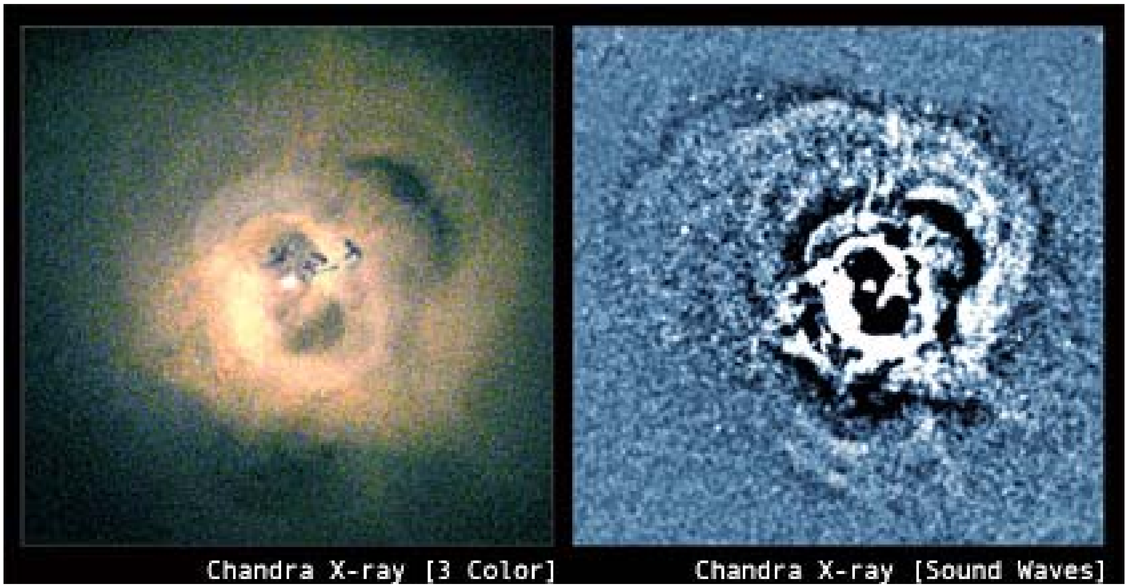
The table above shows the reflectivity of various common materials. For example, snow reflects 80% of the light that falls on it, which means that it absorbs 20% and so 80% + 20% = 100%. This also means that if I have 100 watts of light energy falling on the snow, 80 watts will be reflected and 20 watts will be absorbed.

Problem 1 - If 1000 watts falls on a body, and you measure 300 watts reflected, what is the reflectivity of the body, and from the Table, what might be its composition?

Problem 2 - You are given the reflectivity map at the top of this page. What are the likely compositions of the areas in the map?

Problem 3 - What is the average reflectivity of these four equal-area regions combined?

Problem 4 - Solar radiation delivers 1300 watts per square meter to the surface of Earth. If the area in the map is 20 meters on a side; A) how much solar radiation, in watts, is reflected by each of the four materials covering this area? B) What is the total solar energy, in watts, reflected by this mapped area? C) What is the total solar energy, in watts, absorbed by this area?



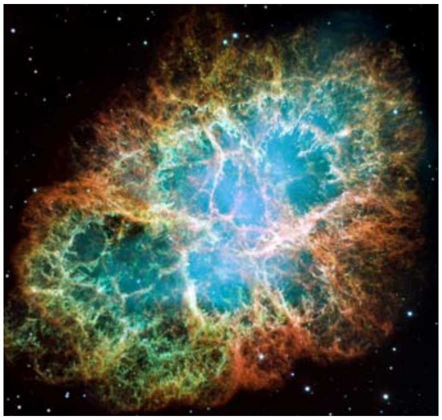
In September 2003, the Chandra Observatory took an x-ray image of a massive black hole in the Perseus Galaxy Cluster located 250 million light years from Earth. Although it could not see the black hole, it did detect the x-ray light from the million-degree gas in the core of the cluster. Instead of a featureless blob, the scientists detected a series of partial concentric rings which they interpreted as sound waves rushing out from the vicinity of the black hole as it swallowed gas in a series of explosions. The image above left shows the x-ray image, and to the right, an enhanced version that reveals the details more clearly.

Problem 1 - The image has a physical width of 350,000 light years. Using a millimeter ruler, what is the scale of the image in light years/millimeter?

Problem 2 - Examine the image on the right very carefully and estimate how far apart the consecutive crests of the sound wave are in millimeters. What is the wave length of the sound wave in light years?

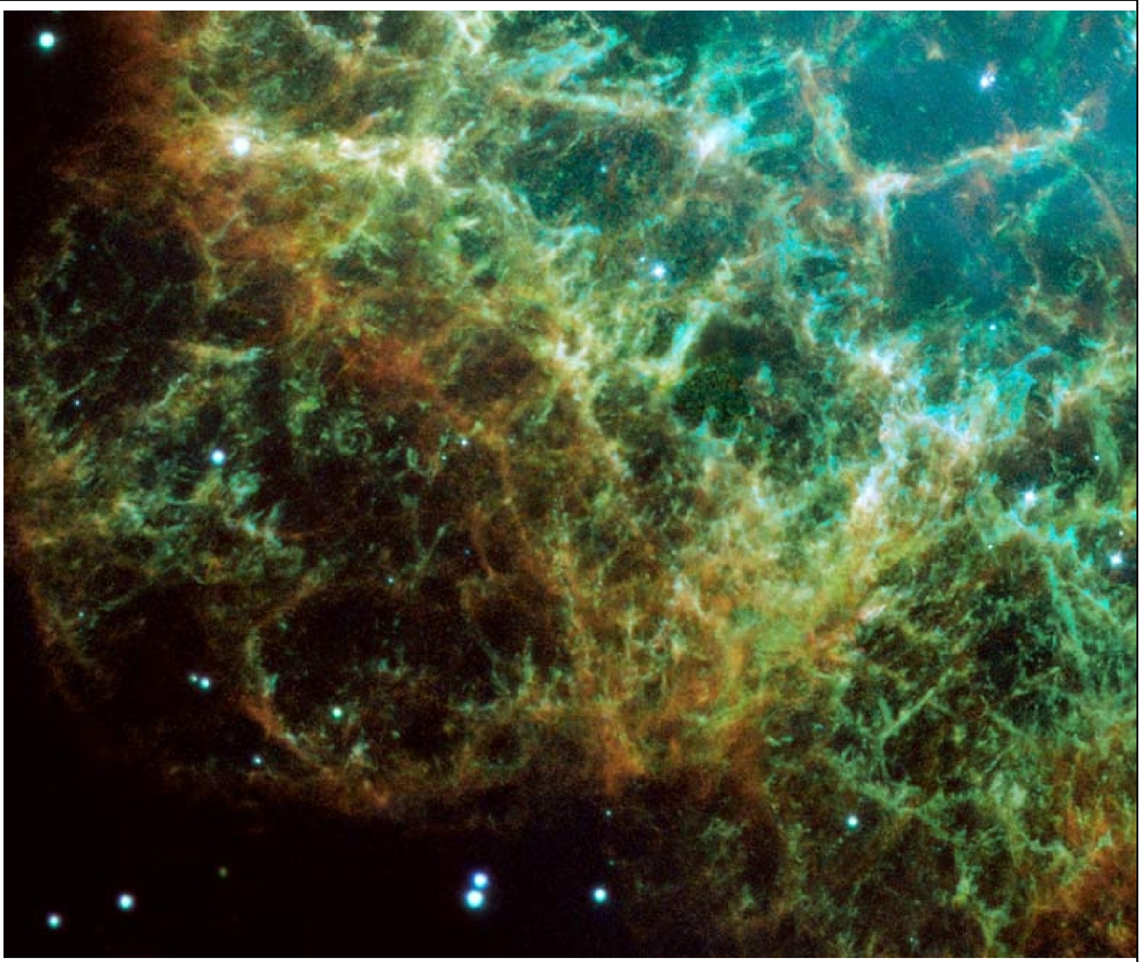
Problem 3 - The wavelength of middle-C on a piano is 1.3 meters. If 1 light year = 9.5 x 10^{15} meters, and if 1 octave represents a change by a factor of 1/2 change in wavelength, how many octaves below middle-C is the sound wave detected by Chandra?

Details from an exploding star



These dramatic images of the Crab Nebula were taken in 2005 by the Hubble Space Telescope. The image on the left has a scale of 0.2 light years/millimeter. The enlargement below has a scale of 0.025 light years/millimeter.

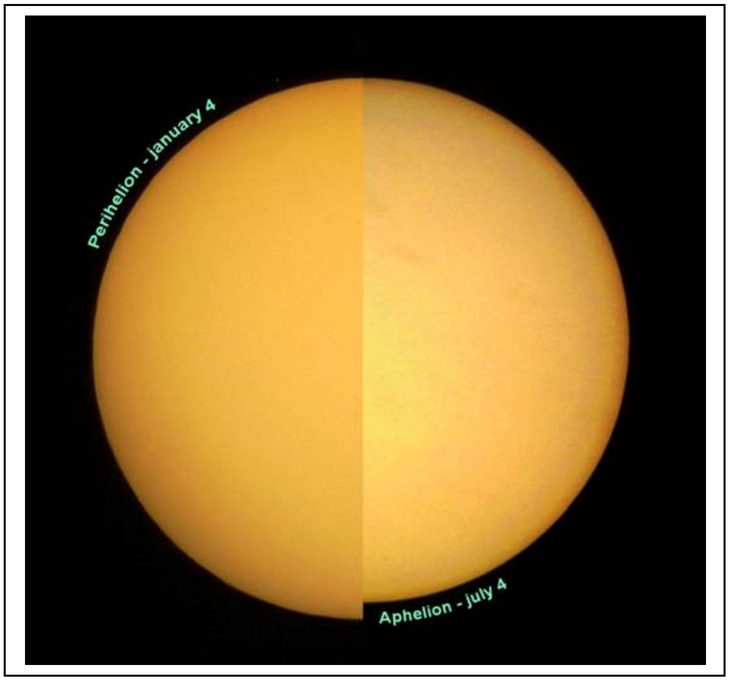
The star that produced this nebula exploded as a supernova in the year 1054 AD, and the expanding gas has been traveling outwards ever since.



Problem 1 – From the information given, what is the average speed of the expanding gas cloud in kilometers/hour? (Note that 1 light year = 62,000 Astronomical Units, and 1 Astronomical Unit = 150 million kilometers, also 1 year = 8760 hours).

Problem 2 – How large are the smallest clumps of the gas in the expanding cloud?

Problem 3 – Draw a sketch, to scale, of the diameter of the solar system (80 Astronomical Units) compared to the size of two or three of the smallest gas clumps.



Earth's orbit is not a perfect circle centered on the sun, but an ellipse! Because of this, in January, Earth is slightly closer to the sun than in June. This means that the sun will actually appear to have a bigger disk in the sky in June than in January…but the difference is impossible to see with the eye, even if you could do so safely!

The figure above shows the sun's disk taken by the SOHO satellite. The left side shows the disk on January 4 and the right side shows the disk on June 4, 2009. As you can see, the diameter of the sun appears to change slightly between these two months.

Problem 1 - What is the average diameter of the Sun, in millimeters, in this figure?

Problem 2 - By what percentage did the diameter of the Sun change between January and June compared to its average diameter?

Problem 3 - If the average distance to the Sun from Earth is 149,600,000 kilometers, how much closer is Earth to the Sun in June compared to January?

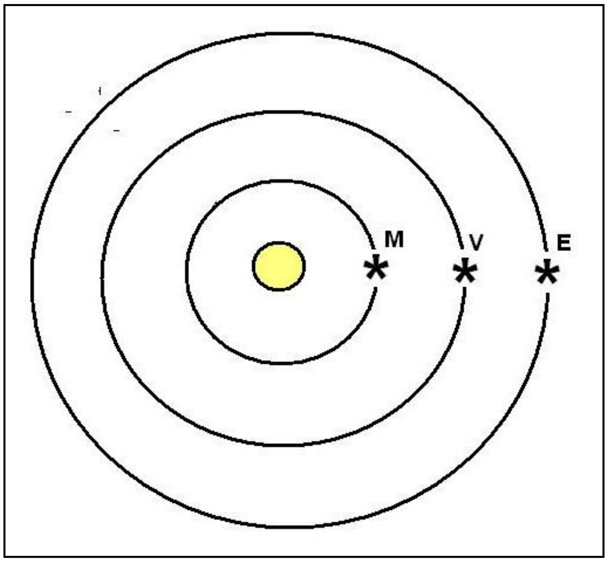
Period	Age (years)	Days per year	Hours per day
Current	0	365	
Upper Cretaceous	70 million	370	
Upper Triassic	220 million	372	
Pennsylvanian	290 million	383	
Mississippian	340 million	398	
Upper Devonian	380 million	399	
Middle Devonian	395 million	405	
Lower Devonian	410 million	410	
Upper Silurian	420 million	400	
Middle Silurian	430 million	413	
Lower Silurian	440 million	421	
Upper Ordovician	450 million	414	
Middle Cambrian	510 million	424	
Ediacarin	600 million	417	
Cryogenian	900 million	486	

We learn that an 'Earth Day' is 24 hours long, and that more precisely it is 23 hours 56 minutes and 4 seconds long. But this hasn't always been the case. Detailed studies of fossil shells, and the banded deposits in certain sandstones, reveal a much different length of day in past eras! These bands in sedimentation and shell-growth follow the lunar month and have individual bands representing the number of days in a lunar month. By counting the number of bands, geologists can work out the number of days in a year, and from this the number of hours in a day when the shell was grown, or the deposits put down. The table above shows the results of one of these studies.

Problem 1 - Complete the table by calculating the number of hours in a day during the various geological eras. It is assumed that Earth orbits the sun at a fixed orbital period, based on astronomical models that support this assumption.

Problem 2 - Plot the number of hours lost compared to the modern '24 hours' value, versus the number of years before the current era.

Problem 3 - By finding the slope of a straight line through the points can you estimate by how much the length of the day has increased in seconds per century?



One of the most interesting things to see in the night sky is two or more planets coming close together in the sky. Astronomers call this a conjunction. As seen from their orbits, another kind of conjunction is sometimes called an 'alignment' which is shown in the figure to the left and involves Mercury, M, Venus, V, and Earth, E. As viewed from Earth's sky, Venus and Mercury would be very close to the sun, and may even be seen as black disks 'transiting' the disk of the sun at the same time, if this alignment were exact. How often do alignments happen?

Earth takes 365 days to travel one complete orbit, while Mercury takes 88 days and Venus takes 224 days, so the time between alignments will require each planet to make a whole number of orbits around the sun and return to the pattern you see in the figure above. Let's look at a simpler problem. Suppose Mercury takes 1/4 Earth-year and Venus takes 2/3 of an Earth-year to make their complete orbits around the sun. You can find the next line-up from one of these two methods:

Method 1: Work out the three number series like this:

Earth = 0, 1, **2**, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12,…
Mercury = 0, 1/4, 2/4, 3/4 ,4/4, 5/4, 6/4, 7/4, **8/4**, 9/4, 10/4, 11/4, 12/4,…
Venus = 0, 2/3, 4/3, **6/3**, 8/3, 10/3, 12/3, 14/3, 16/3, 18/3, 20/3, …

Notice that the first time they all coincide with the same number is at **2 years**. So Mercury has to go around the Sun 8 times, Venus 3 times and Earth 2 times for them to line up again in their orbits.

Method 2: We need to find the Least Common Multiple (LCM) of 1/4, 2/3 and 1. First render the periods in multiples of a common time unit of 1/12, then the sequences are:

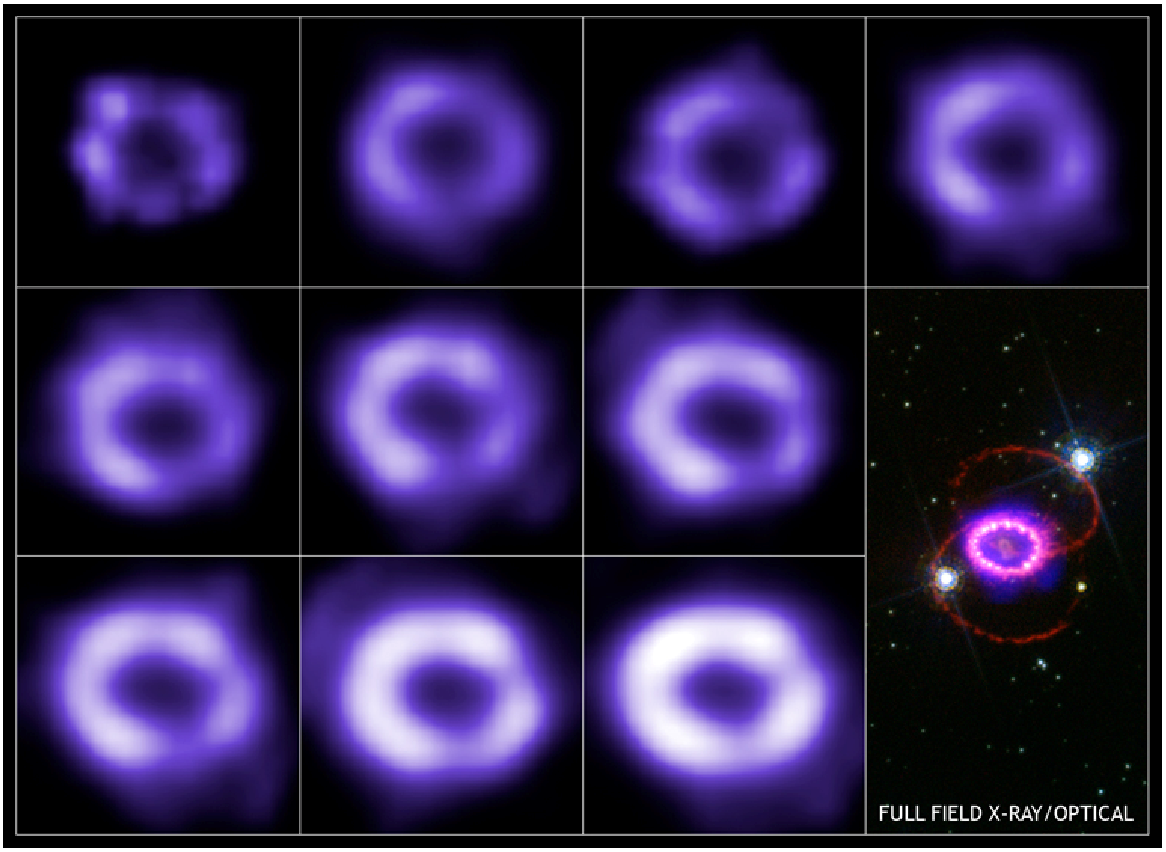
Mercury = 0, 3, 6, 9, 12, 15, 18, 21, **24**,
Venus = 0, 8, 16, **24**, 32, 40, …
Earth, 0, 12, **24**, 36, 48, 60, …

The LCM is 24 which can be found from prime factorization:

Mercury: 3 = 3
Venus: 8 = 2 x 2 x 2
Earth: 12 = 2 x 2 x 3

The LCM the product of the highest powers of each prime number or 3 x 2 x 2 x 2 = 24. and so it will take 24/12 = **2 years.**

Problem 1 - Suppose a more accurate estimate of their orbit periods is that Mercury takes 7/30 Earth-years and Venus takes 26/42 Earth-years. After how many Earth-years will the alignment shown in the figure above reoccur?



In March, 1987 a supernova occurred in the Large Magellanic Cloud; a nearby galaxy to the Milky Way about 160,000 light years away from Earth. The site of the explosion was traced to the location of a blue supergiant star called Sanduleak -69º 202 (SK -69 for short) that had a mass estimated at approximately 20 times our own sun.The series of image above, taken by the Chandra X-ray Observatory, shows the expansion of the million-degree gas ejected by the supernova between January, 2000 (top left image) to January, 2005 (lower right image). The width of each image is 1.9 light years.

Problem 1 - Using a millimeter ruler, what is the scale of each image in light years/millimeter?

Problem 2 - If 1 light year = 9.5×10^{12} kilometers, and 1 year = 3.1×10^{7} seconds, what was the average speed of the supernova gas shell between 2000 and 2005?

The STS-125 Atlantis astronauts retrieved the Hubble Space Telescope Wide-field Planetary Camera 2 (WFPC2) during a very successful and final servicing mission in May 2009. The radiator (above photo) attached to WFPC2 has dimensions of 2.2 meters by 0.8 meters. Its outermost layer is a 4-mm-thick aluminum, curved plate coated with white thermal paint. This radiator has been exposed to space since the deployment of WFPC2 in 1993. During this time, it received numerous impacts by micrometeoroids and man-made particles (flecks of paint, etc). The circles drawn on the radiator plate show the locations of these impacts.

Problem 1 - What is the total surface area of the WFPC2 radiator plate in square meters?

Problem 2 - How many impacts were counted over this area?

Problem 3 - What is the surface density of impacts in units of impacts per square meter?

Problem 4 - How many years was this panel in space?

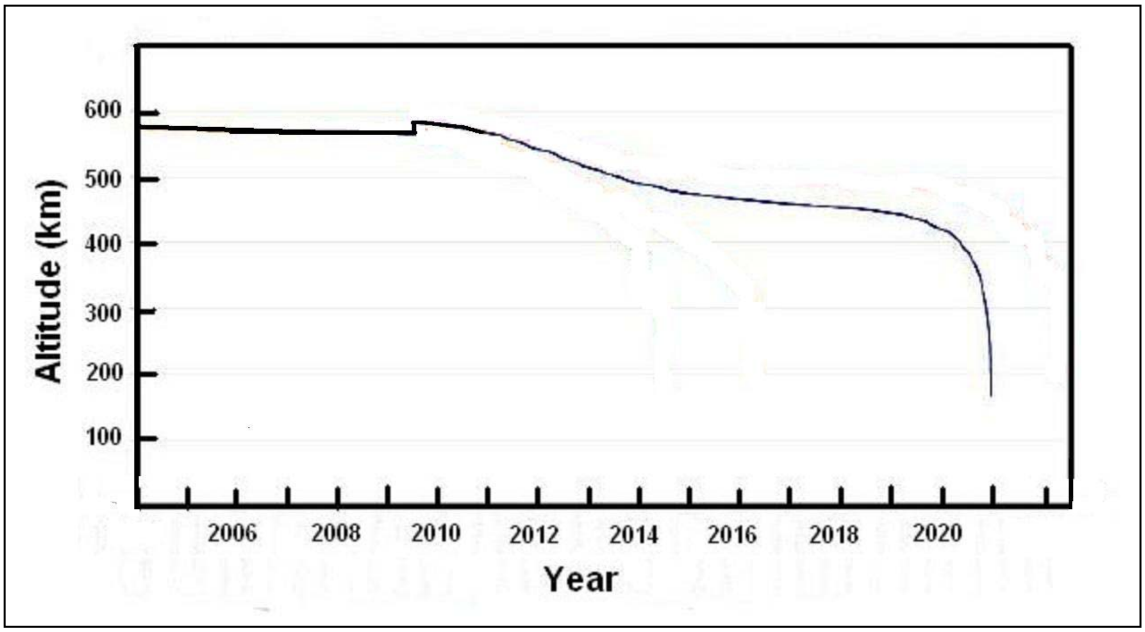
Problem 5 - What is the impact rate in units of impacts per meter^2 per year?

Problem 6 - The solar panels on the International Space Station have a total surface area of 1,632 meters^2. A) How many impacts per year will these solar panels experience? B) What will be the average time, in hours, between impact?

Satellite Drag and the Hubble Space Telescope

The Hubble Space Telescope was never designed to operate forever. What to do with the observatory remains a challenge for NASA once its scientific mission is completed in 2012. Originally, a Space Shuttle was proposed to safely return it to Earth, where it would be given to the National Air and Space Museum in Washington DC. Unfortunately, after the last Servicing Mission, STS-125, in May, 2009, no further Shuttle visits are planned. As solar activity increases, the upper atmosphere heats up and expands, causing greater friction for low-orbiting satellites like HST, and a more rapid re-entry.

The curve below shows the predicted altitude for that last planned re-boost in 2009 of 18-km. NASA plans to use a robotic spacecraft after ca 2015 to allow a controlled re-entry for HST, but if that were not the case, it would re-enter the atmosphere sometime after 2020.

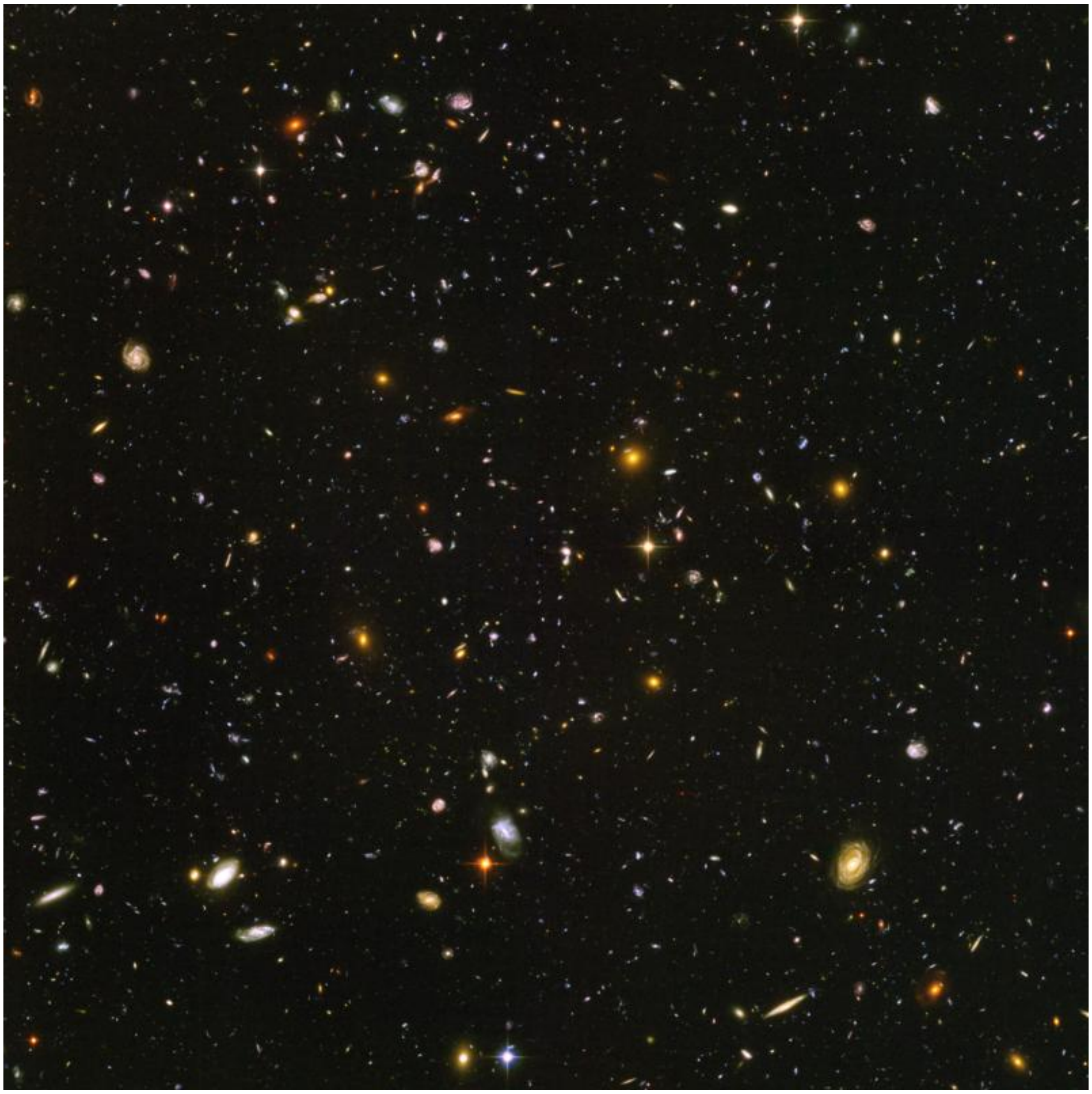


Problem 1 – The last Servicing Mission in 2009 will only extend the science operations by another 5 years. How long after that time will the HST remain in orbit?

Problem 2 – Once HST reaches an altitude of 400 km, with no re-boosts, about how many weeks will remain before the satellite burns up? (Hint: Use a millimeter ruler.)

Counting Galaxies with the Hubble Space Telescope

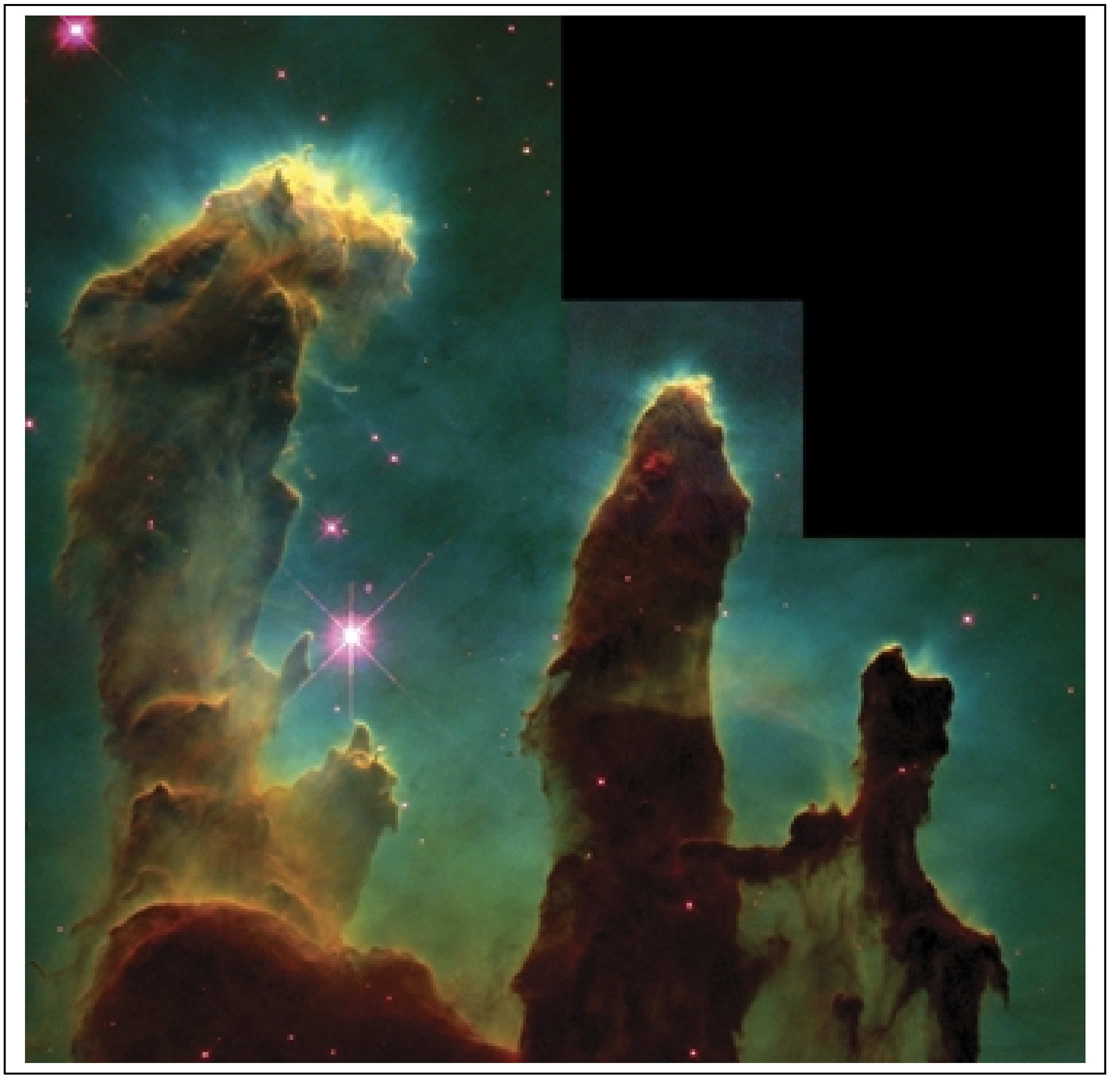
In 2004, the Hubble Space Telescope took a million-second exposure of a small part of the sky to detect as many galaxies as possible. Here's what they saw!



Problem 1 – Divide the field into 16 equal areas. Label the grids alphabetically from A to P starting from the top left cell. Count the number of 'smudges' in four randomly selected cells. What is the average number of galaxies in of one of the cells in the picture? What uncertainties can you identify in counting these galaxies?

Problem 2 – One square degree equals 3,600 square arcminutes. If Hubble Ultra Deep Field picture is 3 arcminutes on a side, what is the area of one of your cells in square degrees? (Note: 1 arcminute equals 1/60 of a degree of angle measure).

Problem 3 – There are 41,250 square degrees in the sky, about how many galaxies are in the full sky as faint as the faintest galaxy that Hubble detected in the Deep Field image?



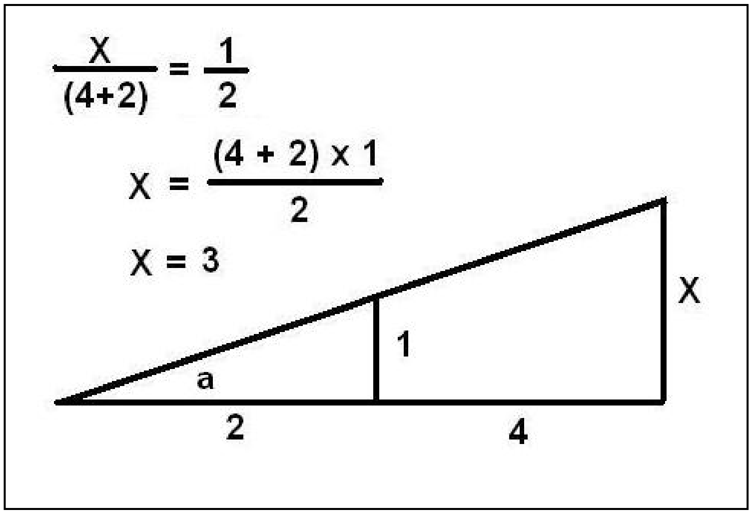
The Hubble Space Telescope took the image of the Eagle Nebula (M-16). This star-forming region is in the constellation Serpens, and located 6,500 light years from Earth. It is only about 6 million years old. The dense clouds of interstellar gas are still collapsing to form new stars. This image is 2.5 arcminutes across. (Note: There are 60 arcminutes in one degree)

Problem 1 – If an angular size of 200 arcseconds corresponds to 1 light year at a distance of 1000 light years. What is the size of this field at the distance of the nebula? (Note: There are 60 arcseconds in one arcminjute.)

Problem 2 – What is the scale of this image in light years/millimeter?

Problem 3 – Our Solar System is about 1/400 of a light year across. How big is it, in millimeters, at the scale of this photo?

Problem 4 - How many times the size of our solar system is the smallest nebula feature you can see in the photo?

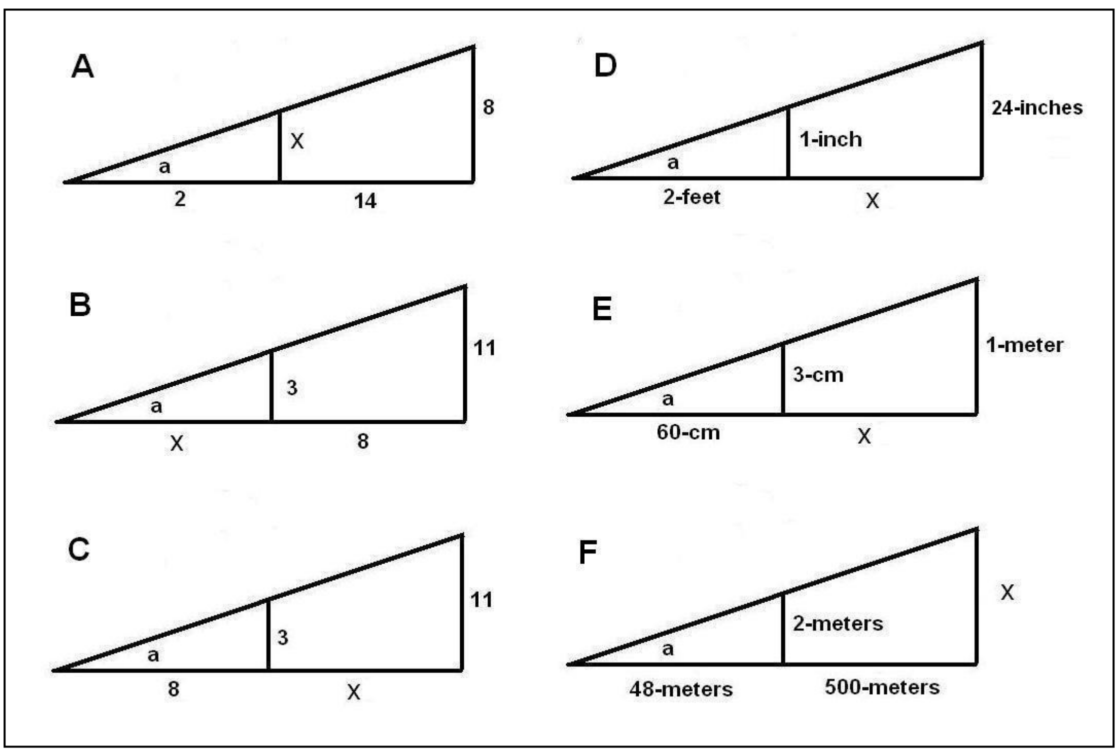


The corresponding sides of similar triangles are proportional to one another as the illustration to the left shows. Because the vertex angle of the triangles are identical in measure, two objects at different distances from the vertex will subtend the same angle, **a** . The corresponding side to 'X' is '1' and the corresponding side to '2' is the combined length of '2+4'.

Problem 1: Use the properties of similar triangles and the ratios of their sides to solve for 'X' in each of the diagrams below.

Problem 2: Which triangles must have the same measure for the indicated angle **a** ?

Problem 3: The sun is 400 times the diameter of the moon. Explain why they appear to have about the same angular size if the moon is at a distance of 384,000 kilometers, and the sun is 150 million kilometers from Earth?



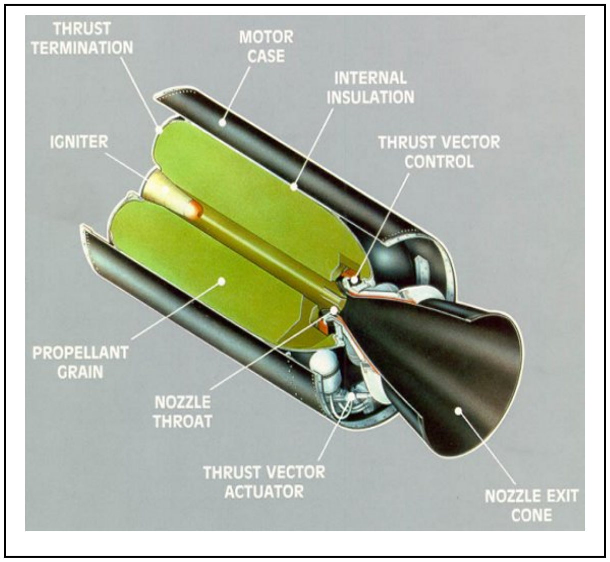
ISS - Orbit Altitude Changes

The International Space Station is a 400-ton, $160 billion platform that supports an international team of 3-5 astronauts for tours of duty lasting up to 6 months at a time. Like all satellites that orbit close to Earth, the atmosphere causes the ISS orbit to decay steadily every day, so the ISS has to be 're-boosted' every few months to prevent it from burning up in the atmosphere.

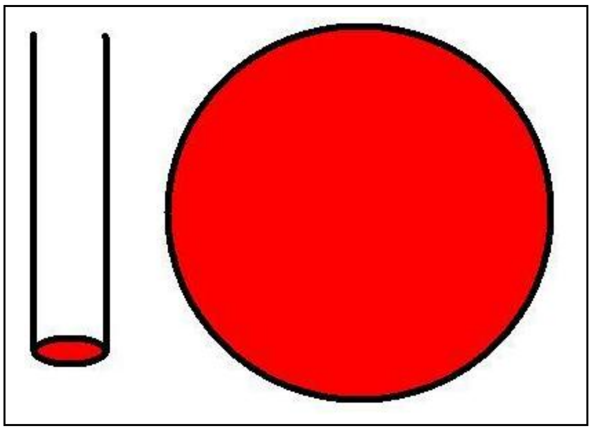
Problem 1 - Based on the following information, what is the altitude of the ISS by April 2009?

"In January, the altitude was 340 kilometers. By March it has lost 8 kilometers before the Progress-59 supply ship raised its altitude by 5 kilometers. In May, the ISS lost 4 1/2 kilometers and was re-boosted by the Progess-60 supply ship by 5 1/2 kilometers. Again the ISS continued to lose altitude by 5 1/2 kilometers by July when the Progress-61 supply ship raised its orbit by 9 1/2 kilometers. The ISS altitude then fell by 3 kilometers by October when the Soyuz TMA-11 mission re-boosted the station by 5 kilometers. The ISS continued to lose altitude until late December, 2007 when it had lost a total of 8 1/2 kilometers since its last re-boost by Soyuz. Since December 2007, the total of all the declines and re-boosts added up to a net change of + 11 1/2 kilometers by April 2009. "

Solid Rocket Boosters



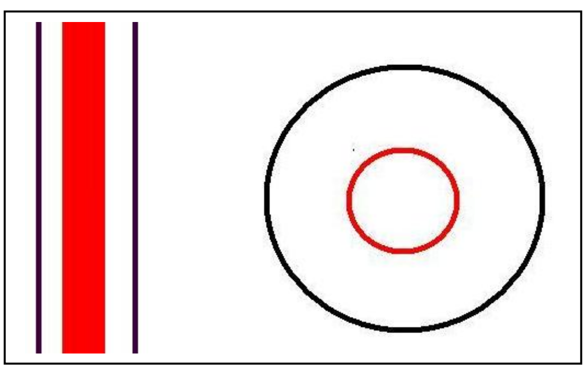
The two solid rocket boosters (SRBs) for the Ares-V launch vehicle will each generate a total thrust of 3.8 million pounds (16.9 megaNewtons). The SRBs from tip to ground are 193 feet long and have a diameter of 12.2 feet. The actual fuel occupies a cylindrical volume about 180 feet (60 meters) long and 11.5 feet (3.7 meters) in diameter. They will 'burn' for a total of 126 seconds. You might think that the fuel burns from the bottom of the cylinder to the top, the way that a candle consumes its wax. The fuel in an SRB is actually designed to burn from the central axis of the cylinder to the outside casing! Let's see why this is a much better way to launch a rocket!



Problem 1 - Thrust is created by burning the exposed surface area of the fuel in the SRB. To launch the 3.4 million pound Ares-V rocket, they have to burn at a rate of 8,740 kg of fuel each second. The density of the fuel is 1770 kg/m^3. If the exposed fuel area is just the circular cross section of the cylinder (see red area in the figure to left), and the burn depth is 0.1 meters each second;

A) What is the total burn rate?

B) Is this enough to launch Ares-V?



Problem 2 - Suppose, instead, that a cylindrical core (red circle) with a diameter of 0.6 meters is cut out along the axis of the booster from top to bottom. The figure to the left shows the red areas where the fuel is burning.

A) What is the surface area of the exposed fuel in the core region?

B) If the burn depth is 0.1-meter each second, what is the mass rate in kg/sec?

C) Is this enough to launch Ares-V?

Evaluating Secondary Physical Constants

Symbol	Name	Value
c	Speed of light	2.9979×10^{10} cm/sec
h	Planck's constant	6.6262×10^{-27} erg sec
m	Electron mass	9.1095×10^{-28} gms
e	Electron charge	4.80325×10^{-10} esu
G	Gravitation constant	6.6732×10^{-8} dyn cm^{2} gm^{-2}
M	Proton mass	1.6726×10^{-24} gms

Also use $\pi = 3.1415926$

Although there are only a dozen fundamental physical constants of Nature, they can be combined to define many additional basic constants in physics, chemistry and astronomy.

In this exercise, you will evaluate a few of these 'secondary' constants to three significant figure accuracy using a calculator and the defined values in the table.

Problem 1 - Bremstrahlung Radiation Constant: $\dfrac{32\pi^2 e^6}{3(2\pi)^{1/2} m^3 c}$

Problem 2 - Photoionization Constant: $\dfrac{32\pi^2 e^6 (2\pi^2 e^4 m)}{3^{3/2} h^3}$

Problem 3 - Stark Line Limit: $\dfrac{16\pi^4 m^2 e^4}{h^4 M^5}$

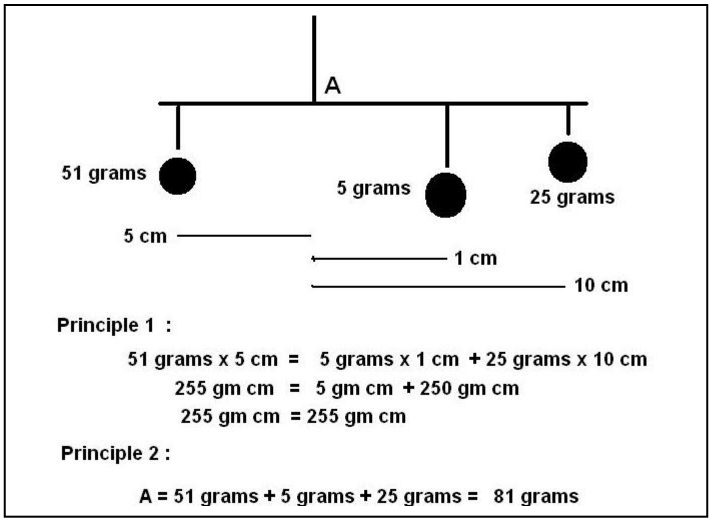
Problem 4 - Thompson Scattering Cross-section: $\dfrac{8\pi}{3}\left(\dfrac{e^2}{mc^2}\right)^2$

Problem 5 - Gravitational Radiation Constant: $\dfrac{32}{5}\dfrac{G^5}{c^{10}}$

Problem 6 - Thomas-Fermi Constant: $\dfrac{324}{175}\left(\dfrac{4}{9\pi}\right)^{2/3}$

Problem 7 - Black Hole Entropy Constant: $\dfrac{c^3}{2hG}$

Space Mobile Puzzle

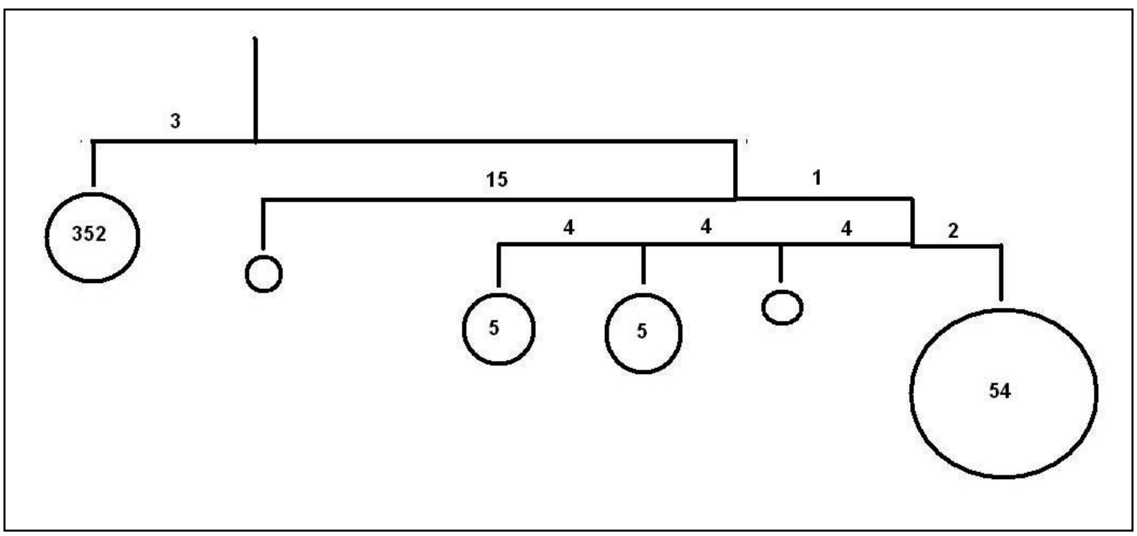


Mobiles are fun to build, but are an example of several important physical principles. The example to the left displays the two basic ones:

1 - The product of the mass x distance from the suspension point of each body must equal zero at the balance point.

2 - The mass at the balance point equals the sum of the masses on the suspended bar.

Using the above two principles, fill-in the missing numbers in the mobile below. The masses of each ball are shown by the numbers inside each circle. The lengths of each cross-bar are indicated by the corresponding numbers. Can you design more elaborate mobiles using these two principles?



An asteroid, or comet, viewed from Earth will be either bright or faint depending on many quantifiable factors. Of course the size of the body and its reflectivity make a big difference. So does its distance from the sun and earth at the time you see it. The brightness also depends on whether, from Earth, it is fully-illuminated like the full moon, or only partly-illuminated like the crescent moon.

Astronomers can put all of these variables together into one single equation which works pretty well to predict a body's brightness just about anywhere inside the solar system!

The streak in the photo above is the asteroid 1999AN10 (Courtesy Palomar Digital Sky Survey). Orbit data suggest that on August 7, 2027 it will pass within 37,000 kilometers of Earth. The formula for the brightness of the asteroid is given by:

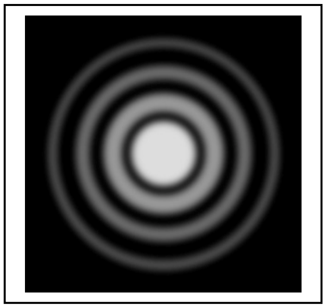
$$R = 0.011\, d\ 10^{-\frac{1}{5}(m)}$$

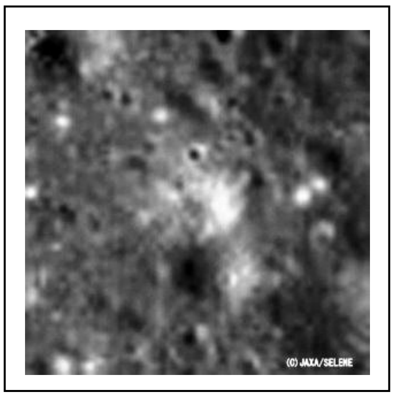
where: R is the asteroid radius in meters, d is the distance to Earth in kilometers, and m is the apparent brightness of the asteroid viewed from Earth. Note, the faintest star you can see with the naked eye is about m = +6.5. The photograph above shows stars as faint as m = +20. The asteroid is assumed to have a reflectivity similar to lunar rock.

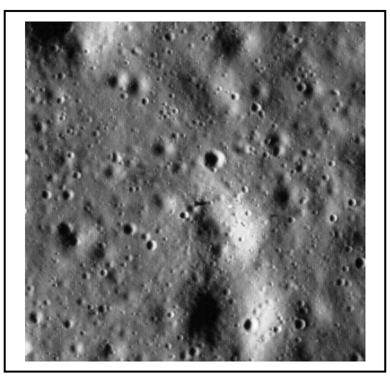
Problem 1 - If the distance to the asteroid at the time of closest approach in 2027 will be d = 37,000 kilometers, what is the formula R(m) for the asteroid?

Problem 2 - If the radius of the asteroid is in the domain between 200 meters and 1000 meters, what is the range of apparent brightnesses?

$$R = 1.22\frac{L}{D}$$







There are many equations that astronomers use to describe the physical world, but none is more important and fundamental to the research that we conduct than the one to the left! You cannot design a telescope, or a satellite sensor, without paying attention to the relationship that it describes.

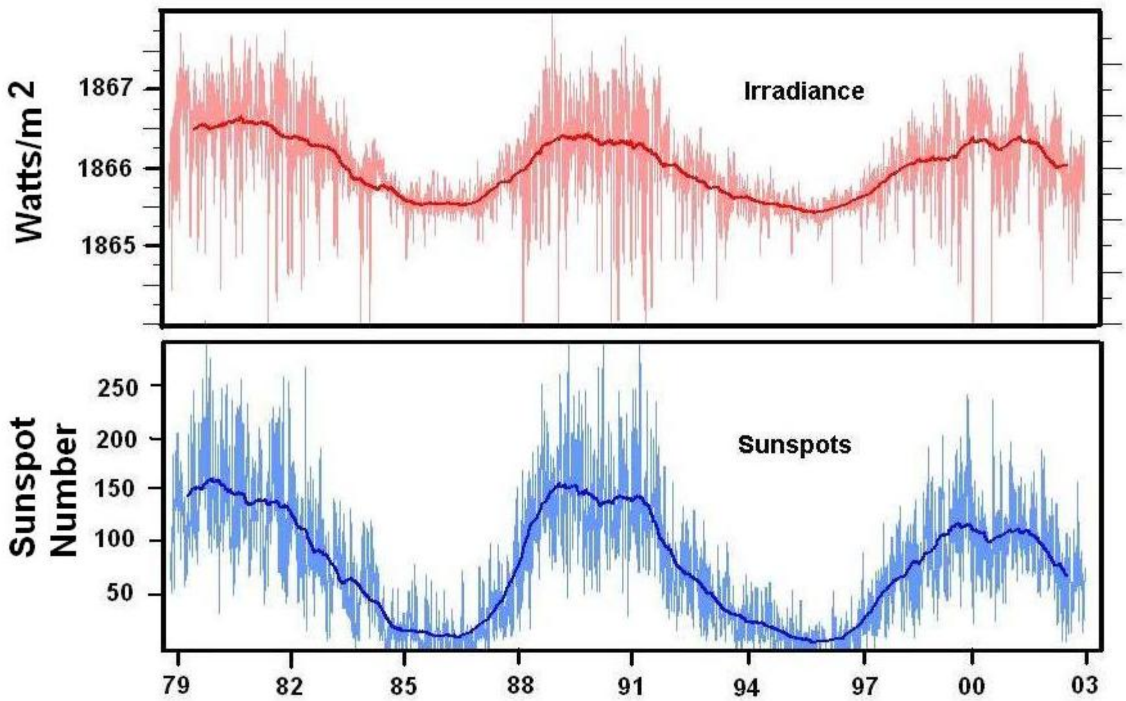
In optics, the best focused spot of light that a perfect lens with a circular aperture can make, limited by the diffraction of light. The diffraction pattern has a bright region in the center called the Airy Disk. The diameter of the Airy Disk is related to the wavelength of the illuminating light, L, and the size of the circular aperture (mirror, lens), given by D. When L and D are expressed in the same units (e.g. centimeters, meters), R will be in units of angular measure called radians (1 radian = 57.3 degrees).

You cannot see details with your eye, with a camera, or with a telescope, that are smaller than the Airy Disk size for your particular optical system. The formula also says that larger telescopes (making D bigger) allow you to see much finer details. For example, compare the top image of the Apollo-15 landing area taken by the Japanese Kaguya Satellite (10 meters/pixel at 100 km orbit elevation: aperture = about 15cm) with the lower image taken by the LRO satellite (1.0 meters/pixel at a 50km orbit elevation: aperture = 0.8 meter). The Apollo-15 Lunar Module (LM) can be seen by its 'horizontal shadow' near the center of the image.

Problem 1 - The Senator Byrd Radio Telescope in Green Bank West Virginia with a dish diameter of D=100 meters is designed to detect radio waves with a wavelength of L= 21-centimeters. What is the angular resolution, R, for this telescope in A) degrees? B) Arc minutes?

Problem 2 - The largest, ground-based optical telescope is the D = 10.4-meter Gran Telescopio Canaris. If this telescope operates at optical wavelengths (L = 0.00006 centimeters wavelength), what is the maximum resolution of this telescope in A) microradians? B) milliarcseconds?

Problem 3 - An astronomer wants to design an infrared telescope with a resolution of 1 arcsecond at a wavelength of 20 micrometers. What would be the diameter of the mirror?



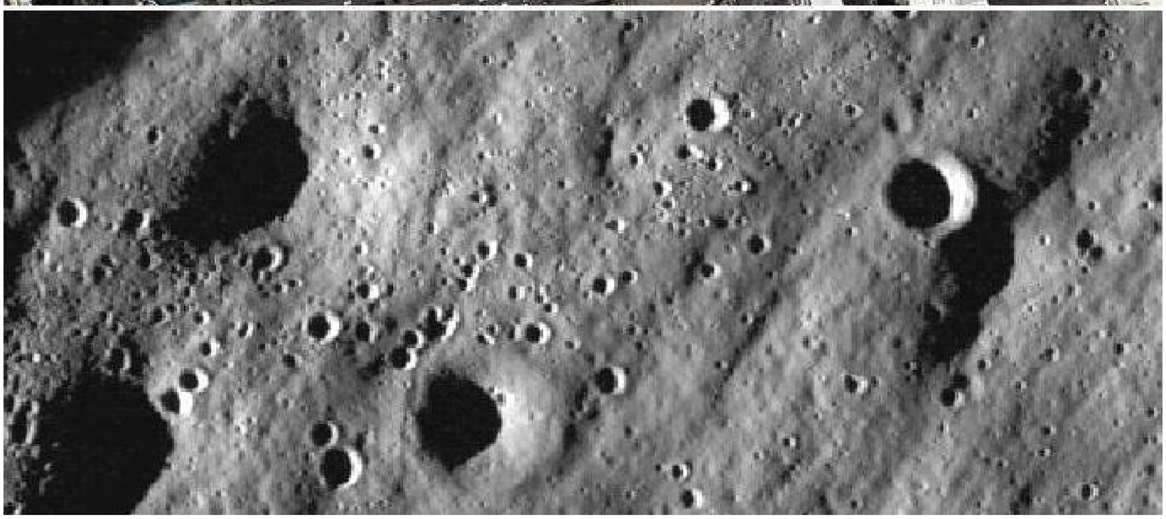
Irradiance is a measure of the amount of energy in sunlight that is hitting an exposed surface each second, and is commonly expressed as watts per square meter. The figure above is a plot of the solar irradiance and sunspot number since January 1979 according to NOAA's National Geophysical Data Center (NGDC). The thin lines indicate the daily irradiance (red) and sunspot number (blue), while the thick lines indicate the running annual average for these two parameters. The total variation in solar irradiance is about 1.3 watts per square meter during one sunspot cycle. The solar irradiance data obtained by the ACRIM satellite, measures the total number of watts of sunlight that strike Earth's upper atmosphere before being absorbed by the atmosphere and ground.

Problem 1 - About what is the average value of the solar irradiance between 1978 and 2003?

Problem 2 - What appears to be the relationship between sunspot number and solar irradiance?

Problem 3 - A homeowner built a solar electricity (photovoltaic) system on his roof in 1985 that produced 3,000 kilowatts-hours of electricity that year. Assuming that the amount of ground-level solar power is similar to the ACRIM measurements, about how much power did his system generate in 1989?

Mare Nubium and Las Vegas



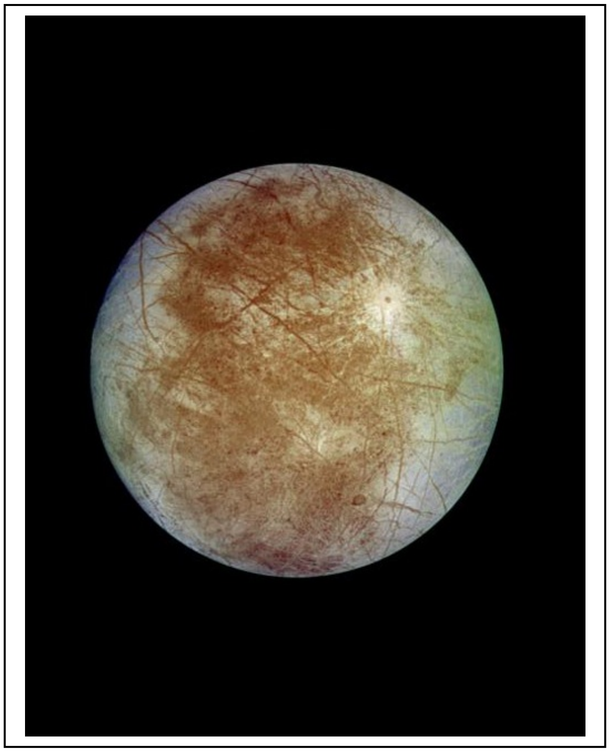
The LRO satellite recently imaged the surface of the moon at a resolution of 1.4 meters/pixel. The above 700-meter wide image shows downtown Las Vegas, Nevada (Top - Courtesy of Digital Globe, Inc.), and Mare Nubium (bottom - LRO) at this same resolution.

Problem 1 - About how big, in meters, are the large, medium and small-sized craters in the LRO image?

Problem 2 - How do the large, medium and small-sized craters compare to familiar objects in Downtown Las Vegas, or in your neighborhood?

Problem 3 - The Space Shuttle measures 37 meters long and has a wingspan of 24 meters. Draw a sketch of the Shuttle in the LRO image. Would you be able to see the Space Shuttle on the moon's surface at this resolution scale? (Note that the Space Shuttle is not equipped to travel to the moon and land!).

Ice or Water?



There are no known terrestrial organisms that can exist at a temperature lower than the freezing temperature of water. It is also believed that liquid water is a crucial ingredient to the chemistry that leads to the origin of life. To change water-ice to liquid water requires energy.

First, you need energy to raise the ice from wherever temperature it is, to 0 Celsius. This is called the Specific Heat and is 2.04 kiloJoules/kilogram C

Then you need enough energy added to the ice near 0 C to actually melt the ice by increasing the kinetic energy of the water molecules so that their hydrogen bonds weaken, and the water stops acting like a solid. This is called the Latent Heat of Fusion and is 333 kiloJoules/kilogram.

Let's see how this works!

Example 1: You have a 3 kilogram block of ice at a temperature of -20 C. The energy needed to raise it by 20 C to a new temperature of 0 C is Eh = 2.04 kiloJoules/kg C) x 3 kilograms x (20 C) = 2.04 x 3 x 20 = 122 kiloJoules.

Example 2: You have a 3 kilogram block of ice at 0 C and you want to melt it completely into liquid water. This requires Em = 333 kiloJoules/kg x 3 kilograms = 999 kiloJoules.

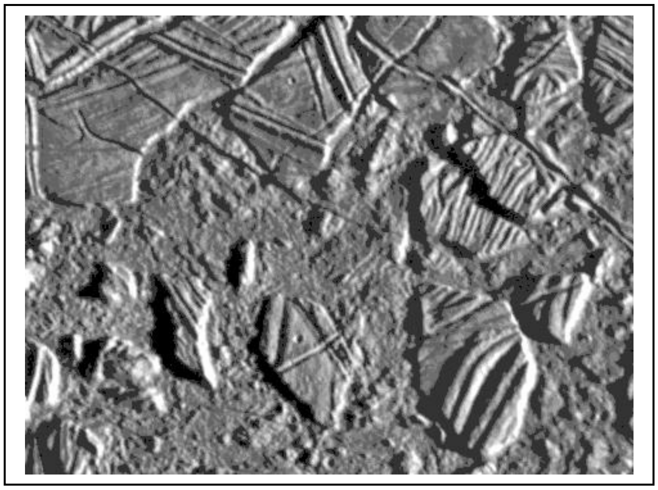
Example 3: The total energy needed to melt a 3 kilogram block of ice from -20 C to 0C is E = Eh + Em = 122 kiloJoules + 999 kiloJoules = 1,121 kiloJoules.

Problem 1 - On the surface of the satellite Europa (see NASA's Galileo photo above), the temperature of ice is -220 C. What total energy in kiloJoules is required to melt a 100 kilogram block of water ice on its surface? (Note: Calculate Eh and Em separately then combine them to get the total energy.)

Problem 2 - To a depth of 1 meter, the total mass of ice on the surface of Europa is 2.8×10^{16} kilograms. How many Joules would be required to melt the entire surface of Europa to this depth? (Note: Calculate Eh and Em separately then combine them to get the total energy. Then convert kiloJoules to Joules)

Problem 3 - The sun produces 4.0×10^{26} Joules every second of heat energy. How long would it take to melt Europa to a depth of 1 meter if all of the sun's energy could be used? (Note: The numbers are BIG, but don't panic!)

Water on Planetary Surfaces



Space is very cold! Without a source of energy, like a nearby star, water will exist at a temperature at nearly -270 C below zero and frozen solid. To create a permanent body of liquid water in which pre-biotic chemistry can occur, a steady source of energy must flow into the ice to keep it melted and in liquid form. Common sources of energy on Earth are volcanic activity, oceanic vents and fumaroles, and sunlight.

The picture above was taken by NASA's Galileo spacecraft of the surface of Jupiter's moon Europa. Its icy crust is believed to hide a liquid-water ocean beneath. The energy for keeping the water in a liquid state is probably generated by the gravity of Jupiter, which distorts Europa's shape through tidal action. The tidal energy may be enough to keep the oceans liquid for billions of years.

A common measure of energy flow or usage is the Watt. One Watt equals one Joule of energy emitted or consumed in one second.

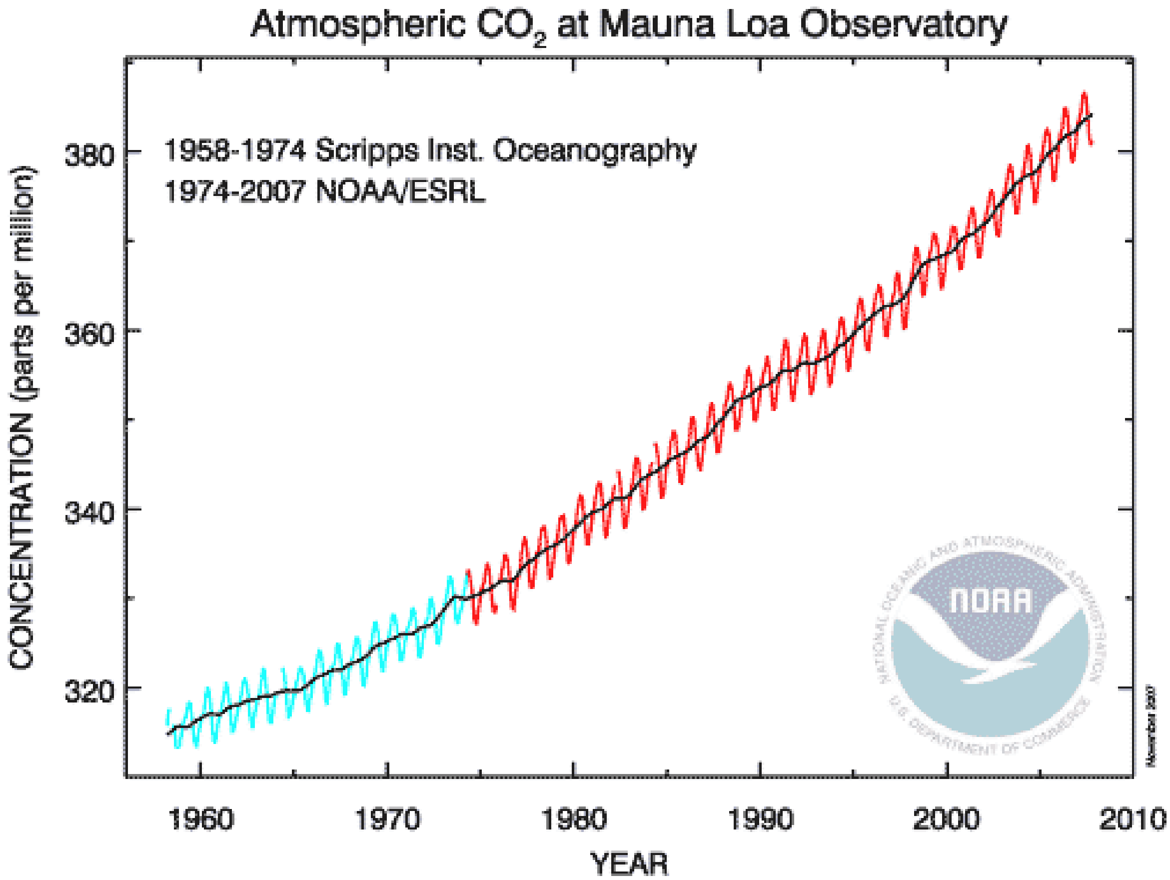
Problem 1: How much energy, in Joules, does a 100 watt incandescent bulb consume if left on for 1 hour?

Problem 2: A house consumes about 3,000 kilowatts in one hour. How many Joules is this?

Problem 3: A homeowner has a solar panel system that produces 3,600,000 Joules every hour. How many watts of electrical appliances can be run by this system?

Water ice at 0 C requires 330,000 Joules of energy to become liquid for each kilogram of ice. Suppose the ice absorbed all the energy that fell on it. Ice doesn't really work that way, but let's suppose that it does just to make a simple mathematical model!

Problem 4: A student wants to melt a 10 kilogram block of ice with a 2,000-watt hair dryer. How many seconds will it take to melt the ice block completely? How many minutes?



This is the Keeling Curve, derived by researchers at the Mauna Kea observatory from atmospheric carbon dioxide measurements made between 1958 - 2005. The accompanying data in Excel spreadsheet form for the period between 1982 and 2008 is provided at

http://spacemath.gsfc.nasa.gov/data/KeelingData.xls

Problem 1 - Based on the tabulated data, create a single mathematical model that accounts for, both the periodic seasonal changes, and the long-term trend.

Problem 2 - Convert your function, which describes the carbon dioxide volume concentration in parts per million (ppm), into an equivalent function that predicts the mass of atmospheric carbon dioxide if 383 ppm (by volume) of carbon dioxide corresponds to 3,000 gigatons.

Problem 3 - What would you predict as the carbon dioxide concentration (ppm) and mass for the years: A) 2020? B)2050, C)2100?

On July 4, 2005 at 5:45 UT the 362-kilogram Impactor from NASA's Deep Impact mission, collided with the nucleus of the comet Tempel 1, causing a bright flash of light and a plume of ejected gas (see photo).

Traveling at 10.3 km/sec, the Impactor created a crater on the nucleus and ejected about 10,000 tons of material.

The average density of the comet nucleus is 400 kg/m^3 and its size can be approximated as a sphere with a radius of 3 kilometers.

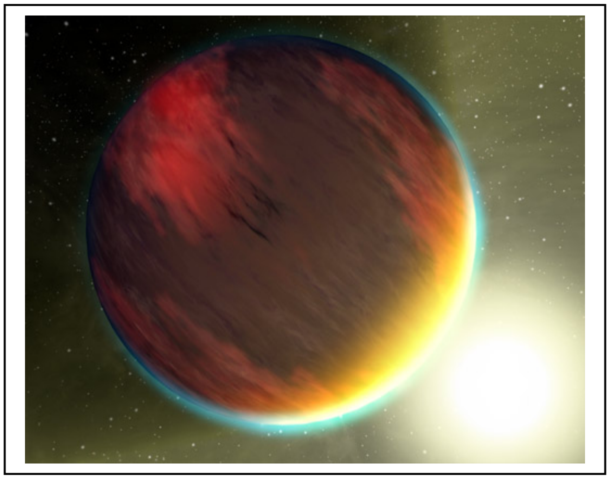
Problem 1 – From the information given, what was the approximate mass of the comet nucleus in kilograms?

Problem 2 - If the Impactor's path was perpendicular to the path taken by the Comet's nucleus, conservation of momentum requires that the product of the mass of the Impactor and its speed perpendicular to the orbit must equal the product of the comet's mass and the comet's speed perpendicular to the orbit after the impact assuming no mass loss. Although the impact ejected 10,000,000 kilograms of comet material, we will ignore this effect since the comet's mass was over 45 trillion kilograms! From the information, what is the final speed of the comet nucleus perpendicular to its orbit in A) kilometers/sec? B) meters/year?

Problem 3 – How far, in kilometers, will the comet nucleus have drifted 'sideways' to its orbit after 1 million years?

Problem 4 – Suppose that the comet had been headed toward Earth, and it was predicted that in 50 years it would collide with Earth. A nuclear bomb with an explosive yield equal to 10 million tons of TNT is launched to intercept the comet nucleus and deliver a blast, whose energy is equal to that of a 7.5 x 10^8 kilogram kilogram Impactor traveling at 10.3 km/sec. Assuming that the nucleus is not pulverized, A) about how far, in kilometers, will the nucleus drift after 20 years? B) Is this enough to avoid hitting Earth (diameter = 12,000 kilometers)?

Organic Molecules Detected on Distant Planet!



The basic chemistry for life has been detected in a second hot gas planet, HD 209458b, depicted in this artist's concept. Two of NASA's Great Observatories – the Hubble Space Telescope and Spitzer Space Telescope, yielded spectral observations that revealed molecules of carbon dioxide, methane and water vapor in the planet's atmosphere. HD 209458b, bigger than Jupiter, occupies a tight, 3.5-day orbit around a sun-like star about 150 light years away in the constellation Pegasus. (NASA Press release October 20, 2009)

Some Interesting Facts: The distance of the planet from the star HD209458 is 7 million kilometers, and its orbit period (year) is only 3.5 days long. At this distance, the temperature of the outer atmosphere is about 1,000 C (1,800 F). At these temperatures, water, methane and carbon dioxide are all in gaseous form. It is also known to be losing hydrogen gas at a ferocious rate, which makes the planet resemble a comet! The planet itself has a mass that is 69% that of Jupiter, and a volume that is 146% greater than that of Jupiter. The unofficial name for this planet is Osiris.

Problem 1 - The mass of Jupiter is 1.9 x 10^{30} grams. The radius of Jupiter is 71,500 kilometers. A) What is the volume of Jupiter in cubic centimeters, assuming it is a perfect sphere? B) What is the density of Jupiter in grams per cubic centimeter (cc), based on its mass and your calculated volume?

Problem 2 - From the information provided; A) What is the volume of Osiris in cubic centimeters, if it is in the shape of a perfect sphere? B) What is the mass of Osiris in grams? C) What is the density of Osiris in grams/cc, and how does this compare to the density of Jupiter?

Problem 3 - The densities of some common ingredients for planets are as follows:

Rock ……………………………….	3.0 grams/cc
Iron ……………………………….	9.0 grams/cc
Water ………………………………	5.0 grams/cc
Ice ………………………………….	1.0 gram/cc
Mixture of hydrogen + helium ……	0.7 grams/cc

Based on the average density of Osiris, from what substances do you think the planet is mostly composed?

How Hot is that Planet?

Planets have been spotted orbiting hundreds of nearby stars, but this makes for a variety of temperatures depending on how far the planet is from its star and the stars luminosity.

The temperature of the planet will be about

$$T=273\left(\frac{(1\text{-}A)L}{D^2}\right)^{1/4}$$

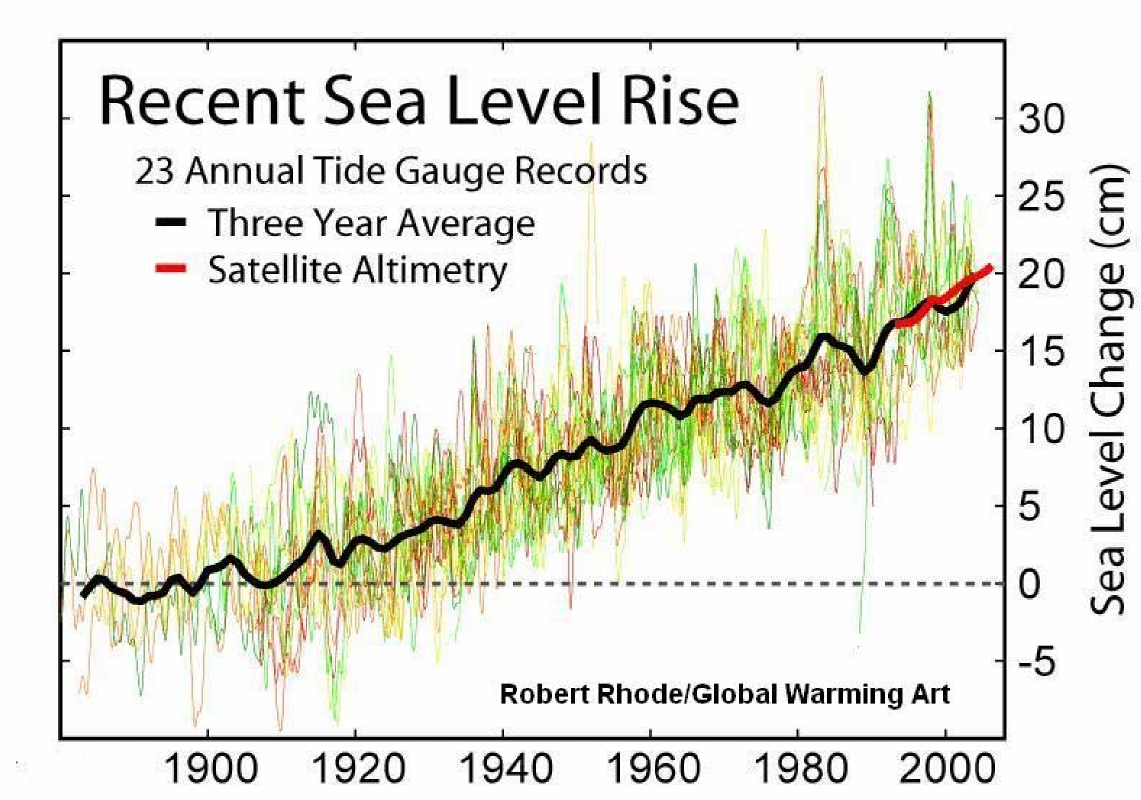
where A is the reflectivity (albedo) of the planet, L is the luminosity of its star in multiples of the sun's power, and D is the distance between the planet and the star in Astronomical Units (AU), where 1 AU is the distance from Earth to the sun (150 million km). The resulting temperature will be in units of Kelvins. (i.e. 0^{o} Celsius = +273 K, and Absolute Zero is defined as 0 K).

Problem 1 - Earth is located 1.0 AU from the sun, for which L = 1.0. What is the surface temperature of Earth if its albedo is 0.4?

Problem 2 - At what distance would Earth have the same temperature as in Problem 1 if the luminosity of the star were increased 1000 times and all other quantities remained the same?

Problem 3 - The recently discovered planet CoRoT-7b (see artist's impression above, from ESA press release), orbits the star CoRoT-7 which is a sun-like star located about 490 light years from Earth in the direction of the constellation Monoceros. If the luminosity of the star is 71% of the sun's luminosity (L = 0.71) and the planet is located 2.6 million kilometers from its star (D= 0.017 AU) what are the predicted surface temperatures of the day-side of CoRoT-7b for the range of albedos shown in the table below?

Surface Material	Example	Albedo (A)	Surface Temperature (K)
Basalt	Moon	0.06	1892
Iron Oxide	Mars	0.16	
Water+Land	Earth	0.40	
Gas	Jupiter	0.70	



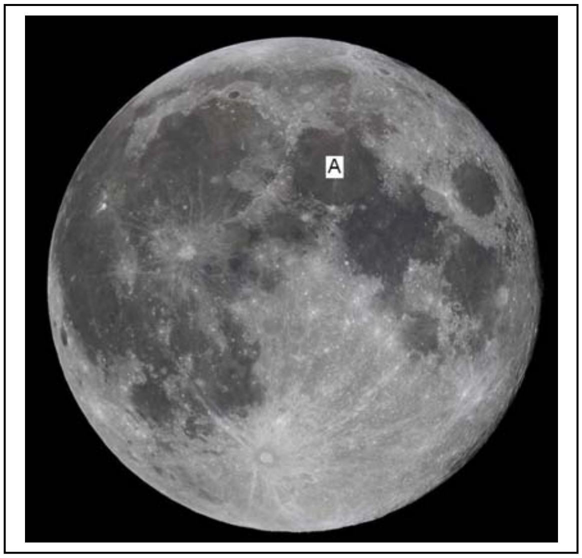
The graph, produced by scientists at the University of Colorado and published in the IPCC Report-2001, shows the most recent global change in sea level since 1880 based on a variety of tide records and satellite data. The many colored curves show the individual tide gauge trends. The black line represents an average of the data in each year.

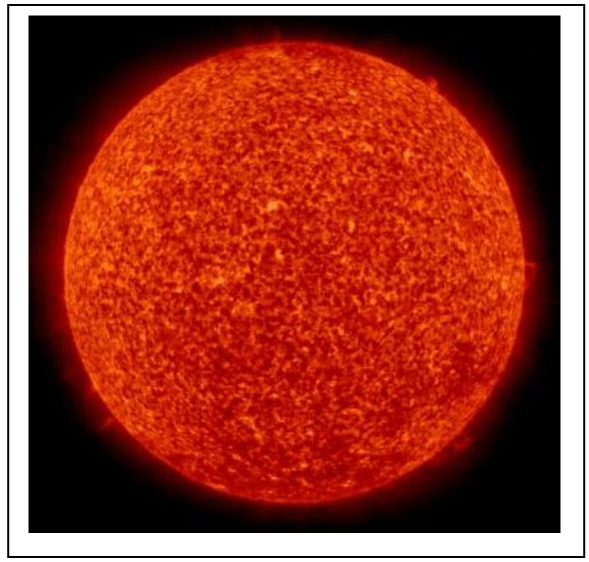
Problem 1 - If you were to draw a straight line through the curve between 1920 and 2000 representing the average of the data, what would be the slope of that line?

Problem 2 - What would be the equation of the straight line in A) Two-Point Form? B) Point-Slope Form? C) Slope-Intercept Form?

Problem 3 - If the causes for the rise remained the same, what would you predict for the sea level rise in A) 2050? B) 2100? C) 2150?

Getting an Angle on the Sun and Moon





The Sun (Diameter = 1,400,000 km) and Moon (Diameter = 3,476 km) have very different physical diameters in kilometers, but in the sky they can appear to be nearly the same size. Astronomers use the angular measure of arcseconds (asec) to measure the apparent sizes of most astronomical objects. (1 degree equals 60 arcminutes, and 1 arcminute equals 60 arcseconds). The photos above show the Sun and Moon at a time when their angular diameters were both about 1,865 arcseconds.

Problem 1 - Using a metric ruler, what is the angular scale of each image in arcseconds per millimeter?

Problem 2 - In arcseconds, what is the size of the smallest feature you can see in the images of the Sun and Moon?

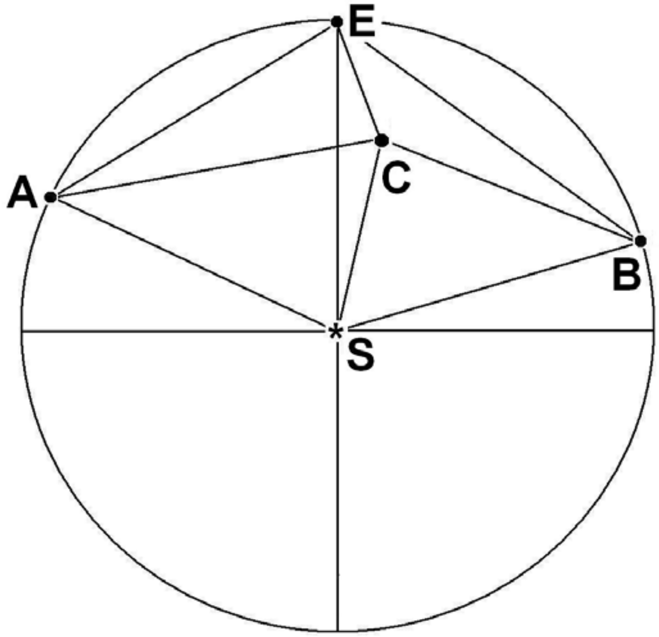
Problem 3 - About what is the area, in square arcseconds ($asec^2$) of the circular Mare Serenitatis (A) region in the photo of the Moon?

Problem 4 - At the distance of the Moon, 1 arcsecond of angular measure equals 1.9 kilometers. The Sun is exactly 400 times farther away than the Moon. On the photograph of the Sun, how many kilometers equals 1 arcsecond of angle?

Problem 5 - What is the area of Mare Serenitatis in square kilometers?

Problem 6 - What would be the physical area, in square-kilometers, of an identical angular area to Mare Serenitatis if it were located on the surface of the sun?

Seeing Solar Storms in STEREO - II



The two STEREO spacecraft are located along Earth's orbit and can view gas clouds ejected by the sun as they travel to Earth. From the geometry, astronomers can accurately determine their speeds, distances, shapes and other properties.

By studying the separate 'stereo' images, astronomers can determine the speed and direction of the cloud before it reaches Earth.

Use the diagram, (angles and distances not drawn to the same scale of the 'givens' below) to answer the following question.

The two STEREO satellites are located at points A and B, with Earth located at Point E and the sun located at Point S, which is the center of a circle with a radius ES of 1.0 Astronomical unit (150 million kilometers). Suppose that the two satellites spot a Coronal Mass Ejection (CME) cloud at Point C. Satellite A measures its angle from the sun mSAC as 45 degrees while Satellite B measures the corresponding angle to be mSBC=50 degrees. In the previous math problem the astronomers knew the ejection angle of the CME, mESC, but in fact they didn't need to know this in order to solve the problem below!

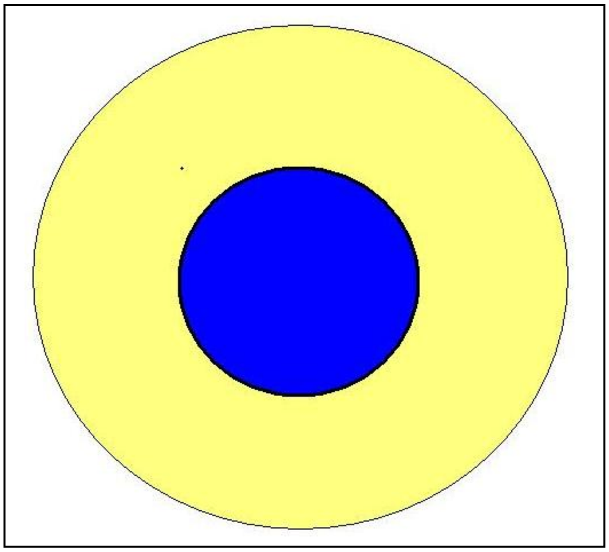
Problem 1 - The astronomers want to know the distance that the CME is from Earth, which is the length of the segment EC. The also want to know the approach angle, mSEC. Use either a scaled construction (easy: using compass, protractor and millimeter ruler) or geometric calculation (difficult: using trigonometric identities) to determine EC from the available data.

Givens from satellite orbits:

SB = SA = SE = 150 million km AE = 136 million km BE = 122 million km
mASE = 54 degrees mBSE = 48 degrees
mEAS = 63 degrees mEBS = 66 degrees mAEB = 129 degrees

Find the measures of all of the angles and segment lengths in thee above diagram rounded to the nearest integer.

Problem 2 - If the CME was traveling at 2 million km/hour, how long did it take to reach the distance indicated by the length of segment SC?



The planet Osiris orbits 7 million kilometers from the star HD209458 located 150 light years away in the constellation Pegasus. The Spitzer Space Telescope has recently detected water, methane and carbon dioxide in the atmosphere of this planet. The planet has a mass that is 69% that of Jupiter, and a volume that is 146% greater than that of Jupiter.

By knowing the mass, radius and density of a planet, astronomers can create plausible models of the composition of the planet's interior. Here's how they do it!

Among the first types of planets being detected in orbit around other stars are enormous Jupiter-sized planets, but as our technology improves, astronomers will be discovering more 'super-Earth' planets that are many times larger than Earth, but not nearly as enormous as Jupiter. To determine whether these new worlds are Earth-like, they will be intensely investigated to determine the kinds of compounds in their atmospheres, and their interior structure. Are these super-Earths merely small gas giants like Jupiter, icy worlds like Uranus and Neptune, or are they more similar to rocky planets like Venus, Earth and Mars?

Problem 1 - A hypothetical planet is modeled as a sphere. The interior has a dense rocky core, and on top of this core is a mantle consisting of a thick layer of ice. If the core volume is 4.18×10^{12} cubic kilometers and the shell volume is 2.92×10^{13} cubic kilometers, what is the radius of this planet in kilometers?

Problem 2 - If the volume of Earth is 1.1×10^{12} cubic kilometers, to the nearest whole number, A) how many Earths could fit inside the core of this hypothetical planet? B) How many Earths could fit inside the mantle of this hypothetical planet?

Problem 3 - Suppose the astronomer who discovered this super-Earth was able to determine that the mass of this new planet is 8.3 times the mass of Earth. The mass of Earth is 6.0×10^{24} kilograms. What is A) the mass of this planet in kilograms? B) The average density of the planet in kilograms/cubic meter?

Problem 4 - Due to the planet's distance from its star, the astronomer proposes that the outer layer of the planet is a thick shell of solid ice with a density of 1000 kilograms/cubic meter. What is the average density of the core of the planet?

Problem 5 - The densities of some common ingredients for planets are as follows:

Granite 3,000 kg/m^3 ; Basalt 5,000 kg/m^3 ; Iron 9,000 kg/m^3

From your answer to Problem 4, what is the likely composition of the core of this planet?

Energy is measured in a number of ways depending on what property is being represented.

Total Energy - Joules and ergs - The total amount of energy in various forms (kinetic, potential, magnetic, thermal, gravitational)

Power - Watts, Joules/second or ergs/second – the rate at which energy is produced or consumed in time. Power = Energy/Time

Flux - Watts/meter^2, Joules/sec/meter^2 or ergs/sec/meter^2 – the rate with which energy flows through a given area in given amount of time: Flux=Power/Area

1 Joule = 10 million ergs	1 kilowatt = 1,000 watts
1 Watt = 1 Joule/1 second	1 megaJoule = 1,000,000 Joules
1 hour = 3600 seconds	3 feet = 1.0 meters

Example: A 5-watt flashlight is left on for 1 hour: Convert its energy consumption of 5 watt-hours to Joules.

$$5 \text{ Watt-hours} \times \frac{1 \text{ Joule}}{1 \text{ sec } 1 \text{ watt}} \times \frac{3{,}600 \text{ sec}}{1 \text{ hour}} = 18{,}000 \text{ Joules}$$

Notice how the compound unit 'watt' is handled so that the appropriate units in the unit conversion ladder cancel.

Problem 1 – The flux of sunlight at Earth's surface is 1300 Watts/meter^2. Convert this flux to ergs/sec/cm^2.

Problem 2 – A 100-watt bulb shines light over a wall with a surface area of 25 meters^2. What is the flux of light energy in Joules/sec/meter^2?

Problem 3 – The common energy unit for electricity is the watt-hour (Wh), which can be written as 1 watt x 1 hour. How many megajoules equal 1 kilowatt-hour (1 kWh)?

Problem 4 – How many ergs of energy are collected from a solar panel on a roof, if the sunlight provides a flux of 300 Joules/sec/meter^2, the solar panels have an area of 27 square feet, and are operating for 8 hours during the day?

The Elementary Particle Masses

Particle	Mass	Status
Higgs Boson	>110 GeV	Predicted
Top Quark	170 GeV	Found
Z Boson	91 GeV	Found
W Boson	80 GeV	Found
Bottom Quark	4.2 GeV	Found
Tauon	1.8 GeV	Found
Charm Quark	1.2 GeV	Found
Strange Quark	120 MeV	Found
Muon	105 MeV	Found
Down Quark	4.0 MeV	Found
Up Quark	2.0 MeV	Found
Tau Neutrino	< 16 MeV	Found
Electron	0.5 MeV	Found
Muon Neutrino	< 0.2 MeV	Found
Electron Neutrino	< 2 eV	Found
Axion	< 1 eV	Predicted
Graviton	0	Predicted
Gluon	0	Found
Photon	0	Found

Most people are familiar with the Periodic Table of the Elements, which summarizes the properties of the 110 known elements starting from Hydrogen. These elements are composed of elementary particles called electrons, protons and neutrons, which combine in various numbers to build up all of the elements.

Since the 1950's, physicists have discovered an even more fundamental collection of particles that seem to be truly elementary, and which combine to produce not only all of the elements, but the very forces that hold them together. The table to the left shows the names of these particles and their mass. Some particles, such as the Higgs Boson are being proposed to exist because some theories require them in order to complete our understanding of how forces and particles interact.

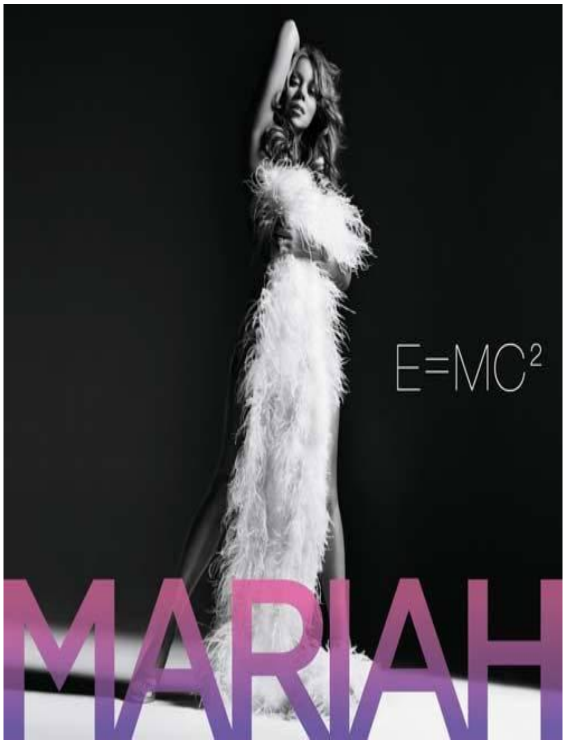
A basic property of a particle is its mass. Because the masses of the fundamental particles are vastly smaller than a gram or a kilogram, physicists use another unit called the electron Volt. This is actually a unit of energy, but because Albert Einstein demonstrated that energy, E, and mass, m, are basically the same things ($E = mc^2$), physicists conveniently state mass as $M = E/c^2$, and then drop the speed-of-light factor c^2 because it is understood to be part of the definition of mass when energy units are used as in the table above. For example, 1 billion electron Volts (1 GeV) are equal to 1.8×10^{-27} kilograms, which is about equal to the mass of a single proton, which is 1.7×10^{-27} kilograms (938 MeV). (Note also that 1 MeV = 1 million electron Volts.)

Problem 1 - What is the mass, in kilograms, of A) An electron? B) A Strange Quark? C) A Top Quark? and D) A Muon?

Problem 2 - A proton consists of two Up Quarks and one Down Quark held tightly together by the strong nuclear force. What is the total quark mass of a proton in A) MeV? B) In kilograms?

Problem 3 - From your answer to Problem 2, what is the mass difference between 1 proton and the combination of the two Up Quarks and one Down Quark in units of A) MeV? B) kilograms?

Problem 4 - An oxygen atom contains a total of 16 protons and neutrons and has a mass of 2.7×10^{-26} kilograms. What is the mass of a Top Quark in terms of the number of oxygen atoms?



Einstein's formula $E = mc^2$ is almost legendary, and sometimes appears even on T-shirts or even Mariah Carey's pop music album cover! But what does the formula really mean?

It means that, what we consider to be energy in its many forms (like light and heat), which we measure in Joules, can also be considered equivalent to mass in its many forms (like grains of sand or mountains), which we measure in units of kilograms!

The problem is that it takes a LOT of energy to make a kilogram of mass! That's why we never see matter suddenly just appearing out of nowhere in our daily lives. That little formula says that 1 kilogram of mass, m, is exactly equal to 9.0×10^{16} Joules of energy, E. That's the energy released by just one, 20 megaton hydrogen bomb!

The formula depends on the speed of light-squared, c^2, to do the conversion from energy units (E in Joules) to matter units (m in kilograms). If c = 300 million meters/sec, we have the simpler formula: E(Joules) = 9×10^{16} x (mass in kilograms). Use this formula to perform the following conversions using a calculator, and providing answers to 2 significant figures in scientific notation:

Problem 1 1-proton mass = 1.6726×10^{-27} kg equals --------------- Joules

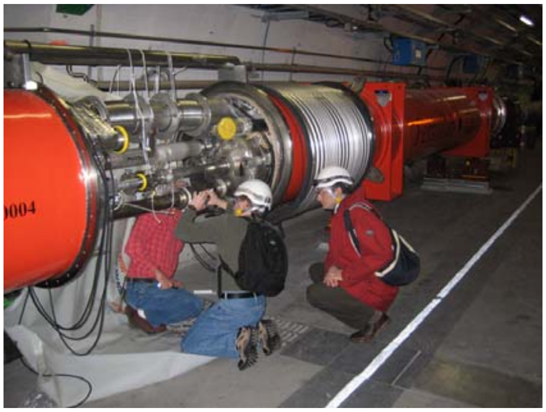
Problem 2 1 Joule of Energy = ------------------- kilograms

Problem 3 1 electron Volt = 1.6×10^{-19} Joules = ------------------ kilograms

Problem 4 1 neutron = 1.6749×10^{-27} kg equals --------------- Joules

Problem 5 1 deuterium nucleus (1 proton+1 neutron) = 3.3444×10^{-27} kg equals --------------- Joules

Problem 6 What is: A) The difference in mass between a single proton plus a single neutron, and the deuterium nucleus? B) The difference in energy? C) The difference in Volts?



A small section of the LHC (Photo: Peter Limon)

During November, 2009 the Large Hadron Collider experiment at CERN began a slow, step-by-step process of being turned on. Its goal is to accelerate two beams of protons and anti-protons to very high energies, and collide them to form new sub-atomic particles for physicists to discover and study.

Although it is not a NASA research program, it will directly impact the findings of many NASA research satellites such as Chandra and WMAP in searching for Dark Matter and Dark Energy. To understand the operation of LHC, we must first learn how to measure and discuss very small amounts of energy!

The 27-kilometer diameter LHC ring, buried deep underground, uses thousands of magnets to steer two beams of protons so that they collide at specific points along the ring. The beams are no bigger than a human hair in diameter, but contain millions of protons and anti-protons traveling at nearly the speed of light and circulating in the 27-kilometer ring in opposite directions. When these particles collide, their energies combine to create a momentary explosion out of which is created hundreds of particles including electrons, quarks and even more massive particles. Thanks to Albert Einstein's $E=mc^2$, the energy of the two protons can create a burst of new particles, m, literally created out of the raw energy, E, of the collisions.

Physicists use the 'electron volt' (eV) as a convenient unit of energy. One 'eV' is the amount of energy that an electron gains as it falls through a voltage difference of exactly 1 volt. This energy is equal to 1.6×10^{-19} Joules. Physicists find it far easier to remember and write energy measured in units of eV than in Joules! The following problems exercise your ability to translate between eV units and Joules.

Problem 1 - The mass of an electron is equivalent to 511,000 electron-Volts of energy (also written as 511 keV). How many Joules is this?

Problem 2 - The mass of a Top Quark is 175 billion eV (or 175 Giga-electron Volts abbreviated as 175 GeV). How many Joules is this?

Problem 3 - A proton has an energy equivalent to 1.5×10^{-10} Joules. How many electron volts is this?

Problem 4 - A small gnat in flight carries about 1.6×10^{-7} Joules of energy. About how many electron Volts is this equivalent to?

Problem 5 - When the LHC began operation, on November 30, 2009 it achieved an energy of 1.2 trillion electron Volts (1.2 Terra eV or 1.2 TeV). How many Joules of energy is this?

Problem 6 - When fully operational, the LHC will carry 100 trillion protons along each beam, with each proton carrying an energy of 15 TeV. What is the total energy of each proton beam in the LHC compared to: A) A baseball in flight (120 Joules)? B) A small car traveling at 60 mph (300,000 Joules)? C) A Boeing 767 in flight (4 billion Joules)?

$$T = \frac{10^{10}}{\sqrt{t}} \text{ Kelvin}$$

$$E = \frac{860{,}000}{\sqrt{t}} \text{ eVolts}$$

T is the temperature in degrees Kelvin that was reached **t** seconds after the Big Bang.

E is the energy in electron Volts that was reached **t** seconds after the Big Bang.

Moments after the Big Bang, the universe was brilliantly hot, but steadily cooled with every passing second. All of the matter we see around us today was once fragmented into its individual parts, and we can re-create many of the original Big Bang conditions in our laboratories today. All we have to do is heat up matter so that the parts collide with the same energy as they would have during the Big Bang.

Physicists do this by using 'atom smashers' that accelerate individual particles such as electrons and protons, and then collide them. Thanks to Relativity, and Einstein's famous equation $E = mc^2$, all of the energy of the collision is then available for creating new kinds of particles…if they exist.

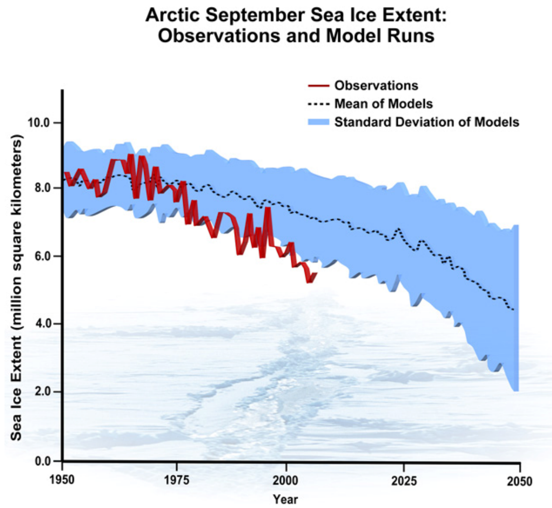
Astronomers can also use the Big Bang model to calculate at exactly what time after the Big Bang particles of matter typically had the energies being explored under laboratory conditions. There are two different formula, depending on whether you want to predict the temperature of the gas, or the average energy of the particles in the gas, as shown on the left.

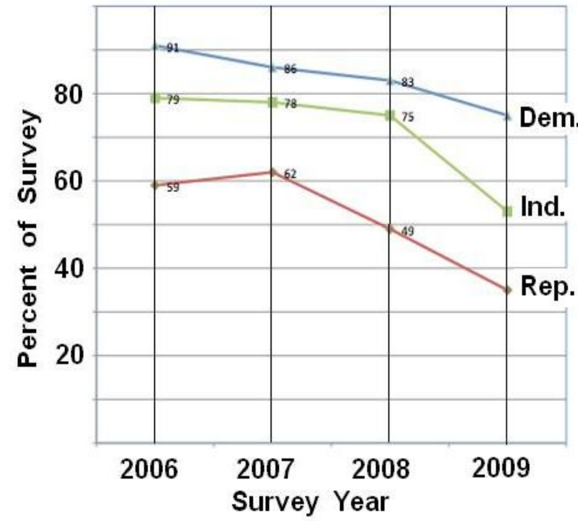
Problem 1 - At 100 seconds after the Big Bang, what was the temperature of the universe, and what was the average collision energy, in kilovolts, of the particles at that time?

Problem 2 - Collisions between particles with a combined energy of 2 billion Volts (2 GeV) can produce a pair of particles, one proton and one anti-proton, out of pure energy. How many seconds after the Big Bang were particles colliding with these energies?

Problem 3 - How hot did the Big Bang have to be in order for it to create particles as massive as a pair of Top Quarks (E = 175 GeV for one top quark), and how long after the Big Bang was this temperature achieved?

Problem 4 - The Large Hadron Collider at CERN in Switzerland was recently 'powered-up' and achieved collision energies of 1.2 TeV, with an ultimate goal of about 15 TeV when it is fully operational in 2010. If 1 TeV = 1 trillion electron Volts (1 Terra eV), A) how many seconds after the Big Bang will the LHC be able to explore the state of matter at the lower and upper energy limits achieved in 2009 and 2010? B) What will be the temperature of matter at these two times?





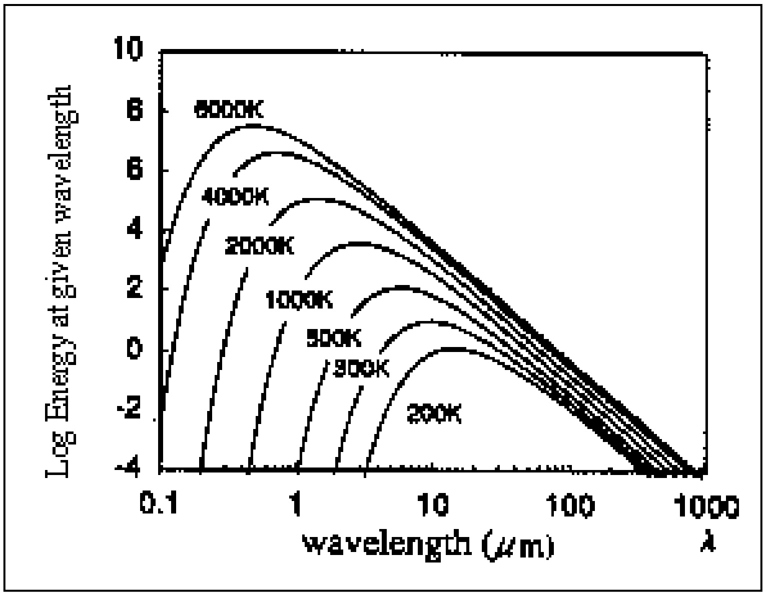
The graph above, based upon research by the National Sea Ice Data Center (Courtesy Steve Deyo, UCAR), shows the amount of Arctic sea ice in September (coldest Arctic month) for the years 1950-2006, based on satellite data (since 1979) and a variety of direct submarine measurements (1950 - 1978). The blue region indicates model forecasts based on climate models.

The figure on the right shows the results of polls conducted between 2006 and 2009 of 1,500 adults by the Pew Research Center for the People & the Press. The graph indicates the number of people, in all three major political parties, believing there is strong scientific evidence that the Earth has gotten warmer over the past few decades.

Problem 1 - Based on the curve in the sea ice graph, which gives the number of millions of square kilometers of Arctic sea ice identified between 1950 and 2006, what is a linear equation that models the average trend in the data between 1950-2006?

Problem 2 - From your linear model for Arctic ice cover, about what year will the Arctic Ice Cap have lost half the sea ice that it had in 1950-1975?

Star Light…Star Bright



Astronomers can 'take the temperature' of a star by measuring the star's brightness through two filters that pass radiation in the 'blue' and 'visual' regions of the visible spectrum.

From the ratio of these brightnesses, a simple cubic relationship yields the temperature of the star, in Kelvins as follows:

$$T(x) = 9391 - 8350\,x + 5300\,x^2 - 1541\,x^3$$

This formula works for star temperatures between 9,000 and 3,500 Kelvin.

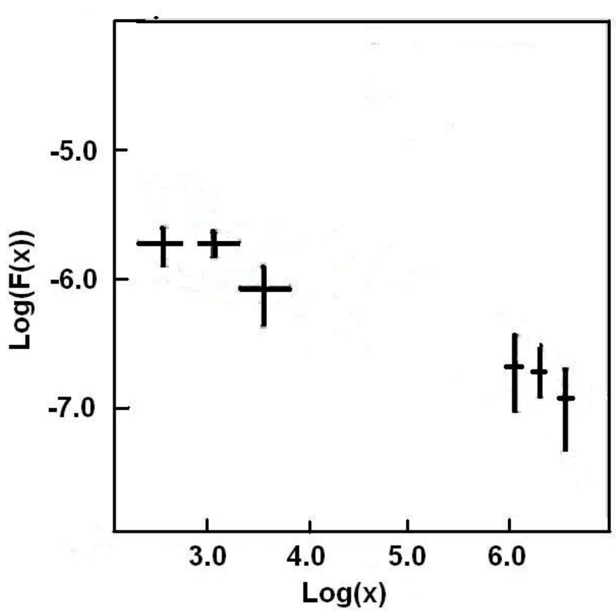
Problem 1 - From the indicated temperature range, what is the domain of this function?

Problem 2 – The sun has a temperature of 5770 K. What is the corresponding value for x?

Problem 3 – To save computation time, an astronomer uses the approximation for T(x) based on a quadratic formula given by $\mathbf{F(x) = 1844x^2 - 6410x + 9175}$. What is the formula that gives the percentage difference, P(x), between F(x) and T(x)?

M82, or the Cigar Galaxy, is a starburst galaxy about 12 million light-years away from Earth. In the galaxy's center, stars are being born 10 times faster than they are inside the entire Milky Way galaxy.

In this false-color image, X-ray data recorded by the Chandra X-ray observatory is blue; infrared light recorded by the Spitzer infrared telescope is red; Hubble space telescope observations of hydrogen line emission is orange, and the bluest visible light is yellow-green. (Credit: NASA/JPL-Caltech, STScI, CXC, Uof A, ESA, AURA, JHU)



The Fermi Gamma-Ray Space Telescope has recently confirmed that the nearby galaxy, Messier 82, is the major source of high-energy gamma-rays seen at Earth: over 12 million light years away!

This galaxy has an active core in which a massive black hole is absorbing matter and turning it into energy at a ferocious rate.

The gamma-rays arrive at Earth, at a rate of about one or two per hour, and span a range of energies (x given in MeV) that are shown in the Log-Log plot to the left. F(x) is related to the number of gamma-rays detected per second over an area of 1 square centimeter.

Power-Laws

A surprising number of physical phenomena can be mathematically represented, at least over a part of their range, in terms of a power-law function **$F(x) = ax^n$**. We are going to explore some interesting, and convenient, features of power-law functions in analyzing data.

Problem 1 - Show that the graph of Log(F(x)) vs Log(x) is a straight line.

Problem 2 - The graph to the left shows the gamma-ray energy spectrum measured by Fermi. The data points are presented as crosses (called error bars), with the measured value being at the center of the cross. The size of each error bar shows the acceptable range of the measurement. Using a ruler, what linear equation passes through the crosses for the entire collection of data?

Problem 3 - From your answer to Problem 2, what is the 'best fit' power-law, F(x), defined by the linear equation you derived from the data?

Problem 4 - Evaluate F(4.0) and F(5.0) to determine the number of gamma rays/sec/cm^2 at energies of 10,000 and 100,000 MeV.


Life-size model of the Webb Space Telescope.

In 2014, the new Webb Space Telescope will be launched. This telescope, designed to detect distant sources of infrared 'heat' radiation, will be a powerful new instrument for discovering distant dwarf planets far beyond the orbit of Neptune and Pluto.

Scientists are already predicting just how sensitive this new infrared telescope will be, and the kinds of distant bodies it should be able to detect in each of its many infrared channels. This problem shows how this forecasting is done.

Problem 1 - The angular diameter of an object is given by the formula:

$$\theta(R) = 0.0014\frac{L}{R} \text{ arcseconds}$$

Create a single graph that shows the angular diameter, θ(R), for an object the size of dwarf planet Pluto (L=2,300 km) spanning a distance range, R, from 30 AU to 100 AU, where 1 AU (Astronomical Unit) is the distance from Earth to the sun (150 million km). How big will Pluto appear to the telescope at a distance of 90 AU (about 3 times its distance of Pluto from the sun)?

Problem 2 - The temperature of a body that absorbs 40% of the solar energy falling on it is given by

$$T(R) = \frac{250}{\sqrt{R}}$$

where R is the distance from the sun in AU. Create a graph that shows θ(R) vs R for objects located in the distance range from 30 to 100 AU. What will be the predicted temperature of a Pluto-like object at 90 AU?

Problem 3 - A body with an angular size θ(R) given in arcseconds emits 40% of its light energy in the infrared and has a temperature given by T(R) in Kelvin degrees. Its brightness in units of Janskys, F, at a wavelength of 25 microns (2500 nanometers) will be given by:

$$F(T) = \frac{11500}{(e^x - 1)}\theta(R)^2 \text{ Janskys} \qquad \text{where} \qquad x = \frac{580}{T(R)}$$

From the formula for θ(R) and T(R), create a curve F(R) for a Pluto-like object. If the Webb Space Telescope cannot detect objects fainter than 4 nanoJanskys, what will be the most distant location for a Pluto-like body that this telescope can detect? (Hint: Plot the curve with a linear scale in R and a log10 scale in F.)

Computing the Orbit of a Comet

This historic image of the nucleus of Halley's Comet by the spacecraft Giotto in 1986 reveals the gases leaving the icy body to form the tail of the comet.

Once astronomers discover a new comet, a series of measurements of its location allows them to calculate the orbit of the comet and predict when it will be closest to the Sun and Earth.

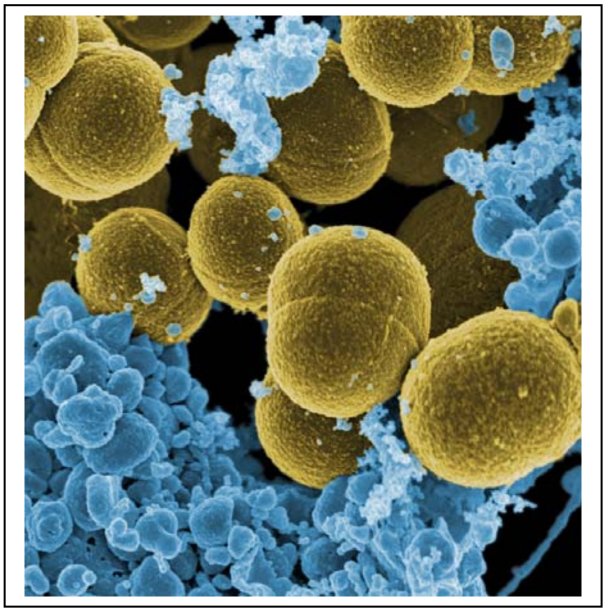
Problem 1 – Astronomers measured two positions of Halley's Comet along its orbit. The x and y locations in its orbital plane are given in units of the Astronomical Unit, which is a unit equal to the distance from Earth to the sun (150 million km). The positions are (+10, +4) and (+14, +3). What are the two equations for the elliptical orbit based on these two points, written as quadratic equations in a and b, which are the lengths of the semimajor and seminor axis of the ellipse?

Problem 2 – Solve the system of two quadratic equations for the ellipse parameters a and b.

Problem 3 – What is the orbit period of Halley's Comet from Kepler's Third Law if $P^2 = a^3$ where a is in Astronomical Units and P is in years?

Problem 4 – The perihelion of the comet is defined as d = a – c where c is the distance between the focus of the ellipse and its center. How close does Halley's Comet come to the sun in this orbit in kilometers?

Estimating Maximum Cell Sizes



NASA's astrobiology program is exploring the basic ingredients of living systems. One of these basic elements is the cell.

A simple living cell generates wastes from the volume inside its cell wall, and passes the wastes outside its cell wall by a process called passive diffusion.

Photo: *S. aureus* bacteria escaping destruction by human white blood cells. (Credit: NIAID / RML)

If a cell cannot remove the waste fast enough, toxins will build up that eventually kill the cell. The balance between waste generation and diffusion, therefore, determines how much volume a cell may have and therefore its typical size.

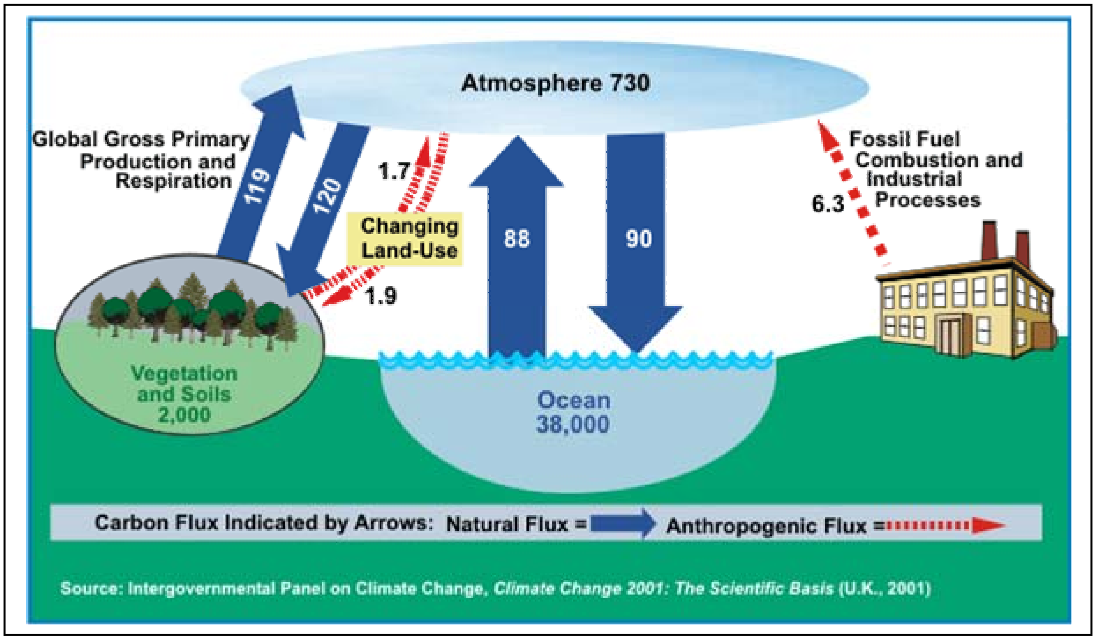
Suppose the cell has a spherical volume, and that it generates waste at a rate of **a** molecules per cubic micron per second. Suppose it removes the waste through its surface at a rate of b molecules per square micron per second, where 1 micron is 0.000001 meters.

Problem 1 - What is the equation that defines the rate, R, at which the organism changes the amount of its net waste products?

Problem 2 - For what value for the cell's radius will the net change be zero ,which means the cell is in equilibrium?

Problem 3 - A hypothetical cell metabolism is measured to be a = 800 molecules/μm^3/sec and b = 2000 molecules/μm^2/sec , about how large might such a cell be if it removed waste products only by passive diffusion?

Problem 4 - Two organisms are discovered that have a size of 1 micron and 10 microns. A) How does the ratio of their diffusion rates compare? B) If the surface waste diffusion rates are the same, how do their metabolic rates of waste production compare?



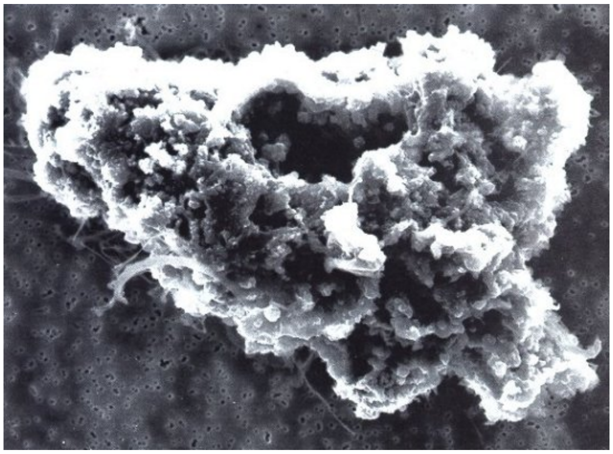
The figure above shows a simplified view of the sources and sinks of the element carbon on Earth. Note that, for every 44 gigatons of the carbon dioxide molecule, there are 12 gigatons of the element carbon.

Problem 1 - What are the sources of carbon increases to the atmosphere in the above diagram? What are the sinks of carbon?

Problem 2 - From the values of the sources and sinks, and assuming they are constant in time, create a simple differential equation that gives the rate-of-change of atmospheric carbon in gigatons.

Problem 3 - Integrate the equation in Problem 2, assuming that C(2005) = 730 gigatons, and derive the simple equation describing the total amount of carbon in the atmosphere as a function of time.

Problem 4 - What does your model predict for the amount of carbon in the atmosphere in 2050 if the above source and sink rates remain the same?


A cosmic dust grain about 0.1mm across captured by a NASA high-altitude aircraft. Probably debris from a passing comet

Planets are built in several stages. The first of these involves small, micron-sized interstellar dust grains that collide and stick together to eventually form centimeter-sized bodies. A simple model of this process can tell us about how long it takes to 'grow' a rock-sized body starting from microscopic dust. This process occurs in dense interstellar clouds, which are known to be the birth places for stars and planets.

Problem 1 – Assume that the forming rock is spherical with a density of 3 gm/cc, a radius R, and a mass M. If the radius is a function of time, R(t), what is the equation for the mass of the rock as a function of time, M(t)?

Problem 2 – The rock grows by absorbing incoming dust grains that have an average mass of m grams and a density of N particles per cubic centimeter in the dust cloud. The particles collide with the surface of the rock at a speed of V cm/sec, what is the equation that gives the rate of growth of the rock's mass in time (dM/dt)?

Problem 3 – From your answer to Problem 1 and 2, re-write dM/dt in terms of M not R.

Problem 4 – Integrate your answer to problem 3 so determine M(t).

Problem 5 – What is the mass of the rock when it reaches a diameter of 1 centimeter if its density is 3 grams/cc?

Problem 6 – The rock begins at t=0 with a mass of 1 dust grain, m = 8×10^{-12} grams. The cloud density N = 3.0×10^{-5} dust grains/cc and the speed of the dust grains striking the rock, without destroying the rock, is V=10 cm/sec. How many years will the growth phase have to last for the rock to reach a diameter of 1 centimeter?


Asteroid Gaspra about 15 km across. Image taken by the NASA Galileo spacecraft.

Planets are built in several stages. The first of these involves small, interstellar dust grains that collide and stick together to form centimeter-sized bodies. This can take millions of years. The second stage involves the formation of kilometer-sized asteroids from the centimeter-sized rocks. A simple model of this process can tell us about how long it takes to 'grow' an asteroid from rock-sized bodies.

Problem 1 – Assume that the forming asteroid is spherical with a density of 3 gm/cc, a radius R, and a mass M. If the radius is a function of time, R(t), what is the equation for the mass of the asteroid as a function of time, M(t)?

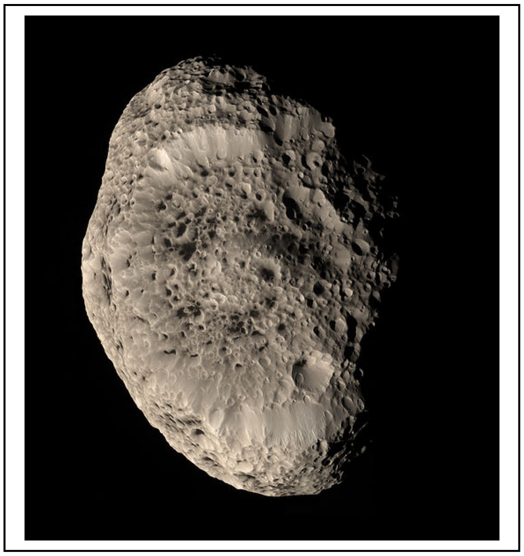
Problem 2 – The asteroid grows by absorbing incoming rocks that have an average mass of 5.0 grams and a density of N rocks per cubic centimeter in the cloud. The rocks collide with the surface of the forming asteroid at a speed of V cm/sec, what is the equation that gives the rate of growth of the asteroid's mass in time (dM/dt)?

Problem 3 – From your answer to Problem 1 and 2, re-write dM/dt in terms of M not R.

Problem 4 – Integrate your answer to problem 3 so determine M(t).

Problem 5 – What is the mass of the asteroid when it reaches a diameter of 1 kilometer if its density is 3 grams/cc?

Problem 6 – The asteroid begins at t=0 with a mass of m=5 grams. The cloud density N = 1.0×10^{-8} rocks/cc and the speed of the rocks striking the asteroid, without destroying the asteroid, is V=1 kilometer/sec. How many years will the growth phase have to last for the asteroid to reach a diameter of 1 kilometer?



Planets are built in several stages. Dust grains grow to large rocks in a million years, then rocks accumulate to form asteroids in a few years or so. The third stage combines kilometer-wide asteroids to make rocky planets. A simple model of this process can tell us about how long it takes to 'grow' a planet by accumulating asteroid-sized bodies through collisions. Saturn's moon Hyperion (see image) is 300 km across and is an example of a 'small' planet-sized body called a planetoid.

Problem 1 – Assume that the forming planet is spherical with a density of 3 gm/cc, a radius R, and a mass M. If the radius is a function of time, R(t), what is the equation for the mass of the planet as a function of time, M(t)?

Problem 2 – The planet grows by absorbing incoming asteroids that have an average mass of 10^{15} grams and a density of N asteroids per cubic centimeter in the cloud. The asteroids collide with the surface of the forming planet at a speed of V cm/sec, what is the equation that gives the rate of growth of the planet's mass in time (dM/dt)?

Problem 3 – From your answer to Problem 1 and 2, re-write dM/dt in terms of M not R.

Problem 4 – Integrate your answer to Problem 3 so determine M(t).

Problem 5 – What is the mass of the planet when it reaches a diameter of 5000 kilometers if its density is 3 grams/cc?

Problem 6 – The planetoid begins at t=0 with a mass of m= 2 x 10^{15} grams. The cloud density N = 1.0 x 10^{-24} asteroids/cc (1 asteroid per 1000 cubic kilometers), and the speed of the asteroids striking the planet, without destroying the planet, is V=1 kilometer/sec. How many years will the growth phase have to last for the planet to reach a diameter of 5000 kilometers?

The Higgs Boson and the Mystery of Mass

The search is on for the Higgs Boson at the Large Hadron Collider, which began operation on November 23, 2009. This sub-atomic particle plays a major role in explaining why some particles have a measurable mass (electrons and quarks) while others do not (photons, gluons). This remarkable ability to cause particles to have mass can be described by the properties of the following equation:

$$\boxed{V(x) = 2x^4 - (1 - T^2)x^2 + \frac{1}{8}}$$

This equation describes the energy, V, stored in what physicists call the Higgs field. The variable x is the mass of the Higgs Boson, and T is the energy that particles such as electrons and protons carry when they are colliding with each other. The value of V determines how much mass a colliding particle will have, and this depends on the mass of the Higgs Boson.

Problem 1 - What is the shape of the function V(x) over the domain [0,+1] for a collision energy of; A) T=0? B) T=0.5? C) T = 0.8 and D) T=1.0?

Problem 2 - The observed mass of the Higgs Boson is defined by the location of the minimum of V(x) over the domain [0, +1]. How does the predicted mass of the Higgs Boson change as the value of T increases from 0 to 1?

Problem 3 - The measured mass of an electron is $M_0=9.1 \times 10^{-31}$ kilograms. According to the current models of the Higgs field, the electron's mass is determined by the simple equation $M = M_0\ x$, where x is the mass of the Higgs Boson. What would be the predicted electron masses for T = 0.5?

Problem 4 - The LHC will achieve collision energies of about 5 trillion electron-Volts (5 TeV). At this energy, T = 33. What will the function V(x) look like, and what will be the predicted mass of the electron at these energies?

The Energy of Empty Space

An important concept in cosmology is that the 'empty space' between stars and galaxies is not really empty at all! Today, the amount of invisible energy hidden in space is just enough to be detected as Dark Energy, as astronomers measure the expansion speed of the universe. Soon after the Big Bang, this Dark Energy caused the universe to expand by huge amounts in less than a second. Cosmologists call this early period of the Big Bang Era, Cosmic Inflation.

Physicists have developed a number of theories to quantify how Cosmic Inflation occurred. The basis for many of these theories is a mathematical equation of the form shown below. This equation connects the energy of empty space, V(x), to the existence of a new field in nature whose strength is determined by the value for x in the equation.

An interesting property of this new field is the way in which it interacts with all other particles in the universe as the temperature of the universe changes. As the universe cools from very high temperatures (T=1) near the Big Bang to very low temperatures (T=0) today, the function V(x) changes its shape. This causes the value of x where the function has its minimum to also change. The consequence of changing the value for x in the universe is that particles such as electrons and quarks will have different masses than what we observe today.

$$\boxed{V(x) = 2x^4 - (1-T^2)x^2 + \frac{1}{8}}$$

Problem 1 - What are the domain and range of the function V(x)?

Problem 2 - What is the axis of symmetry of V(x)?

Problem 3 - Is V(x) an even or an odd function?

Problem 4 - For T=0, what are the critical points of the function in the domain [-2, +2]?

Problem 5 - Over the domain [0,+2] where are the local minima and maxima located for T=0?

Problem 6 - Using a graphing calculator or an Excel spreadsheet, graph V(x) for the values T=0, 0.5, 0.8 and 1.0 over the domain [0,+1]. Tabulate the x-value of the local minimum as a function of T. In terms of its x location, what do you think happens to the end behavior of the minimum of V(x) in this domain as T increases?

Problem 7 - During the Cosmic Inflation Era, what is the vacuum energy difference defined as V = V(0) - V(1/2)?

Problem 8 - The actual energy stored in 'empty space' given by V(x) has the physical units of the density of energy in multiples of 10^{35} Joules per cubic meter. What is the available energy density during the Cosmic Inflation Era in these physical units?

The Volume of a Hypersphere

This spherical propellant tank is an important component of testing for the Altair lunar lander, an integral part of NASA's Constellation Program. It will be filled with liquid methane and extensively tested in a simulated lunar thermal environment to determine how liquid methane would react to being stored on the moon.

The volume of a sphere is a mathematical quantity that can be extended to spaces with different numbers of dimensions with some very interesting, and surprising, consequences!

The mathematical formula for the volume of a sphere in a space of N dimensions is given by the recursion relation

$$V(N) = \frac{2\pi R^2}{N} V(N-2)$$

For example, for 3-dimensional space, N = 3 and since from the table to the left, V(N-2) = V(1) = 2R, we have the usual formula

$$V(3) = \frac{4}{3}\pi R^3$$

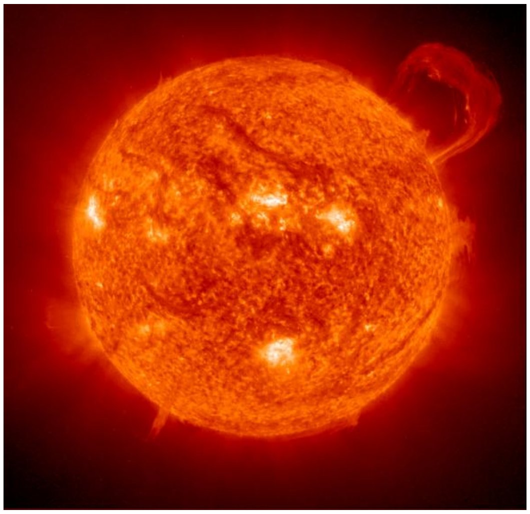
Dimension	Formula	Volume
0	1	1.00
1	2R	2.00
2	πR^2	3.14
3	$\frac{4}{3}\pi R^3$	4.19
4		
5		
6		
7		
8		
9		
10		

Problem 1 - Calculate the volume formula for 'hyper-spheres' of dimension 4 through 10 and fill-in the second column in the table.

Problem 2 - Evaluate each formula for the volume of a sphere with a radius of 1.00 and enter the answer in column 3.

Problem 3 - Create a graph that shows V(N) versus N. For what dimension of space, N, is the volume of a hypersphere its maximum possible value?

Problem 4 - As N increases without limit, what is the end behavior of the volume of an N-dimensional hypersphere?



Detailed mathematical models of the interior of the sun are based on astronomical observations and our knowledge of the physics of stars. These models allow us to explore many aspects of how the sun 'works' that are permanently hidden from view.

The Standard Model of the sun, created by astrophysicists during the last 50 years, allows us to investigate many separate properties. One of these is the density of the heated gas throughout the interior. The function below gives a best-fit formula, D(x) for the density (in grams/cm^3) from the core (x=0) to the surface (x=1) and points in-between.

$$D(x) = 519x^4 - 1630x^3 + 1844x^2 - 889x + 155$$

For example, at a radius 30% of the way to the surface, x = 0.3 and so D(x=0.3) = 14.5 grams/cm^3.

Problem 1 - What is the estimated core density of the sun?

Problem 2 - To the nearest 1% of the radius of the sun, at what radius does the density of the sun fall to 50% of its core density at x=0? (Hint: Use a graphing calculator and estimate x to 0.01)

Problem 3 - What is the estimated density of the sun near its surface at x=0.9 using this polynomial approximation?

Problem 4 - Integrate D(x) throughout the volume of the solar interior to estimate the total mass of the sun in grams. (Use the volume element dV = $4\pi x^2$ dx, and use the fact that for x=1, the physical radius of the sun is 6.9×10^{10} centimeters.)

How many stars are in the sky?

One of the very first things that astronomers studied was the number of stars in the sky. From this, they hoped to get a mathematical picture of the shape and extent of the entire Milky Way galaxy. This is perhaps why some cartoons of 'astronomers' often have them sitting at a telescope and tallying stars on a sheet of paper! Naked-eye counts usually number a few thousand, but with increasingly powerful telescopes, fainter stars can be seen and counted, too.

Over the decades, 'star count' sophisticated models have been created, and rendered into approximate mathematical functions that let us explore what we see in the sky. One such approximation, which gives the average number of stars in the sky, is shown below:

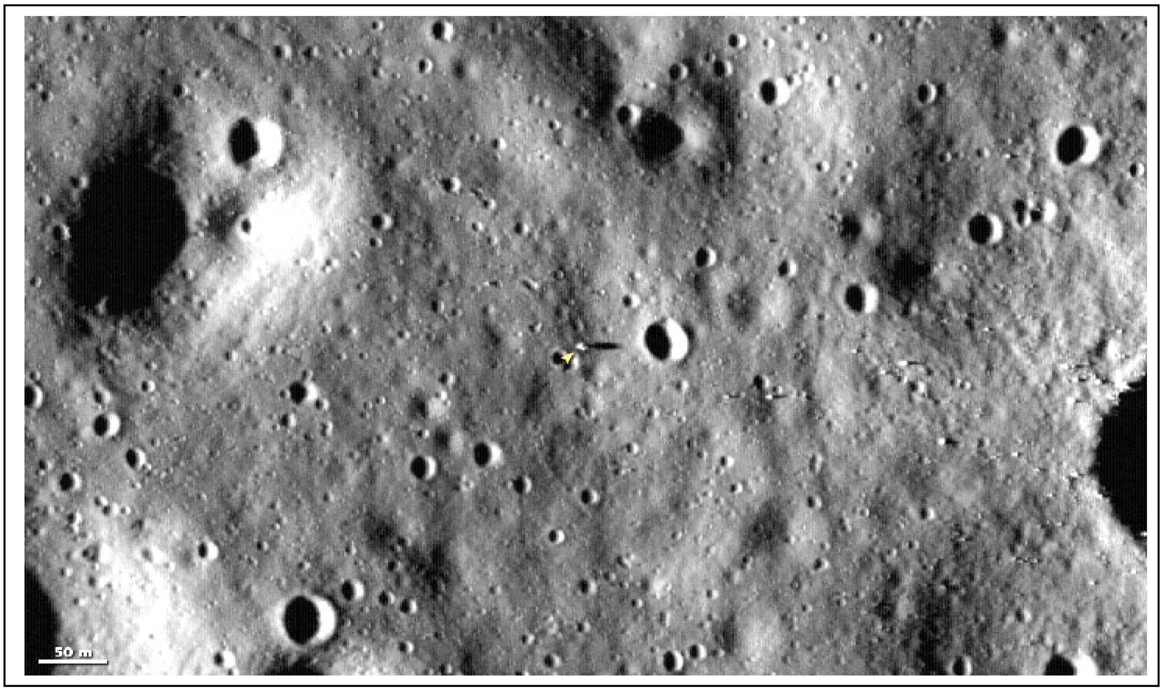
$$\mathrm{Log}_{10}N(m) = -0.0003\, m^3 + 0.0019\, m^2 + 0.484\, m - 3.82$$

This polynomial is valid over the range [+4.0, +25.0] and gives the Log_{10} of the total number of stars per square degree fainter than an apparent magnitude of m. For example, at an apparent magnitude of +6.0, which is the limit of vision for most people, the function predicts that $\mathrm{Log}_{10}N(6) = -0.912$ so that there are $10^{-0.912} = 0.12$ stars per square degree of the sky. Because the full sky area is 41,253 square degrees, there are about 5,077 stars brighter than, or equal to, this magnitude.

Problem 1 - A small telescope can detect stars as faint as magnitude +10. If the human eye-limit is +6 magnitudes, how many more stars can the telescope see than the human eye?

Problem 2 - The Hubble Space Telescope can see stars as faint as magnitude +25. About how many stars can the telescope see in an area of the sky the size of the full moon (1/4 square degree)?

Problem 3 - A photograph is taken of a faint star cluster that has an area of 1 square degree. If the astronomer counts 5,237 stars in this area of the sky with magnitudes in the range from +11 to +15, how many of these stars are actually related to the star cluster?



This image of the 800-meter x 480-meter region near the Apollo-11 landing pad was taken by the Lunar Reconnaissance Orbiter (LRO). It reveals hundreds of craters covering the landing area with sizes as small as 5 meters. The Apollo-11 landing pad is at the center of the image, and is casting a long horizontal shadow to the right of the pad, in the direction of a small crater.

Astronomers use counts of the number of craters per kilometer^2 as a function of crater diameter to determine the age of a given lunar landscape, and the distribution of the sizes of the impactors. Crater counts are also used to determine which areas are safe to land. The power-law function below is based upon the above image from LRO and gives the surface density of craters near the Apollo-11 landing site in terms of craters per kilometer^2 of a given diameter, x, in meters. The range of validity is from 2 meters to 40 meters for this particular lunar area. Apollo-11 astronauts did not find any craters smaller than 2-meters near the landing area.

$$S(x) = 22000\, x^{-2.4} \text{ craters/km}^2$$

Problem 1 – Integrate the function S(x) to get the function N(x>m) which gives the number of craters per kilometer^2 with diameters greater than m-meters.

Problem 2 - Integrate the function S(x) to get the function N(x<m) which gives the number of craters per kilometer^2 with diameters smaller than m-meters.

Problem 3 – The Apollo-11 astronauts surveyed the area shown in the image above in order to find a landing site that was not part of a crater. To two significant figures, what is the maximum fraction of the area in the above image covered by craters larger than 2 meters in diameter? (Assume that the craters do not overlap, which is a good approximation to what the image shows.)

Rotation Velocity of a Galaxy

Spiral galaxy M-101 showing its bright nucleus and spiral arms. The radius of M-101 is about 90,000 light years, which corresponds to x=9 in the formula for V(x). (Hubble image)

Stars orbit the center of a galaxy with speeds that decrease as their orbital distances increase. A simple function, V(x) can model the orbital speeds of stars as a function of their distance, x, from the nucleus of the galaxy:

$$V(x) = \frac{350x}{\left(1+x^2\right)^{\frac{3}{4}}}$$

For example: At a distance of 10,000 light years from the center, x = 1.0 and the rotation speed is V(1.0) = 208 kilometers/sec.

Problem 1 – For small x (i.e. x < 1), what is the limiting form of V(x)?

Problem 2 – For large x, (i.e. x > 1) what is the limiting form of V(x)?

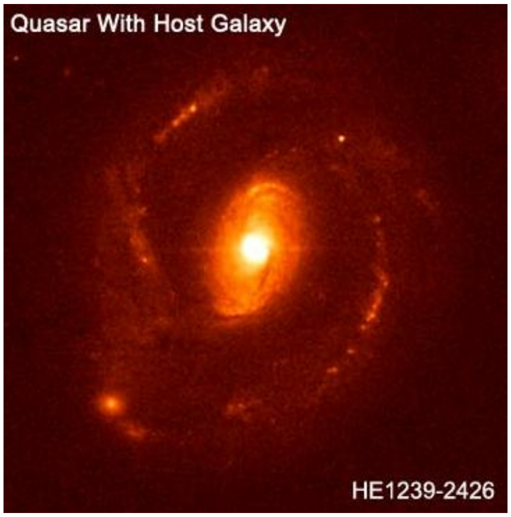
Problem 3 - The radius of M-101 is 90,000 light years. How fast are stars orbiting the center of M-101 according to V(x)? (Hint: At a radius of 90,000 light years, x=9.0. If the units of V(x) are kilometers/sec, what is V(x) at x = 9.0?)

Problem 4 – For what value of x is V(x) maximum?

Problem 5 – For x=1 the physical distance is 10,000 light years. How many years does it take a star to complete one circular orbit at x=1.0 if 1 light year equals 9.5 x 10^{12} km, and there are 3.1 x 10^{7} seconds in a year?

Note: This example of V(x) is for galaxies in which most of the mass is concentrated within their central regions (x < 1), however, astronomers know that this model is not completely accurate. Beyond x = 1, the rotation speeds for some galaxies, including the Milky Way, do not decrease rapidly as suggested by V(x), but actually remain constant. This implies that some galaxies contain substantial amounts of 'Dark Matter' that is not in the form of stars or other known forms of matter.

How Many Quasars Are There?



This Hubble Space Telescope image shows the brilliant 'quasar' core of a distant galaxy located 1.5 billion light years from Earth in the direction of the constellation Corvus the Crow.

Quasars are among the most luminous galaxies in the universe, and because of this, astronomers can detect them to great distances exceeding nearly 10 billion light years from Earth. Originally discovered as peculiar faint 'blue stars' in 1964, astronomers have counted up their numbers as a function of their brightness, expressed according to the logarithmic stellar apparent magnitude scale, m. The function that provides a good match to the quasar surface density, N(m), defined over the domain [+13.0, +23.0], is given by:

$$\frac{dN(m)}{dm} = 10^{f(m)}$$

where f(m) is a piecewise function defined as follows:

f(m) = +0.7m -12.0 for $+13.0 \le m \le +18.0$
f(m) = +0.5m -9.0 for $+19.0 \le m \le +23.0$

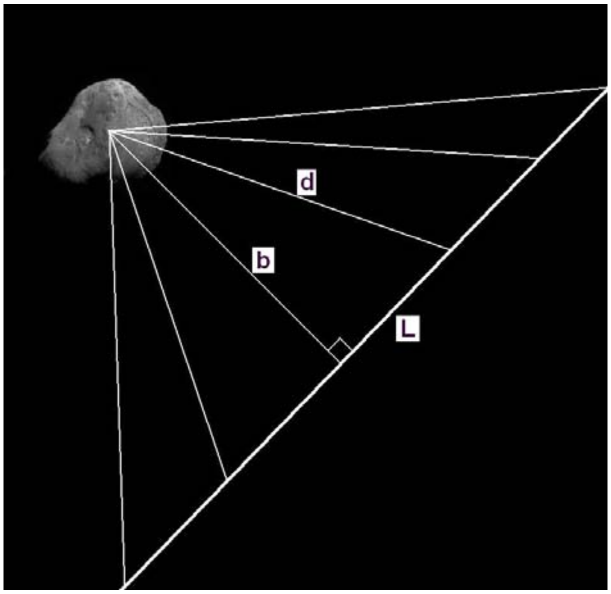
Problem 1 - The integral of dN(m)/dm is the total number of quasars present in one square degree of the sky. As a comparison, the full moon subtends an area of 1/4 square degree. How many quasars does the function predict that have magnitudes between +13.0 and +18.0 inclusively, over an area of the sky similar to five times the area of the full moon?

Problem 2 - How many quasars does the function predict that have magnitudes between +19.0 and +23.0 inclusively, over an area of the sky similar to five times the area of the full moon?

Problem 3 - An astronomer wants to study quasars in the brightness interval +13.0 < m < +18.0. What is the minimum sky area, in square degrees, that she needs to photograph in order to have one quasar present in the photograph?

Problem 4 - Combining the quasar sky densities from Problem 1 and 2, and the fact that the sky has a total angular area of 41,253 deg^2, how many quasars are there with magnitudes in the range from +13 to +23, inclusively?

Deep Impact - Comet Flyby



On July 4, 2005, the Deep Impact spacecraft flew by the comet Tempel-1 along a path shown in the figure to the left, at a speed of V=10 km/sec. Its closest distance to the comet was b = 500 kilometers at a time, t=0. The distance traveled along the path is given by L = Vt.

The diameter of the comet is D = 8 kilometers, and the distance to the comet in kilometers is d(t), so the angular diameter of the comet in arcminutes is given by

$$\Theta(t) = 3438\frac{D}{d(t)}$$

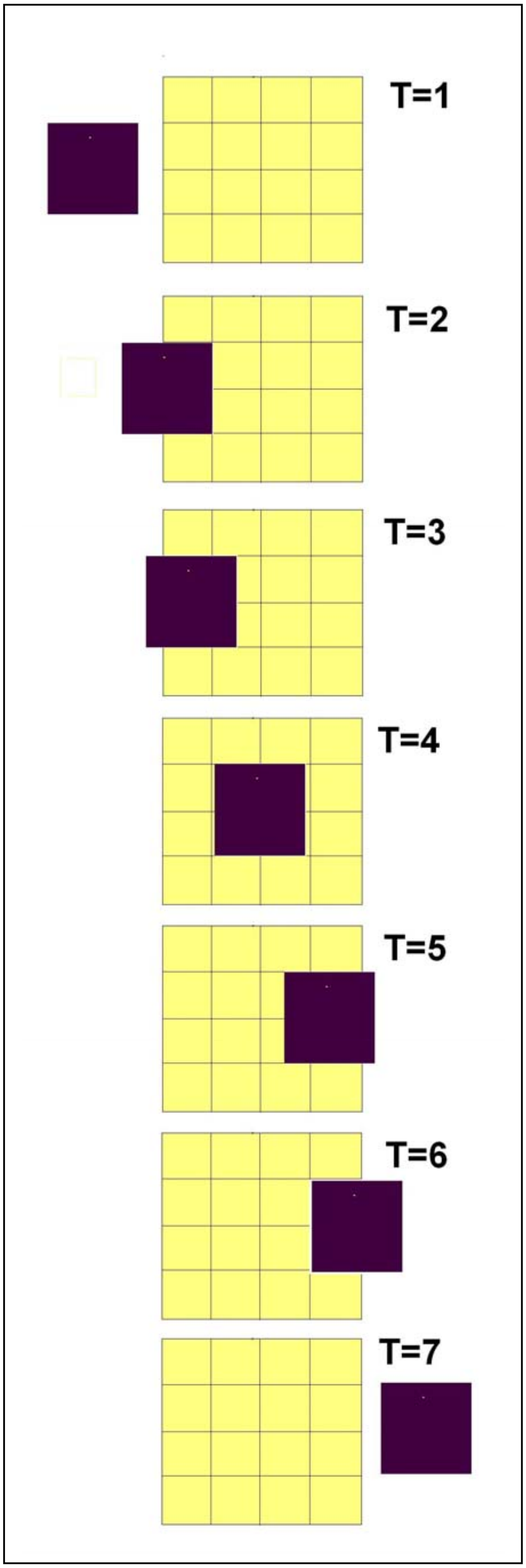
Problem 1 - What is the formula for the distance to the comet from the spacecraft defined as *d(t)*?

Problem 2 - What is the formula for the angular diameter of the comet as seen from the spacecraft at any time, t, along the trajectory, defined by the variables V, b and L? Simplify the formula by defining two constants B = 3438L/b and c = V^2/b^2.

Problem 3 - What is the exact numerical formula for the rate-of-change in time of the angular size of the Tempel-1 as viewed by the spacecraft as it flys by?

Problem 4 - What was the angular diameter of Tempel-1 at the closest approach, and how fast will the angular size be decreasing when Tempel-1 reaches one-half its maximum angular size?

Kepler Spies Five New Planets

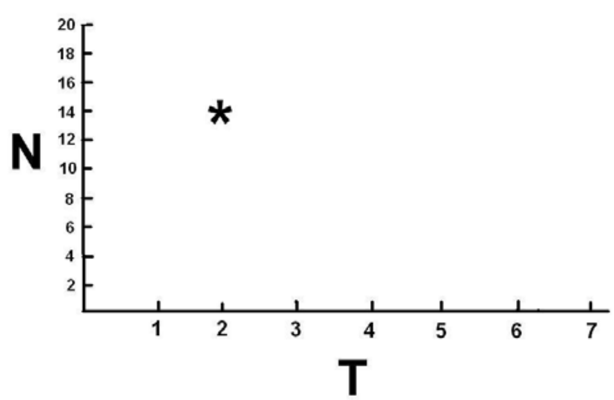


NASA's Kepler spacecraft recently announced the discovery of five new planets orbiting distant stars. The satellite measures the dimming of the light from these stars as planets pass across the face of the star as viewed from Earth. To see how this works, lets look at a simple model.

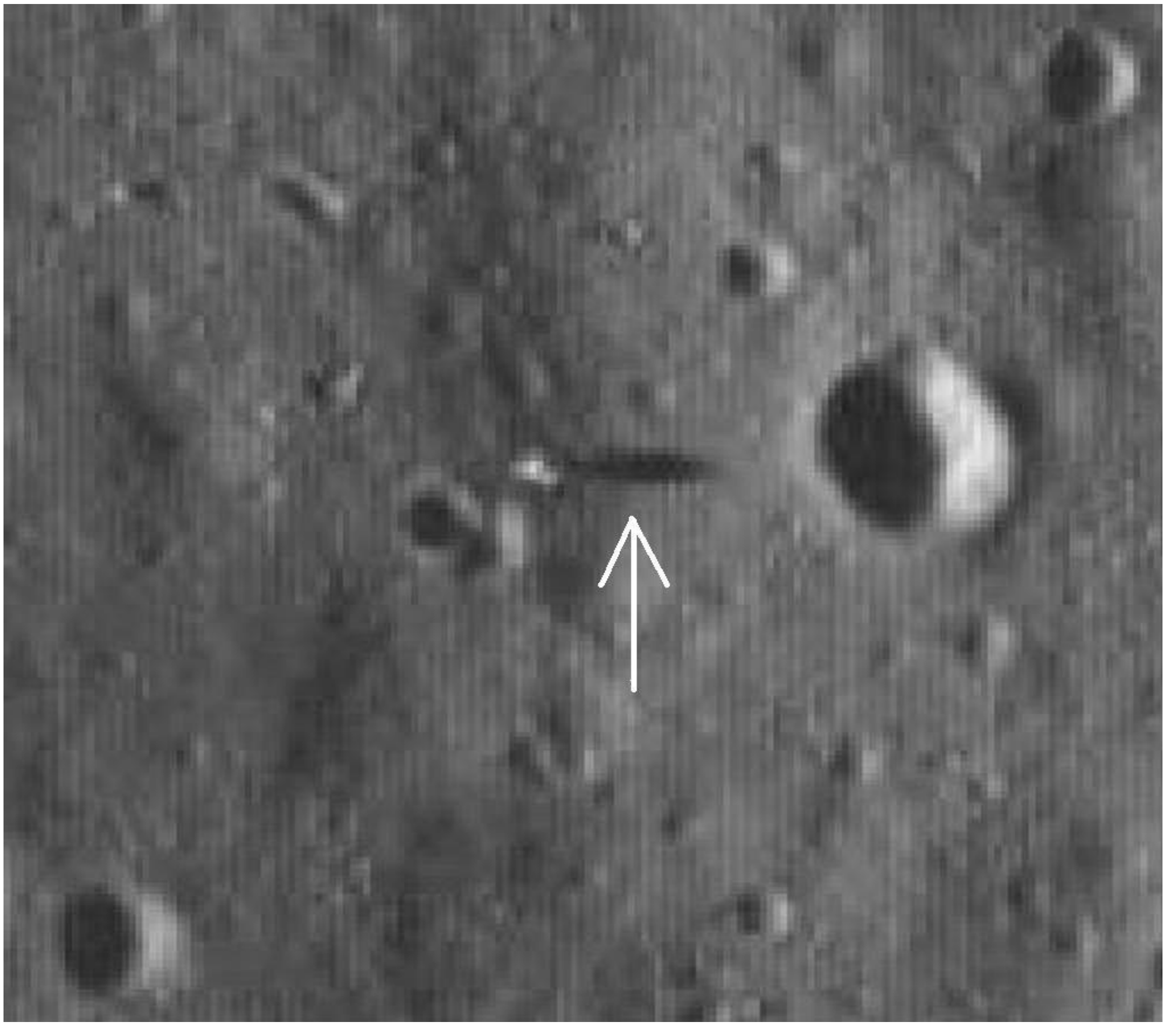
In the Bizarro Universe, stars and planets are cubical, hot spherical. Bizarro astronomers search for distant planets around other stars by watching planets pass across the face of the stars and cause the light to dim.

Problem 1 - The sequence of figures shows the transit of one such planet, Osiris (black). Complete the 'light curve' for this star by counting the number of exposed 'star squares' not shaded by the planet. At each time, T, create a graph of the number of star brightness squares. The panel for T=2 has been completed and plotted on the graph below.

Problem 2 - If you knew that the width of the star was 1 million kilometers, how could you use the data in the figure to estimate the width of the planet?



LRO Sees Apollo 11 on the Moon!

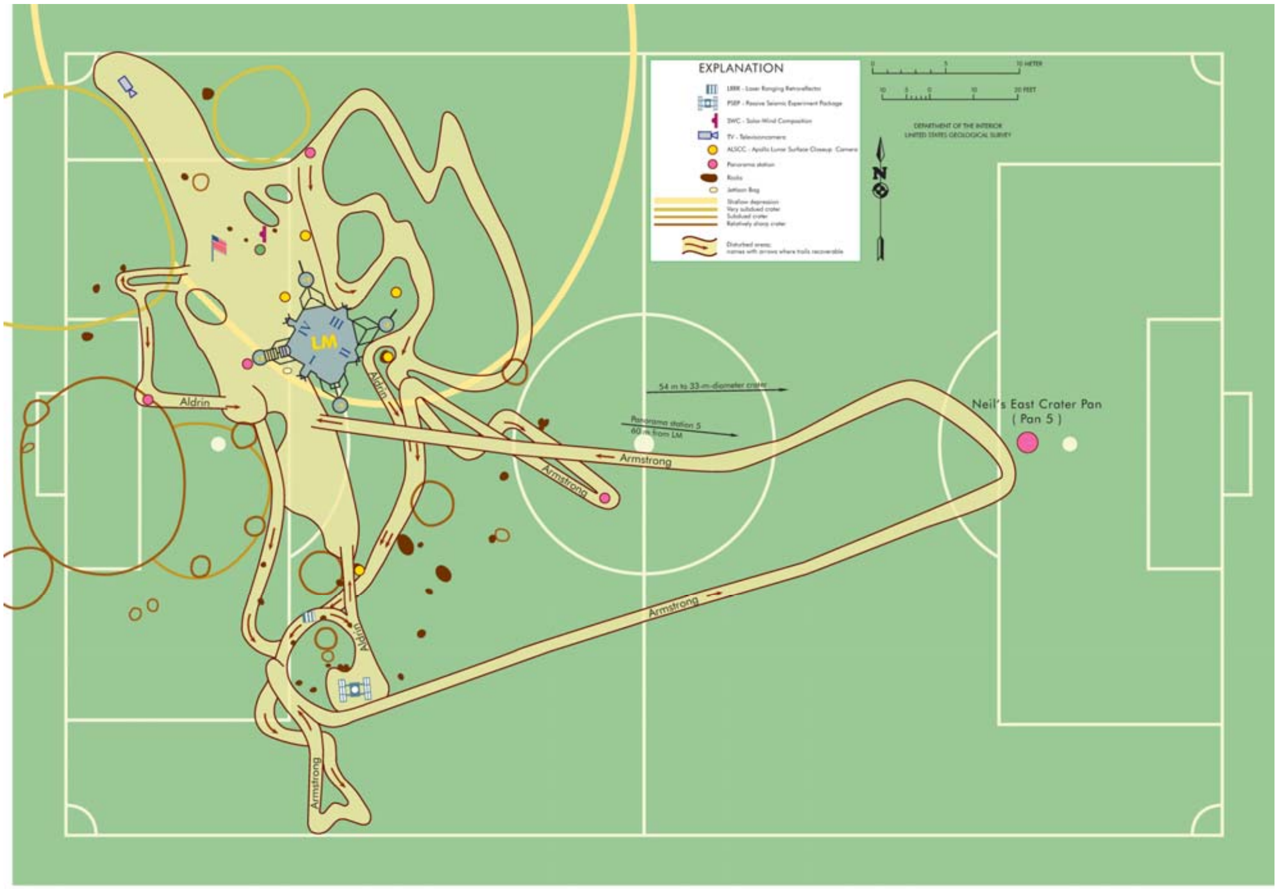


The LRO satellite recently imaged the Apollo 11 landing area on the surface of the moon. The above (172 pixels wide x 171 pixels high) image shows this area and is 172 meters wide.

Problem 1 - Determine the scale of the image in meters per millimeter and meters per pixel? What is the diameter, in meters, of A) the largest crater? B) the smallest crater?

Problem 2 - The shadow identified by the arrow was cast by the Lunar Landing Module which is about 3.5 meters tall. Using A) trigonometry, or a B) scaled drawing and a protractor, what was the sun angle at the time of the photograph?

Problem 3 - Are there any individual boulders larger than 1 meter across in this area?



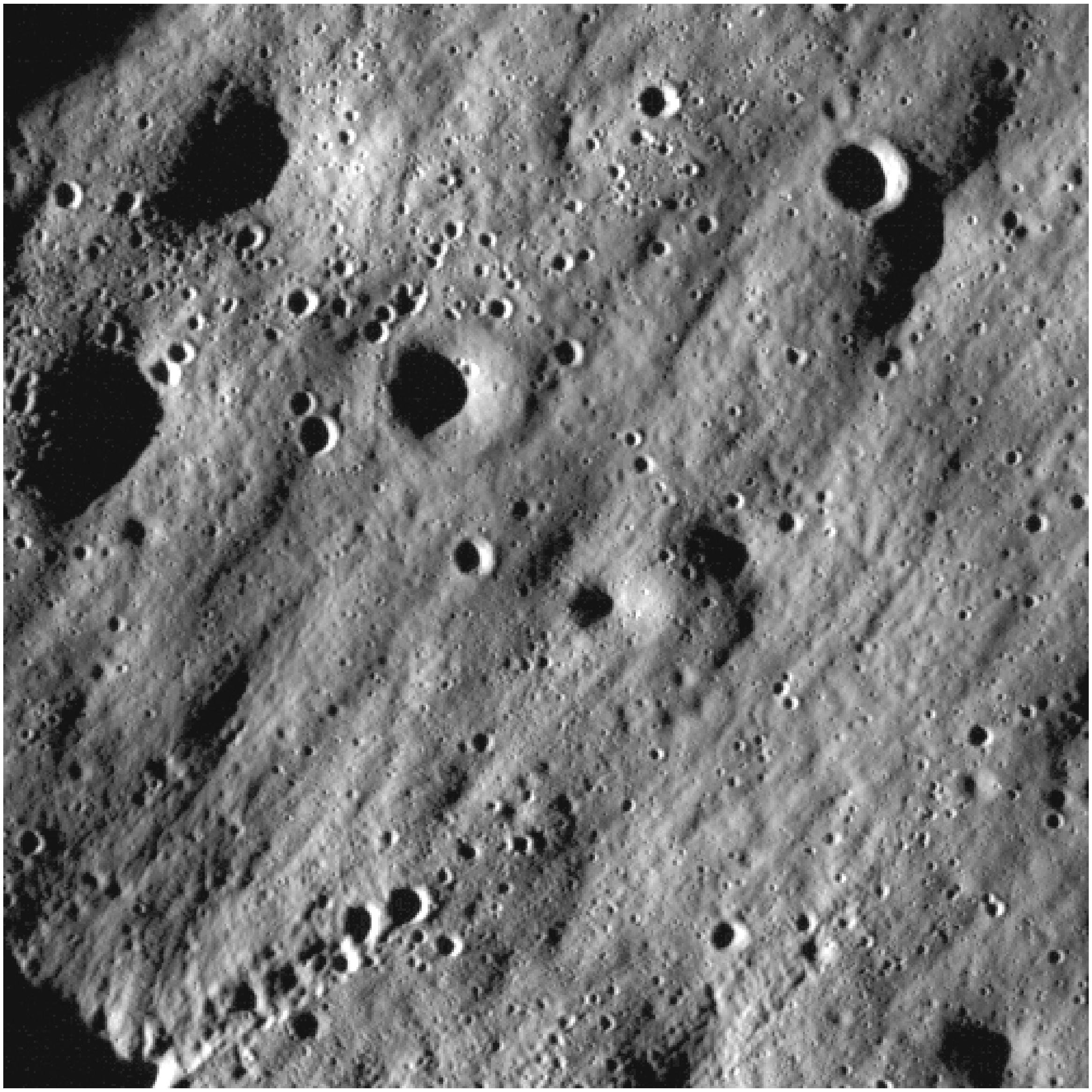
The NASA Lunar Reconnaissance Orbiter (LRO), launched in 2009, will be able to see details on the lunar surface at much higher resolution than any previous lunar mapping mission. The images will have a resolution of about 0.7 meters per pixel. The map shows the area surrounding the Apollo 11 landing site where astronauts deployed experiments and walked around the landing site to gather rock and soil samples. The grid lines show a standard soccer field in comparison, with a distance between the white goal boundaries of 110 meters.

Problem 1 - Use a millimeter ruler and the information provided to determine the scale of the figure in meters per millimeter.

Problem 2 - Draw an overlay of 10 rows and 10 columns on the above figure at a location near the Apollo-11 landing area, with individual cells representing the individual LRO pixels.

Problem 3 - Will LRO be able to see: A) the Lunar Module 'LM' marked on the map? B) The discolorations (shown in yellow) of the lunar soil caused by the paths taken by the astronauts? C) Details on the LM?

LRO's First Image of Mare Nubium

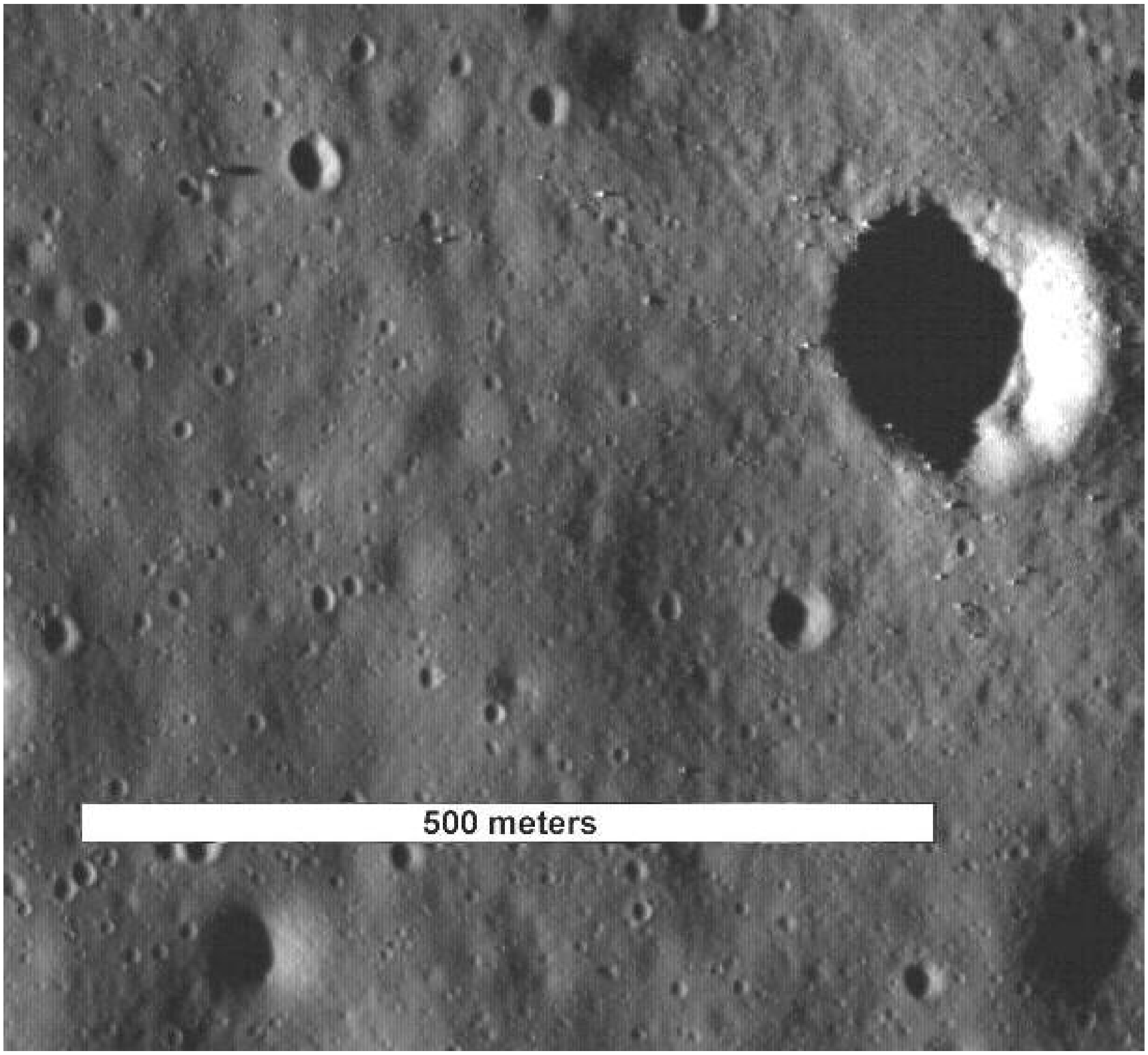


This is one of the first images taken by LRO showing details in Mare Nubium. The width of the image is 700 meters (500 pixels).

Problem 1 - Use a millimeter ruler to determine the scale of the image in meters per millimeter, and meters per pixel.

Problem 2 – What is the diameter, in meters, of the smallest recognizable crater you can find?

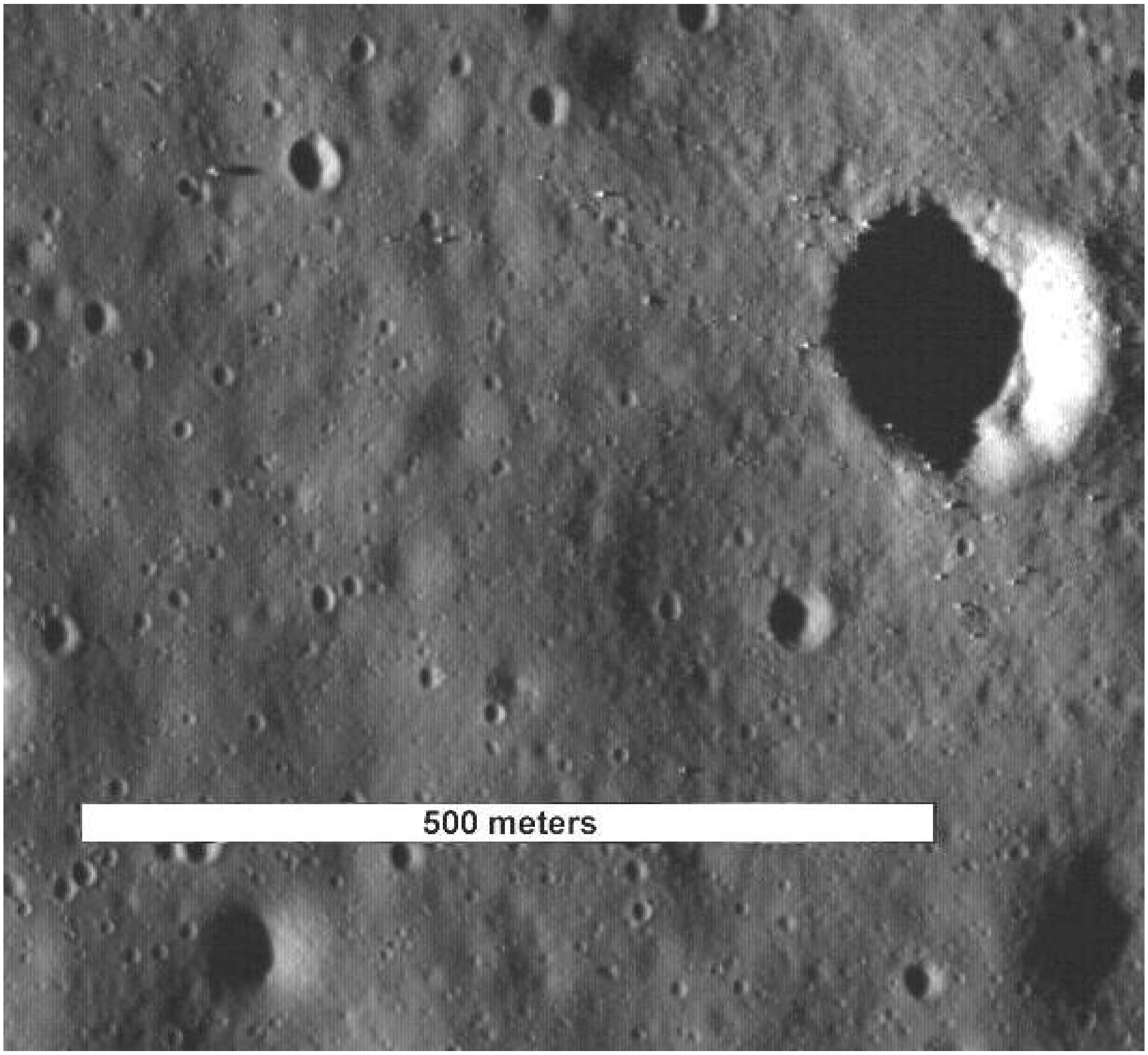
Problem 3 – Suppose your house is 42 feet wide and 60 feet long, and its sits on a property that is 75 feet wide and 96 feet long. Draw two squares at the same pixel scale as the LRO image. (Assume 1 meter = 3 feet)



The LRO satellite recently imaged the surface of the moon at a resolution of 1.4 meters/pixel. The above image shows the region near the Apollo-11 landing area. The Lunar Module (LM) and its shadow are shown to the left of the large crater in the upper right corner. Sunlight comes from the left, and so craters will have their shadow zones on the left-hand side of their depressions. Objects above the surface, like the Apollo LM, will be bright on the left side, and have their right-side in shadow.

Problem 1 - From the information given, and using a millimeter ruler: A) determine the scale of the image (meters per millimeter); and B)the length of the Apollo LM shadow.

Problem 2 - Find as many boulders as you can, and determine their approximate size using the height of the LM (3.5 meters) and the length of the LM shadow to establish their sizes. Do you think there are smaller boulders that the ones you can easily spot?



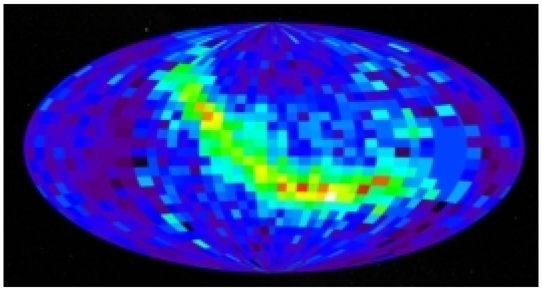
The LRO satellite recently imaged the surface of the moon at a resolution of 1.4 meters/pixel. The above image shows a region near the Apollo-11 landing site. The Lunar Module (LM) can be seen from its very long shadow near the large crater in the upper left corner of the image.

Problem 1 - With a millimeter ruler, determine the scale of this image in meters/mm. What is the total area of this image in square-kilometers?

Problem 2 - Measure all of the craters larger than or equal to 9 meters and create a histogram of the numbers of the craters. Divide the number of craters in each bin, by the total area of the field, to get $\mathbf{A_c}$: the Areal Crater Density (craters/km^2).

Problem 3 - The average distance between craters of a given size is found by taking the square-root of the reciprocal of $\mathbf{A_c}$. About what is the average distance between craters with a diameter close to 5 meters?

IBEX Creates an Unusual Image of the Sky!



NASA's IBEX satellite recently made headlines by creating a picture of the entire sky, not using light but by using cosmic particles called ENAs (Energetic Neutral Atoms). These fast-moving atoms flow through the solar system. Some of them reach Earth, where they can be captured by the IBEX satellite. By counting how many of these ENAs the satellite sees in different directions in the sky, IBEX can create a unique 'picture' of where ENAs are coming from in space.

The big surprise was that they were not coming from all over the sky as expected. They were also coming from a specific band of directions as we see in the image to the left. This image has the same kind of geometry as the map of the Earth below it! It is called a Mollweide Projection, except that instead of graphing geographic points on Earth, the IBEX image shows points in outer space!

	A	B	C	D	E
1					
2					
3					
4					
5					

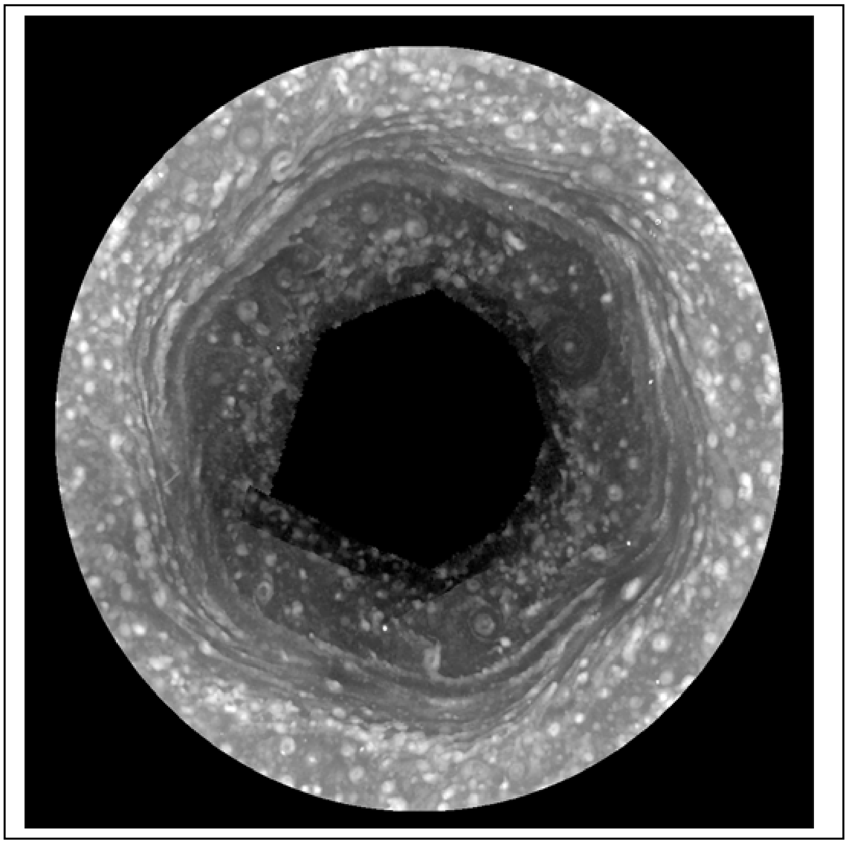
Data String:
A5, E2, B2, D4, C3, A1, E4, C3, D4, B2, D4, B3, C4, E5, D5, D4, C2, D3, B1, E5, A2, C3, D5, C5, D4, E4, D3, C4, B4, D2, E3, C1, B5, A3, E1, A4, D1, B3, C2, E3

The IBEX satellite detected a series of particles entering its ENA instrument, and was able to determine the direction that each particle came from in the sky. The grid above shows a portion of the sky as a 5x5 grid with columns labeled by their letter and rows by their number. The data string to the right shows the detections of individual ENA particles with their direction indicated by their cell. 'A5, E2, B2…' means that the first ENA particle came from the direction of cell 'A5', the second from cell 'E2' and the third from cell 'B2' and so on. In some ways this process is like the 'call out' during a Bingo game, except that you keep track of the particle 'tokens' in each square to build a picture! Let's look at an example of constructing an ENA image.

Problem 1 - From the hypothetical data string, tally the number of particles detected in each sky cell in the grid. Select colors to represent the number of ENAs to create an 'image' of the sky in ENAs! How many particles were reported by this data string?

Problem 2 - Suppose that cell B2 is in the direction of the constellation Auriga, cell C3 is towards Taurus and cell D4 is towards Orion, from which constellation in the sky were most of the ENAs detected?

The Mysterious Hexagon on Saturn!



The Cassini spacecraft recently took this high-resolution image of Saturn's north-polar region, which was observed by the Voyager 1 and 2 spacecraft in the early-1980s to have a remarkable hexagonal jet stream! The white spots are individual clouds, much like fair-weather cumulus clouds on Earth. The interior distance between opposite vertices of the hexagon is 25,000 kilometers, and the estimated speed of the winds along the walls of the hexagon is about 100 meters/sec. Use a millimeter ruler, or your knowledge of hexagons, to answer these questions:

Problem 1 - How long does it take the jet stream to make one complete circuit of the hexagon; A) In hours?, B) In days?

Problem 2 - The Earth has a radius of 6,378 kilometers, from a scaled drawing of the hexagon and Earth, how many Earths can fit inside the hexagonal area?

Problem 3 - Acceleration is a measure of the change in the speed and/or direction of motion per unit time interval. A) From the hexagon figure, how much time, T, elapsed in seconds as the velocity of the jet stream completely changed direction as it crossed a vertex region? B) If the total velocity change, V, passing across one vertex is about 173 meters/sec, what was the average acceleration of the jet stream defined as $a = V/T$ in meters/sec^2?

Scientific Data - The gift that keeps on giving!

NASA launched its Solar Dynamics Observatory mission in late 2009. The three instruments on board the satellite will take lots and lots of pictures of the Sun. So it will require lots and lots of disc space to store the data.

Imagine you are the engineer responsible for purchasing the computer systems to handle that data. You must figure out how many computer discs and tapes you'll need, and what they will cost, so you can ask NASA for the right amount of money in your mission budget.

8 bits	= 1 byte	
1 kilobyte	= 1 thousand bytes	10^3
1 megabyte	= 1 million bytes	10^6
1 gigabyte	= 1 billion bytes	10^9
1 terabyte	= 1 trillion bytes	10^{12}
1 petabyte	= 1 quadrillion bytes	10^{15}

Data rate:	130 megabits / second
Hours of data/day:	24 hrs
Size of disc drive:	1 terabyte
Cost per disc drive:	$300
Size of backup tape:	800 gigabytes
Cost per backup tape:	$60

Problem 1 - How many bytes will your discs need to hold each day? (Express your answer in terabytes)

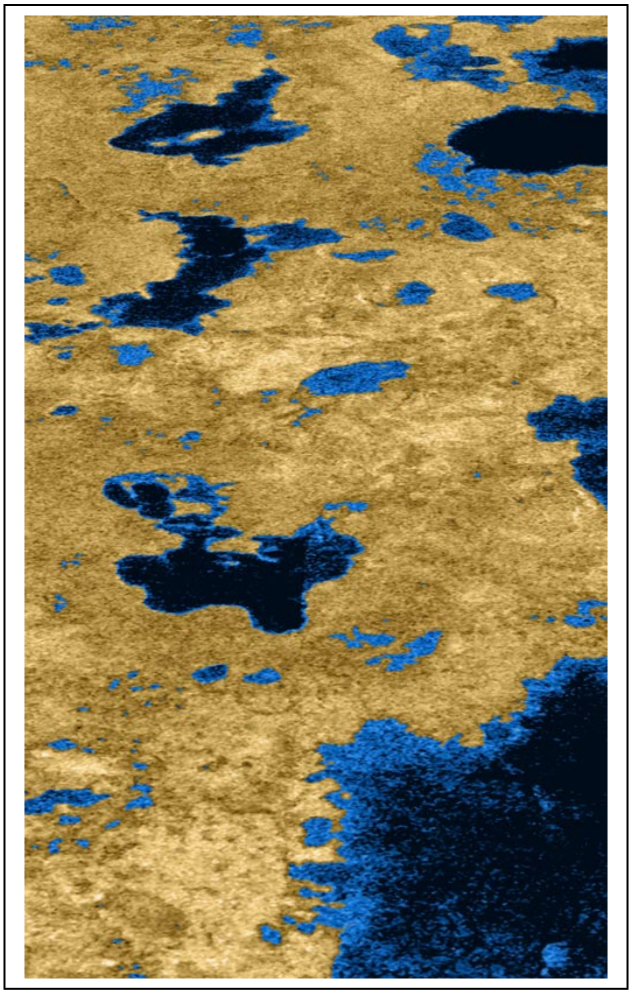
Problem 2 - NASA would like you to keep about 60 days of data online (i.e. on disc). Data older than 60 days will be archived and copied to tapes. How many bytes of drive space will you need to hold 60 days of data?

Problem 3 - How many disc drives will you need to purchase, and how much will they cost?

Problem 4 - How many terabytes of tape storage will you need to archive a year's worth of data?

Problem 5 - How many tapes will you need, and what will they cost you?

Problem 6 - There is a rule of thumb (called Moore's Law) that says the costs for electronics halve every 18 months. If you can wait 18 months before you purchase your discs and tapes, estimate how much money you might save.



A key goal in the search for life elsewhere in the universe is to detect liquid water, which is generally agreed to be the most essential ingredient for living systems that we know about.

The image to the left is a false-color synthetic radar map of a northern region of Titan taken during a flyby of the cloudy moon by the robotic Cassini spacecraft in July, 2006. On this map, which spans about 150 kilometers across, dark regions reflect relatively little of the broadcast radar signal. Images like this show Titan to be only the second body in the solar system to possess liquids on the surface. In this case, the liquid is not water but methane!

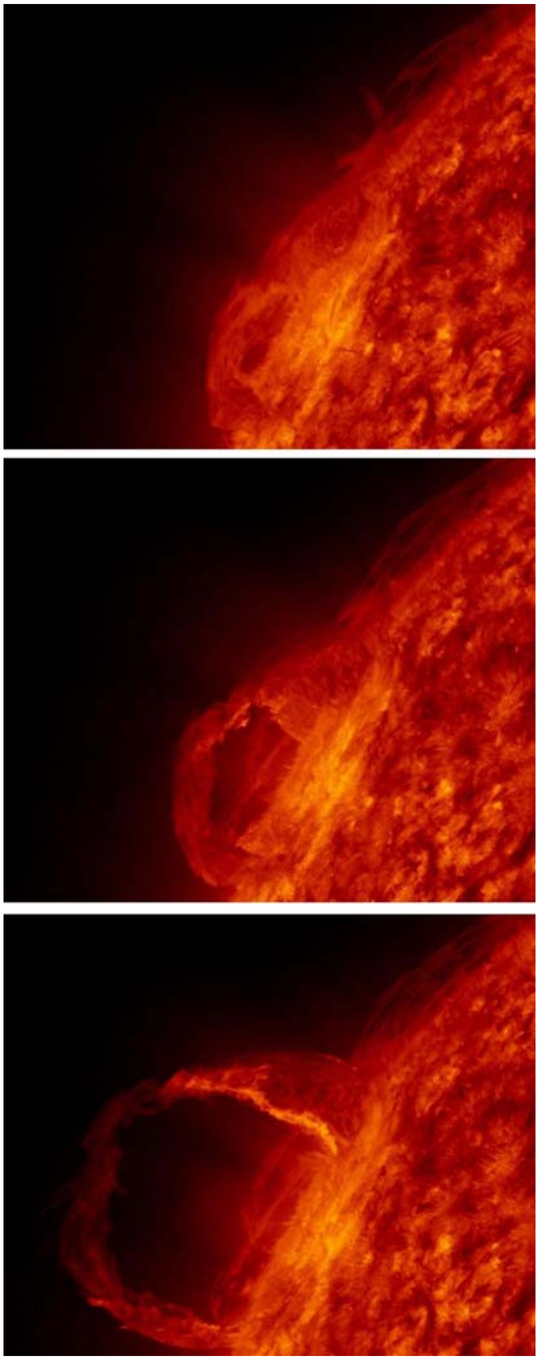
Future observations from Cassini during Titan flybys will further test the methane lake hypothesis, as comparative wind affects on the regions are studied.

Problem 1 – From the information provided, what is the scale of this image in kilometers per millimeter?

Problem 2 – What is the approximate total surface area of the lakes in this radar image?

Problem 3 - Assume that the lakes have an average depth of about 20 meters. How many cubic kilometers of methane are implied by the radar image?

Problem 4 – The volume of Lake Tahoe on Earth is about 150 km^3. How many Lake Tahoes-worth of methane are covered by the Cassini radar image?



On April 21, 2010 NASA's Solar Dynamics Observatory released its much-awaited 'First Light' images of the Sun. Among them was a sequence of images taken on March 30, showing an eruptive prominence ejecting millions of tons of ionized gas (plasma) into space. The three images to the left show selected scenes from the first 'high definition' movie of this event. The top image was taken at 17:50:49, the middle image at 18:02:09 and the bottom image at 18:13:29.

Problem 1 – The width of the image is 300,000 kilometers. Using a millimeter ruler, what is the scale of these images in kilometers/millimeter?

Problem 2 – If the Earth were represented by a disk the size of a penny (10 millimeters), on this same scale how big was the loop of the eruptive prominence in the bottom image if the radius of Earth is 6,378 kilometers?

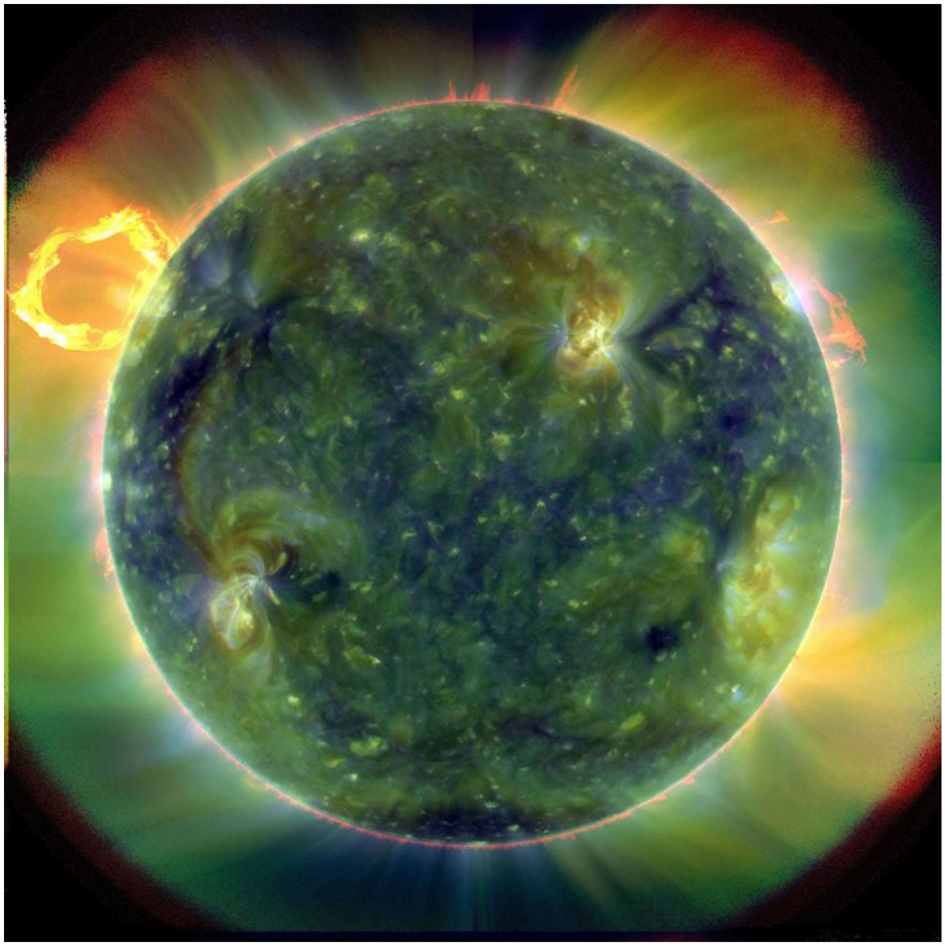
Problem 3 – What was the average speed of the prominence in A) kilometers/second? B) Kilometers/hour? C) Miles/hour?

For additional views of this prominence, see the NASA/SDO movies at:

http://svs.gsfc.nasa.gov/vis/a000000/a003600/a003693/index.html

or to read the Press Release:

http://www.nasa.gov/mission_pages/sdo/news/first-light.html



On April 21, 2010 NASA's Solar Dynamics Observatory released its much-awaited 'First Light' images of the Sun. The image above shows a full-disk, multi-wavelength, extreme ultraviolet image of the sun taken by SDO on March 30, 2010. False colors trace different gas temperatures. Black indicates very low temperatures near 10,000 K close to the solar surface (photosphere). Reds are relatively cool plasma heated to 60,000 Kelvin (100,000 F); blues, greens and white are hotter plasma with temperatures greater than 1 million Kelvin (2,000,000 F).

Problem 1 – The radius of the sun is 690,000 kilometers. Using a millimeter ruler, what is the scale of these images in kilometers/millimeter?

Problem 2 – What are the smallest features you can find on this image, and how large are they in kilometers, and in comparison to Earth if the radius of Earth is 6378 kilometers?

Problem 3 – Where is the coolest gas (coronal holes), and the hottest gas (micro flares), located in this image?

NASA's Terra satellite flew over the Deepwater Horizon rig's oil spill in the Gulf of Mexico on Saturday, May 1 and captured the above natural-color image of the slick from space. The oil slick resulted from an accident at the Deepwater Horizon rig in the Gulf of Mexico. NOAA's estimated release rate of oil spilling into the Gulf is 200,000 gallons per day since April 20 when the accident occurred.

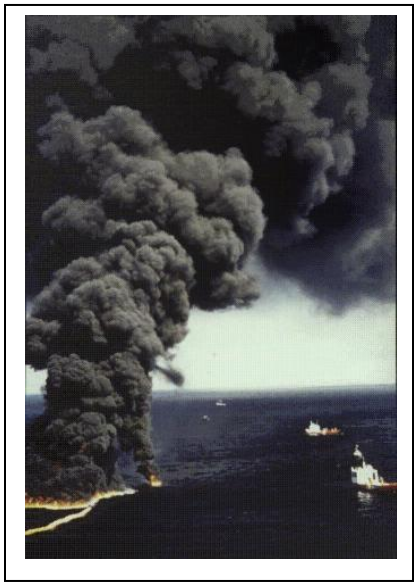
Problem 1 – Using a metric ruler, calculate the scale of this image in kilometers/mm.

Problem 2 – What is the approximate area of this oil leak in A) square kilometers? B) square meters?

Problem 3 - The estimated quantity of oil covering this area is about 2 million gallons. If one gallon of oil has a mass of 3.0 kg, what is the surface density, S, of oil in this patch in A) Gallons/meter^2? B) kg/meter^2?

Problem 4 – The density of crude oil is about D=850 kg/m^3. From your estimate for S, what is the approximate thickness, h, of the oil layer covering the ocean water?

Problem 5 - Suppose that an average 'oil' molecule has a length of about 5 nanometers. About what is the average thickness of this oil layer in molecules if the molecules are lined up end to end?



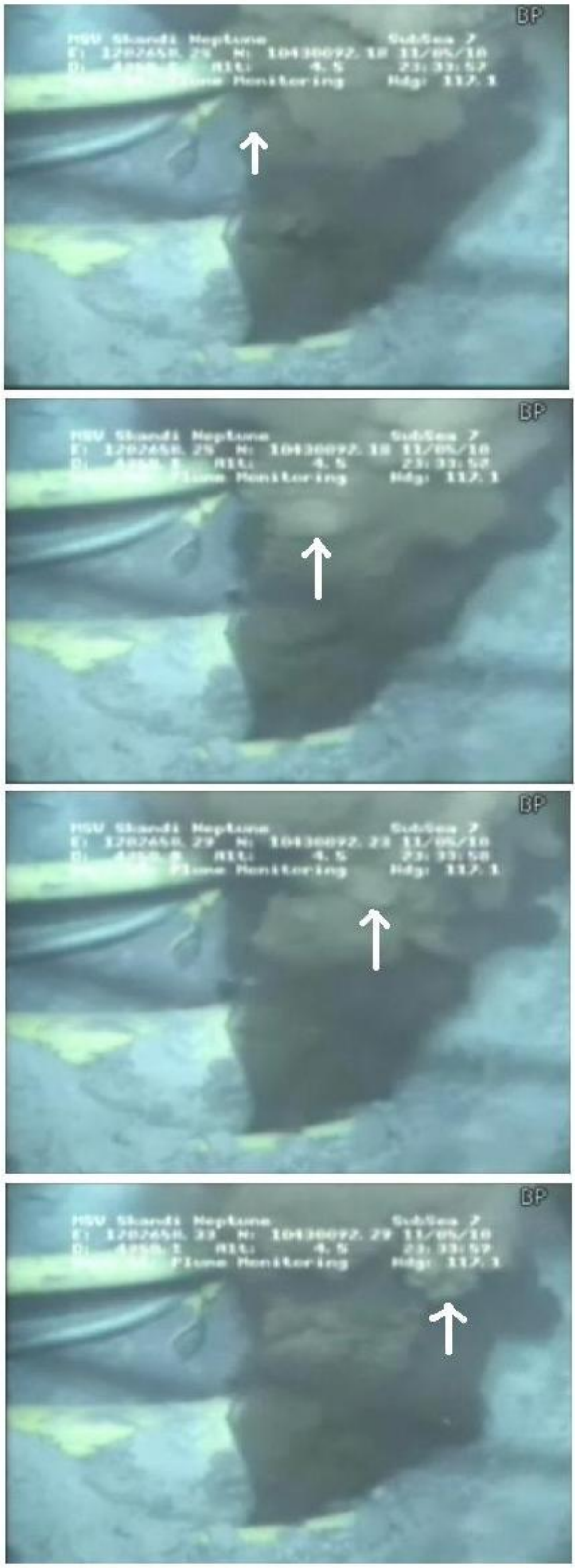
On March 21, 2010 the Eyjafjalla Volcano in Iceland erupted, and the expanding ash cloud grounded over 3,000 flights in Europe. Then on April 20, a major oil spill began in the Gulf of Mexico. Although the preferred method for dealing with the oil spill is to collect it using skimmers, burning it is also a common option (see above left photo). A major concern in burning this oil is the addition of carbon dioxide to the atmosphere during the combustion process.

Problem 1 – The Gulf Oil Spill is predicted to generate 200,000 gallons of crude oil every day. If 50% of this is ultimately burned-off, how many tons/day of carbon dioxide are generated if the combustion of 1 gallon of oil generates 10 kg of carbon dioxide?

Problem 2 – Scientists have estimated that the Iceland volcano generated 15,000 tons of carbon dioxide per day, and this eruption continued for about 28 days. How many days will the Gulf Oil burn-off have to continue before its carbon dioxide contribution equals that of the total carbon dioxide generated by the Eyjafjalla Volcano?

Problem 3 – It has been estimated that the European aviation industry generates 344,000 tons of carbon dioxide each day. If 60% of this industry was shut down by the ash cloud from the Eyjafjalla Volcano, how many tons of carbon dioxide would have been produced by airline flights during the 5-day shut-down of the industry?

Problem 4 – What can you conclude by comparing your answers to Problem 1, 2 and 3?



The April 14, 2010 BP Gulf Oil Leak was in the news for nearly three months, and ranked as one of the most environmentally costly accidents in recent history. Considerable debate continues as to the actual rate at which the leaky British Petrolium (BP) well is leaking oil. Initial estimates from the observed surface oil slick suggested 210,000 gal/day. Following the release of actual videos of the leak, experts estimated a new rate from 3 to 4 million gallons/day.

The images to the left were extracted from the May 12, 2010 video between 23:33:57 and 23:33:58. The arrow shows how far a portion of the billowing oil moved during this time.

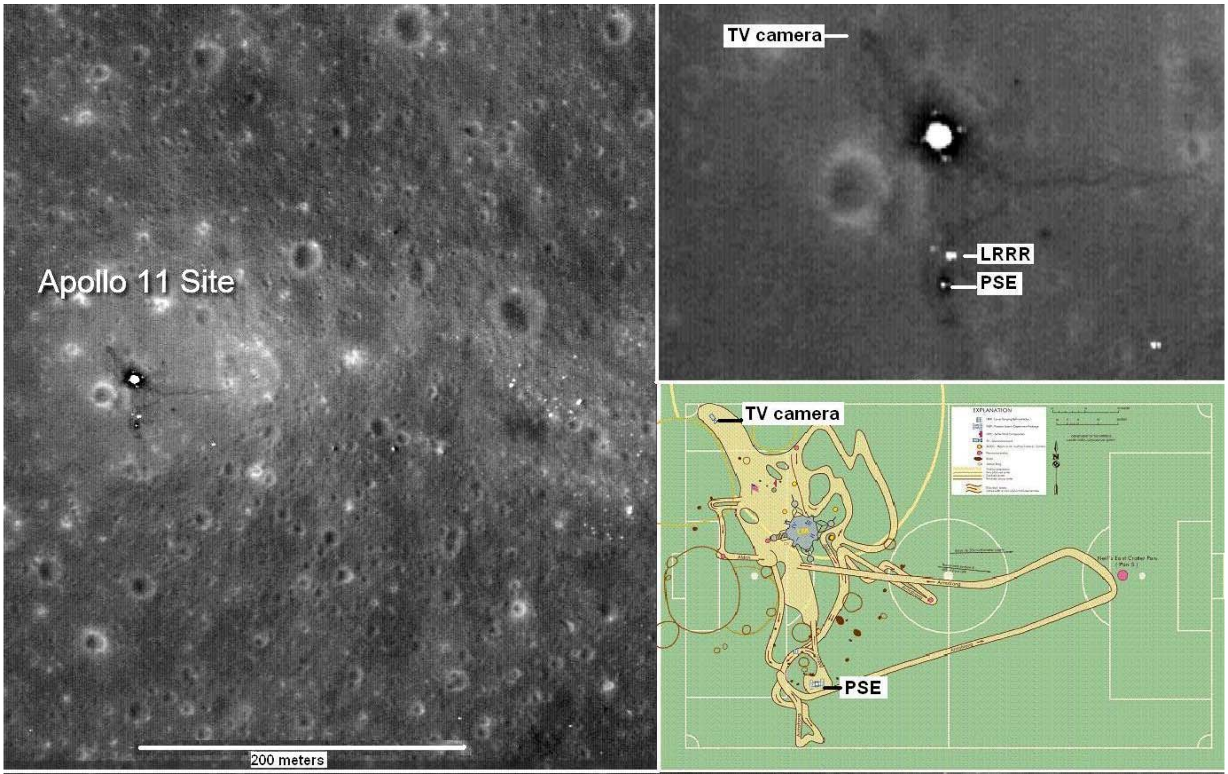
The diameter of the pipe fragment shown in the image is 21 inches.

Problem 1 - From the scale of the images, how many inches did the oil spot move in the time between the first and last images?

Problem 2 - What is the area of the open circular pipe in square-feet?

Problem 3 - If the oil is emerging at the same speed as you derived in Problem 1, how many cubic-feet of oil is leaving the pipe each second?

Problem 4 - If 1 cubic foot equals 7.5 gallons, what do you estimate as the rate in gallons/day at which oil is leaving the pipe if A) 100% of the dark material is oil? B) 50% is oil and 50% is gas?



The Lunar Reconnaissance Orbiter (LRO) recently imaged the Apollo-11 landing area at high-resolution and obtained the image above (Top left). An enlargement of the area is shown in the inset (Top right) and a rough map of the area is also shown (bottom right). The landing pad with three of its four foot-pads is clearly seen, together with the Lunar Ranging Retro Reflector experiment (LRRR), the Passive Seismic Experiment (PSE) and the TV camera area. The additional white spots seen in the left image are boulders from the West Crater located just off the right edge of the image.

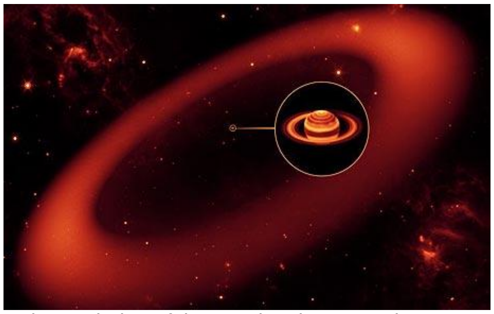
Problem 1 - Using a millimeter ruler and the '200 meter' metric bar, what is the scale of each of the two images and the map?

Problem 2 - About what is the distance between the TV camera and the PSE?

Problem 3 - From the left-hand image; A) What is the height and width of the field? B) What is the area of the field in square-kilometers?

Problem 4 - In the left-hand image, what is the diameter, in meters, of A) the largest crater, and B) the smallest crater?

Problem 5 - By counting craters in the left-hand image, what is the surface density of cratering in this region of the moon in units of craters per square kilometer?



Artist rendering of the new ice ring around Saturn detected by the Spitzer Space Telescope.

"This is one supersized ring," said one of the authors, Professor Anne Verbiscer, an astronomer at the University of Virginia in Charlottesville. Saturn's moon Phoebe orbits within the ring and is believed to be the source of the material.

The thin array of ice and dust particles lies at the far reaches of the Saturnian system. The ring was very diffuse and did not reflect much visible light but the infrared Spitzer telescope was able to detect it. Although the ring dust is very cold -316F it shines with thermal 'heat' radiation. No one had looked at its location with an infrared instrument until now.

"The bulk of the ring material starts about 6.0 million km from the planet, extends outward about another 12 million km, and is 2.6 million km thick. The newly found ring is so huge it would take 1 billion Earths to fill it." (CNN News, October 7, 2009)

Many news reports noted that the ring volume was equal to 1 billion Earths. Is that estimate correct? Let's assume that the ring can be approximated by a washer with an inner radius of r, an outer radius of R and a thickness of h.

Problem 1 - What is the formula for the area of a circle with a radius R from which another concentric circle with a radius r has been subtracted?

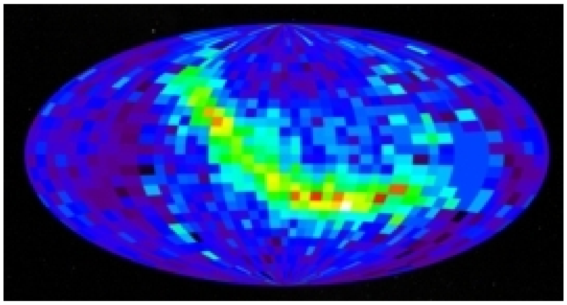
Problem 2 - What is the volume of the region defined by the area calculated in Problem 1 if the height of the volume is h?

Problem 3 - If $r = 6 \times 10^6$ kilometers, $R = 1.2 \times 10^7$ kilometers and $h = 2.4 \times 10^6$ kilometers, what is the volume of the new ring in cubic kilometers?

Problem 4 - The Earth is a sphere with a radius of 6,378 kilometers. What is the volume of Earth in cubic kilometers?

Problem 5 - About how many Earths can be fit within the volume of Saturn's new ice ring?

Problem 6 - How does your answer compare to the Press Release information? Why are they different?



NASA's IBEX satellite has detected fast-moving atoms streaking into the solar system from interstellar space. The energetic neutral atoms (called ENAs) are created in an area of our solar system known as the interstellar boundary region. This region is where charged particles from the sun, called the solar wind, flow outward far beyond the orbits of the planets and collide with material between stars.

The NASA Press release says that the ENAs "…travel inward toward the sun from interstellar space at speeds from 100,000 mph to more than 2.4 million mph."

$$K.E. = \frac{1}{2} m V^2$$

Question: How do scientists know the speeds of these particles?

Answer: It's all about Kinetic Energy!

The IBEX satellite detects energetic neutral atoms with a kinetic energy (K.E.) of 1,000 electron volts (1 keV), where 1 electron volt (eV) equals 1.6 x 10^{-19} Joules of energy. In the formula above, with KE expressed in Joules and the particle mass, m, expressed in kilograms, the speed of the particle, V, will be in meters/sec.

Problem 1 - What is the formula for the particle speed, V, in terms of the particle's mass and kinetic energy?

Problem 2 - Show that, if the particle is a proton with a mass of 1.7 x 10^{-27} kg and it has a speed of 450 kilometers/sec, to two significant figures, its energy is A) 1.7 x 10^{-16} Joules, or B) 1,100 eV.

Problem 3 - The IBEX satellite measures ENAs with an energy of 1 keV in order to make the image shown above. The most common element in the universe is hydrogen (1 proton). If the detected ENAs are all protons, what is the speed of the protons, in kilometers/sec, detected by IBEX to two significant figures?

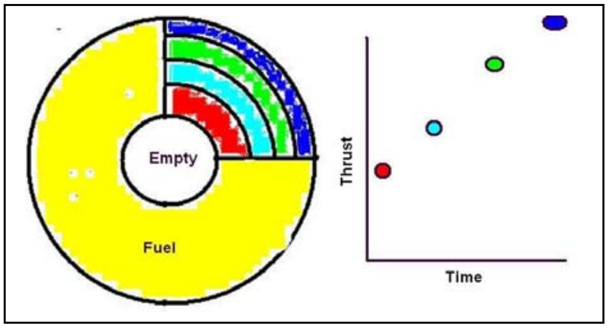
The most distant galaxy cluster yet has been discovered by combining data from NASA's Chandra X-ray Observatory and optical and infrared telescopes. The cluster is located about 10.2 billion light years away, and is observed as it was when the Universe was only about a quarter of its present age. The galaxy cluster, known as JKCS041, beats the previous record holder by about a billion light years. Galaxy clusters are the largest gravitationally bound objects in the Universe.

JKCS041 was originally detected in 2006 in a survey from the United Kingdom Infrared Telescope (UKIRT). The Chandra data were the final - but crucial - piece of evidence that showed JKCS041 was, indeed, a genuine galaxy cluster. Clusters of galaxies have such strong gravitational fields that they can serve as a bottle for very high temperature gas. These gases often emit x-ray light that can be detected by observatories such as Chandra. The discovery of such a high-temperature gas between the galaxies in JKCS041 supports the original idea that the galaxies seen in that direction are, in fact, members of a cluster. From the X-ray information, astronomers can also measure the total mass of the entire cluster that is responsible for creating the gravitational field holding the gas in place.

Problem 1 - The Chandra satellite detected x-rays coming from the region of the sky containing the galaxy cluster JKS041. The electrons in the gas are emitting the X-rays, and colliding at high speed with the protons in the gas. The energy of the x-rays at the time they were emitted by the hot gas was 21,400 electron Volts (eV). This energy is shared equally between the electrons and protons. The speed of a proton is related to its kinetic energy by $E = 1/2mV^2$ where E is the energy in Joules, V is the proton speed in meters/sec, and m is the mass of a proton ($m = 1.7 \times 10^{-27}$ kg). About how fast are the protons moving? (Note: 1 eV = 1.6×10^{-19} Joules)

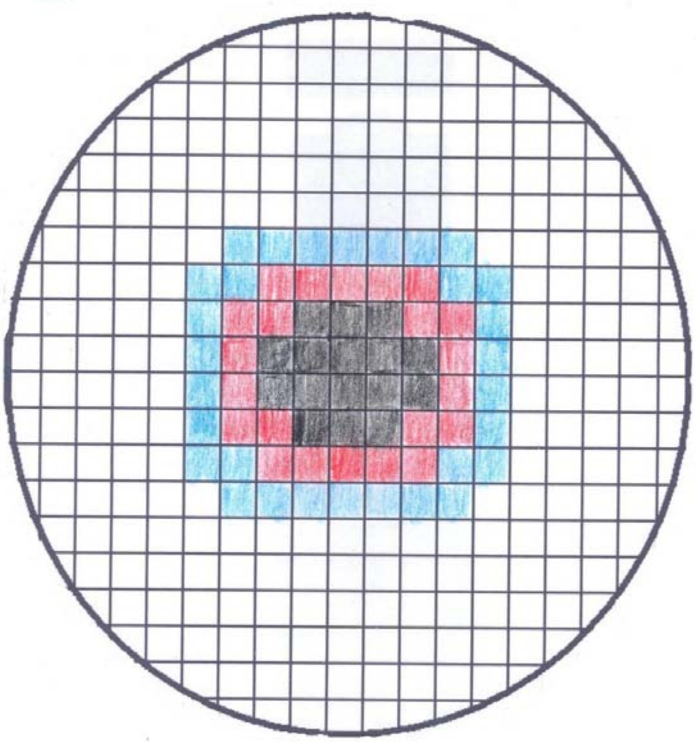
Problem 2 -The escape velocity (in km/s) from a body is given by $V = 0.17 (M/R)^{1/2}$ where M is the mass in multiples of the mass of our sun, and R is the average distance, in light years, between the body and the gas particle. Example, for the Milky Way, R = 50,000 light years and M = 300 billion so V = 420 km/sec. Compared to the sun, about how much mass do you need to confine the gas cloud observed by Chandra, if the cluster of galaxies has a radius of about 1 million light years A) in units of the sun's mass? B) In terms of the number of Milky Way galaxies where 1 Milky Way is about 2×10^{12} solar masses?

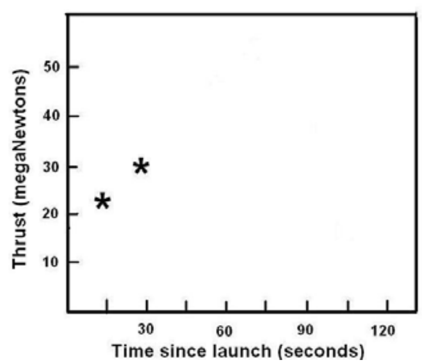
Solid Rocket Boosters - II



The example above shows the thrust curve for fuel burned in shells concentric with a cylindrical empty cavity along the axis of the booster from the inner (red) zone to the outer (blue) zone.

As the fuel in a solid rocket booster burns, it produces gas that exits the nozzle at very high pressure. This produces the thrust needed to launch a rocket. The area under combustion is a hollow core along the long axis of the booster from top to bottom. Depending on the shape of this empty tube, different volumes of gas will be produced from second to second, leading to different patterns of thrust for the rocket during its flight. The curve that describes a rocket engine's 'thrust versus time' is called the **thrust curve**. The more volume of fuel that is burned, the more thrust is produced.





Problem 1 - The grid shows the cross section of a proposed SRB that is a cylinder 80-meters tall. The black squares represent the empty core region. As the fuel burns its way from the core to the outside circle, complete the shading of the rings of combustion at each 15-second time step. Count the number of shaded squares in each ring. The first two rings in red and blue are shown as an example. The graph represents the thrust curve and gives the number of squares shaded (red=22 and blue = 30) at each time step.

Problem 2 - How many seconds after launch does the SRB produce its maximum thrust?

At 11:30 AM EST, NASA successfully launched the Ares 1-X rocket from Cape Canaveral. The top image is an artists illustration of the launch and the bottom photo shows the actual launch from Pad 39B. The flight reached the target sub-orbital altitude of 150,000 feet. The next launch will be in March 2014 of Ares 1-Y. (Images courtesy NASA)

The first half of the flight just before the rocket reached its peak altitude was powered by the first stage. Once the first stage was jettisoned 150 seconds after launch, the capsule traveled under its own inertia under the influence of Earth's gravity. The capsule reached a maximum altitude of 28 miles (45 km) at a point 40 miles (64 km) downrange from the launch pad, traveling at a horizontal speed of 4000 mph (6400 km/hr). We can approximate the capsule's path by a portion of a parabolic arc.

The time that it takes a body to fall to the ground is given by

$$H = \frac{1}{2} g T^2$$

where H is the altitude in meters, T is the time in seconds and g is the acceleration of gravity given by g = 9.8 meters/sec^2.

Problem 1 - To two significant figures, how many seconds did it take the Ares 1-X capsule to fall from an altitude of 45 kilometers?

Problem 2 - At the horizontal speed that the capsule was traveling, and to two significant figures, how far from the launch pad did it come to earth if the capsule reached its maximum altitude 64 kilometers downrange from the launch pad?

The neat thing about ballistic problems (flying baseballs or rockets) is that their motion in the vertical dimension is independent of their motion in the horizontal dimension. This means we can write one equation that describes the movement in time along the x axis, and a second equation that describes the movement in time along the y axis. In function notation, we write these as y(t) and y(t) where t is the independent variable representing time.

To draw the curve representing the trajectory, we have a choice to make. We can either create a table for X and Y at various instants in time, or we can simply eliminate the independent variable, t, and plot the curve y(x).

Problem 1 - The Ares 1X underwent powered flight while its first stage rocket engines were operating, but after it reached the highest point in its trajectory (apogee) the Ares 1X capsule coasted back to Earth for a water landing. The parametric equations that defined its horizontal downrange location (x) and its altitude (y) in meters are given by

$$x(t) = 64,000 + 1800t$$
$$y(t) = 45,000 - 4.9t^2$$

Using the method of substitution, create the new function y(x) by eliminating the variable t.

Problem 2 - Determine how far downrange from launch pad 39A at Cape Canaveral the capsule landed, (y(x)=0), giving your answer in both meters and kilometers to two significant figures.

Problem 3 - Why is it sometimes easier not to work with the parametric form of the motion of a rocket or projectile?

Potential energy is the energy that a body possesses due to its **location** in space, while kinetic energy is the energy that it has depending on its **speed** through space. For locations within a few hundred kilometers of Earth's surface, and for speeds that are small compared to that of light, we have the two energy formulae:

$$P.E = mgh \qquad K.E = \frac{1}{2}mV^2$$

where g is the acceleration of gravity near Earth's surface and has a value of 9.8 meters/sec^2. If we use units of mass, **m**, in kilograms, height above the ground, **h**, in meters, and the body's speed, **V**, in meters/sec, the units of energy (P.E and K.E.) are Joules.

As a baseball, a coasting rocket, or a stone dropped from a bridge moves along its trajectory back to the ground, it is constantly exchanging, joule by joule, potential energy for kinetic energy. Before it falls, its energy is 100% P.E, while in the instant just before it lands, its energy is 100% K.E.

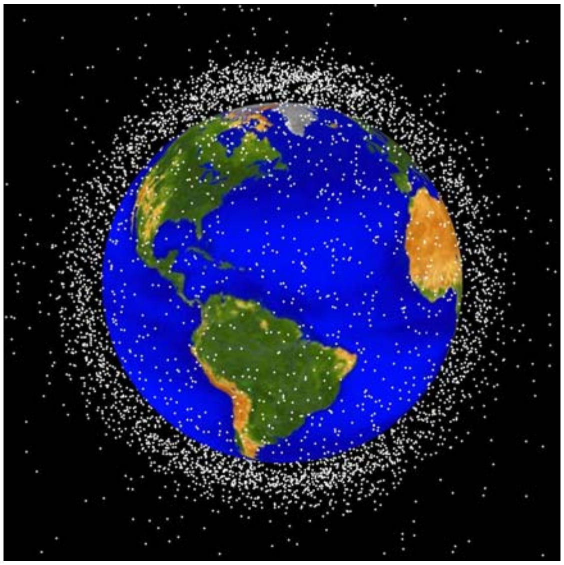
Problem 1 - A baseball with m = 0.145 kilograms falls from the top of its arc to the ground; a distance of 100 meters. A) What was its K.E., in joules, at the top of its arc? B) What was the baseball's P.E. in joules at the top of the arc?

Problem 2 - The Ares 1-X capsule had a mass of 5,000 kilograms. If the capsule fell 45 kilometers from the top of its trajectory 'arc', how much kinetic energy did it have at the moment of impact with the ground?

Problem 3 - Suppose that the baseball in Problem 1 was dropped from the same height as the Ares 1-X capsule. What would its K.E. be at the moment of impact?

Problem 4 - From the formula for K.E. and your answers to Problems 2 and 3, in meters/sec to two significant figures; A) What was the speed of the baseball when it hit the ground? B) What was the speed of the Ares 1-X capsule when it landed? C) Discuss how your answers do not seem to make 'common sense'.

Problem 5 - Use the formulae for P.E. and K.E. to explain your answers to Problem 4. (Hint: Don't worry about air resistance!!)



We live at the bottom of a deep 'gravity well' that takes an enormous amount of energy to climb out of. The figure shows the locations of 12,000 currently in Low Earth Orbit (LEO). To reach any altitude above sea level, we have to work against gravity. The higher we want to climb, the more energy we need to use to reach that altitude.

We can measure this energy in terms of the number of joules per kilogram (J/kg) that is needed to reach an elevation of h meters. The formula is given very simply by

$$E = 9.8\,h \quad \text{J/kg}$$

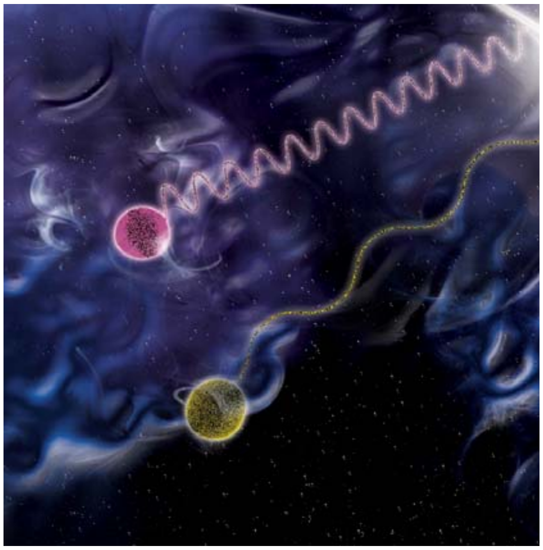
Problem 1 - A mountain climber hikes from sea level to the top of Mt Everest, which has an altitude of 8,848 meters. A) How many Joules/kg will he need for the trip? B) If his total mass is 75 kg, how many Joules of energy will he have to expend to reach the summit?

Problem 2 - The Ares 1-X rocket was launched on Wednesday, October 28 at Cape Canaveral. If the payload reached a maximum altitude of 45 kilometers, A) how many megaJoules/kg were needed for the payload to reach this altitude? B) If the payload mass was 5,000 kg, how many megaJoules were required?

Problem 3 - At what altitudes will the mountain climber and the Ares 1-X need to expend the same number of Joules/kg?

Problem 4 - How many megaJoules/kg will the Ares 1-X need to expend to reach Low Earth Orbit at an altitude of 200 miles? (1 mile = 1.62 kilometers).

Problem 5 - In terms of megaJoules/kg, by what factor is the Ares 1-X energy requirement greater to get into LEO than to reach an altitude of 45 kilometers?


An artistic impression of two gamma-ray photons traveling through a lumpy space (Courtesy: NASA /Sonoma State University/Aurore Simonnet.)

The most advanced theories of how gravity works have proposed that empty space is not smooth, but may be filled by invisible limps and bumps that distort space into a froth of bubbles that are billions of times smaller than atomic nuclei. That's why we have not detected them in laboratory experiments thus far.

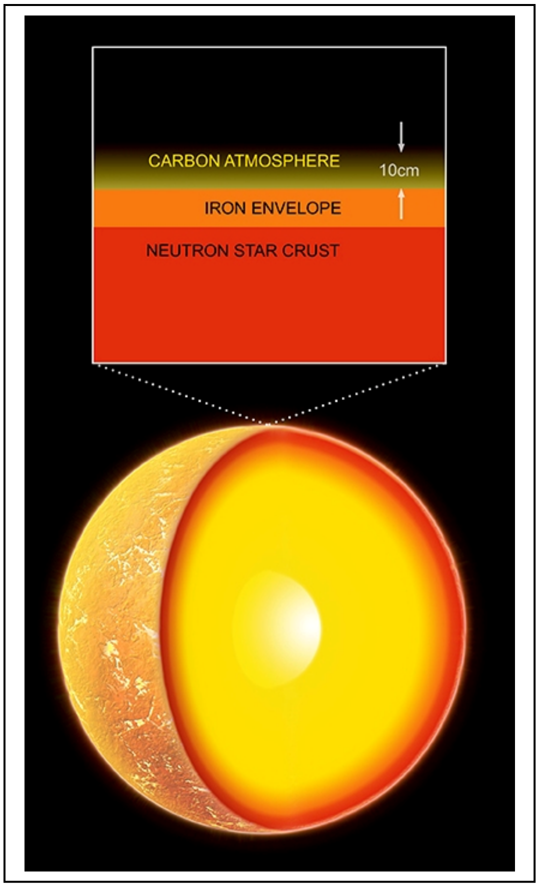
By studying how intense gamma-rays travel through the vast spaces between galaxies in the universe, NASA's Fermi Gamma-ray Observatory may have placed limits on this frothiness that eliminate many of the theories being explored to date.

As the gamma-rays travel through space, the shortest-wavelength gamma-rays take a slightly different path through space than the longer-wavelength gamma-rays. Although all gamma-rays travel at the speed of light, the invisible lumps in space scatter the short-wavelength (high-energy) gamma rays more than the long-wavelength (low-energy) ones so that there is a difference in the travel times of the long and short-wavelength gamma rays. This means that in traveling a distance, L, to Earth, the time should be about t = L/c where c is the speed of light. The theories predict that the lumps in space cause the arrival times for the long and short-wavelength gamma-rays to differ by an amount **T = (L/c) x (d/λ).** In this equation, λ is the wavelength difference between the two gamma-rays (related to their energy-difference), **d** is the length scale corresponding to the lumpiness in space, and **c** is the speed of light, 3×10^{10} cm/sec.

Problem 1 – Suppose two rays of visible light differed in wavelength by 100 nanometers ($\lambda = 10^{-9}$ cm) and traveled from the sun to Earth (L=150 million kilometers) through space with a lumpiness of about the scale of an atomic nucleus (d=10^{-14} cm). What would be the arrival time delay in seconds at Earth between the two visible light photons?

Problem 2 – Suppose there is no indication that arriving visible light photons (10^{-9} cm) experience any delays in arrival time longer than 1 second, from quasars as far away as 5 billion light years. What is the maximum size of the lumps in space that are consistent with this limit?

Problem 3 – The Fermi Telescope measured a gamma-ray pulse from a distant object located 10 billion light years from Earth. The time delay was no more than 0.7 seconds. The wavelength difference in the gamma-rays was 4.0×10^{-12} centimeters (33 billion electron volts of energy). What is the largest size that can be involved in the lumpiness of empty space given this Fermi measurement.



The atmosphere of Earth has a thickness of over 100 kilometers, however the Chandra X-Ray Observatory recently detected an atmosphere on the neutron star Cassiopeia-A that measured only 10 centimeters thick. How can a small planet have a deeper atmosphere than an entire stars-worth of matter?

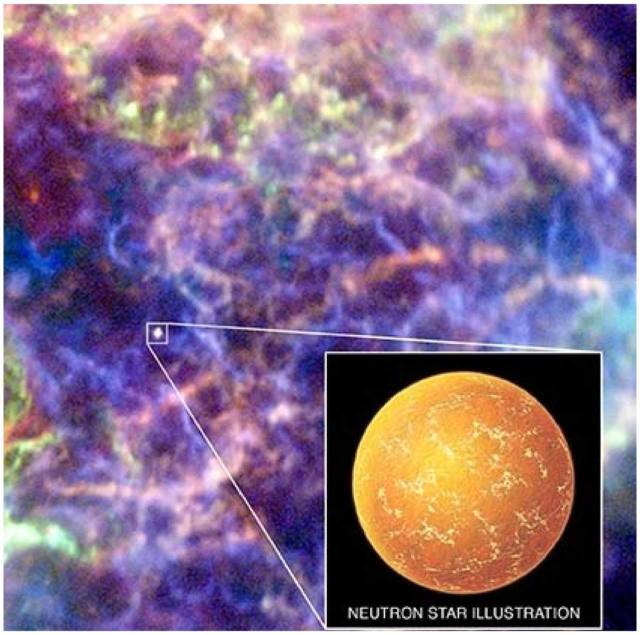
The size of an atmosphere depends on a balance between the force of gravity pulling it towards the center of the planet, and the pressure of the atmosphere due to its temperature and density, pushing in the opposite direction. A pair of simple equations then defines how the density of the atmosphere has to rearrange itself with height above the surface so that gravity and pressure are always in balance. The equations look like this:

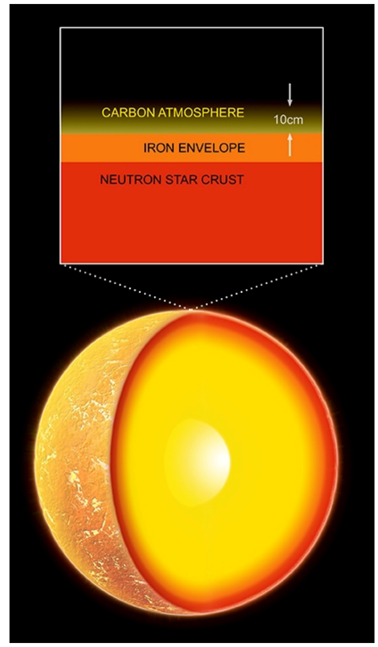
$$n(z) = n_0 e^{-\frac{z}{H}} \quad \text{where} \quad H = \frac{kT}{mg}$$

The exponential equation says that as you get farther from the surface, the density of the gas, N, drops very fast. The quantity, H in meters, is called the 'scale height' and its value is defined by the atmosphere's temperature, T, in Kelvins, and the acceleration of gravity at the surface, g, in multiples of Earth's acceleration (9.8 meters/sec^2). It also depends on the average mass, m, of the particles in the atmosphere. A light atmosphere made from hydrogen (m=1) will produce a value for H that is much larger than one made from pure oxygen (m=16). In this equation, k is Boltzman's Constant and equals 1.38 x 10^{-23} Joules/degree.

Problem 1 – The surface acceleration of the neutron star is 100 billion times that of Earth, the temperature of the gas is 3 million Kelvins compared to Earth's of 300 Kelvins, and the neutron star atmosphere is composed of carbon (A=12) rather than Earth's mixture of nitrogen and oxygen (A=28). From the formula for H, and the way in which it scales with m, T and g, what would you predict as the scale height for the neutron star atmosphere if for Earth, H = 8 kilometers?

Problem 2 – How far from the surface would you have to travel in order for the density of the atmosphere to fall by 1 million times for: A) Earth and B) the neutron star?





NASA Press Release – “The Chandra X-ray Observatory image to the left shows the central region of the supernova remnant Cassiopeia A. This interstellar cloud 14 light years across, is all that remains of a massive star that exploded 330 years ago. A careful analysis of the X-ray data has revealed that the dense neutron star left behind by the supernova has a thin carbon atmosphere as shown in the figure to the right. The neutron star is only 14 miles (23 kilometers) in diameter, and is as dense as an atomic nucleus (100 trillion gm/cc). The atmosphere is only about four inches (10 cm) thick, has a density similar to diamond (3.5 gm/cc), and a temperature of nearly 2 million Kelvin. The surface gravity on the neutron star is 100 billion times stronger than on Earth, which causes the atmosphere to be incrredibly thing even with such a high temperature.”

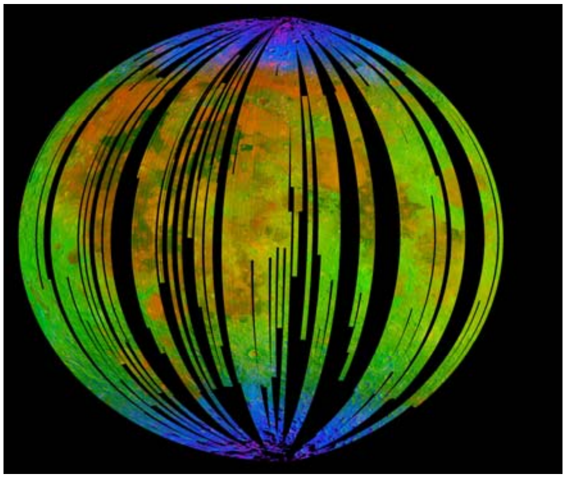
How much carbon is there?

Problem 1 – What are the facts that we know about the atmosphere from the news announcement, and what combination of facts will help us estimate the atmosphere’s mass?

Problem 2 – If the volume of a thin spherical shell is $V = 4 \pi R^2 h$ where R is the radius of the sphere and h is the thickness of the shell, what other formula do you need to calculate the atmosphere’s mass?

Problem 3 – What is your estimate for the mass of the carbon atmosphere in A) kilograms? B) metric tons? C) Earth Atmosphere masses (Ae) where 1 Ae = 5.1×10^{18} kg? (Provide answers to two significant figures)

Water on the Moon !



The debate has gone back and forth over the last 10 years as new data are found, but measurements by Deep Impact/EPOXI, Cassini and most recently the Lunar Reconnaissance Orbiter and Chandrayaan-1 are now considered conclusive. Beneath the shadows of polar craters, billions of gallons of water may be available for harvesting by future astronauts.

The image to the left created by the Moon Minerology Mapper (M3) instrument onboard Chandrayaan-1, shows deposits and sources of hydroxyl molecules. The data has been colored blue and superimposed on a lunar photo.

Complimentary data from the Deep Impact/EPOXI and Cassini missions of the rest of the lunar surface also detected hydroxyl molecules covering about 25% of the surveyed lunar surface. The hydroxyl molecule (symbol OH) consists of one atom of oxygen and one of hydrogen, and because water is basically a hydroxyl molecule with a second hydrogen atom added, detecting hydroxyl on the moon is an indication that water molecules are also present.

How much water might be present? The M3 instrument can only detect hydroxyl molecules if they are in the top 1-millimeter of the lunar surface. The measurements also suggest that about 1 metric ton of lunar surface has to be processed to extract 1 liter (0.26 gallons) of water.

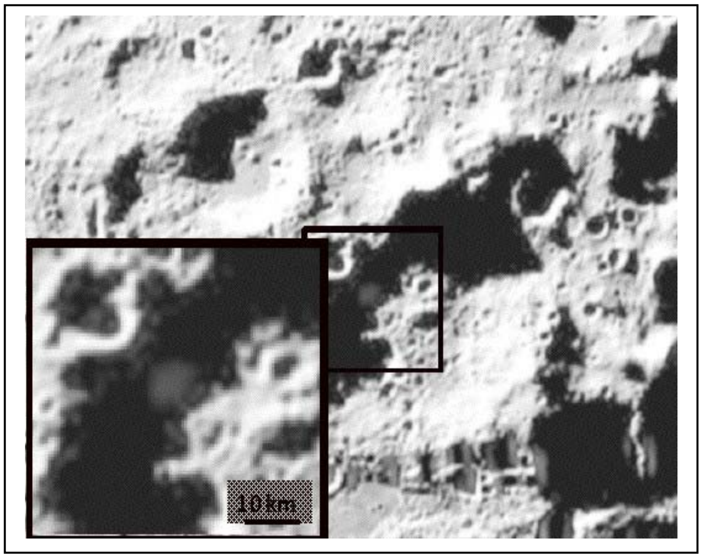
Problem 1 – The radius of the moon is 1,731 kilometers. How many cubic meters of surface volume is present in a layer that is 1 millimeter thick?

Problem 2 – The density of the lunar surface (called the regolith) is about 3000 kilograms/meter^3. How many metric tons of regolith are found in the surface volume calculated in Problem 1?

Problem 3 – The concentration of water is 1 liter per metric ton. How many liters of water could be recovered from the 1 millimeter thick surface layer if 25% of the lunar surface contains water?

Problem 4 – How many gallons could be recovered if the entire surface layer were mined? (1 Gallon = 3.78 liters).

LCROSS 'Sees' Water on the Moon!



On October 9, 2009 the LCROSS spacecraft and its companion Centaur upper stage, impacted the lunar surface within the shadowed crater Cabeus located near the moon's South Pole. The Centaur impact speed was 9,000 km/hr with a mass of 2.2 tons.

The impact created a crater about 20 meters across and about 3 meters deep. Some of the excavated material formed a plume of debris visible to the LCROSS satellite as it flew by. Instruments on LCROSS detected about 25 gallons of water.

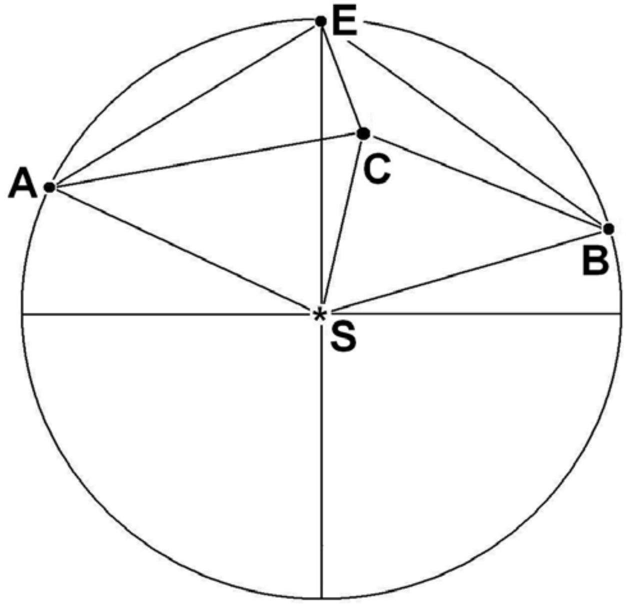
Problem 1 - The volume of the crater can be approximated as a cylinder with a diameter of 20 meters and a height of 3 meters. From the formula $V = \pi R^2 h$, what was the volume of the lunar surface excavated by the LCROSS-Centaur impact in cubic meters?

Problem 2 - If the density of the lunar soil (regolith) is about 3000 kilograms/meter^3, how many tons of regolith were excavated by the impact?

Problem 3 - During an impact, most of the excavated material remains as a ring-shaped ejecta blanket around the crater. For the LCROSS crater, the ejecta appeared to be scattered over an area about 70 meters in diameter and perhaps 0.2 meter thick around the crater. How many tons of regolith from the crater remained near the crater?

Problem 4 - If the detected water came from the regolith ejected in the plume, and not scattered in the ejecta blanket ,what was the concentration of water in the plume in units of tons of regolith per liter of water?

Seeing Solar Storms in STEREO - I



The two STEREO spacecraft are located along Earth's orbit and can view gas clouds ejected by the sun as they travel to Earth. From the geometry, astronomers can accurately determine their speeds, distances, shapes and other properties.

By studying the separate 'stereo' images, astronomers can determine the speed and direction of the cloud before it reaches Earth.

Use the diagram, (angles and distances not drawn to the same scale of the 'givens' below) to answer the following question.

The two STEREO satellites are located at points A and B, with Earth located at Point E and the sun located at Point S, which is the center of a circle with a radius ES of 1.0 Astronomical Unit (1 AU is the distance of Earth from the sun or 150 million kilometers). Suppose that the two satellites spot a Coronal Mass Ejection (CME) cloud at Point C. Satellite A measures its angle from the sun mSAC as 45 degrees while Satellite B measures the corresponding angle to be mSBC=50 degrees. The CME is ejected from the sun at the angle mESC=14 degrees.

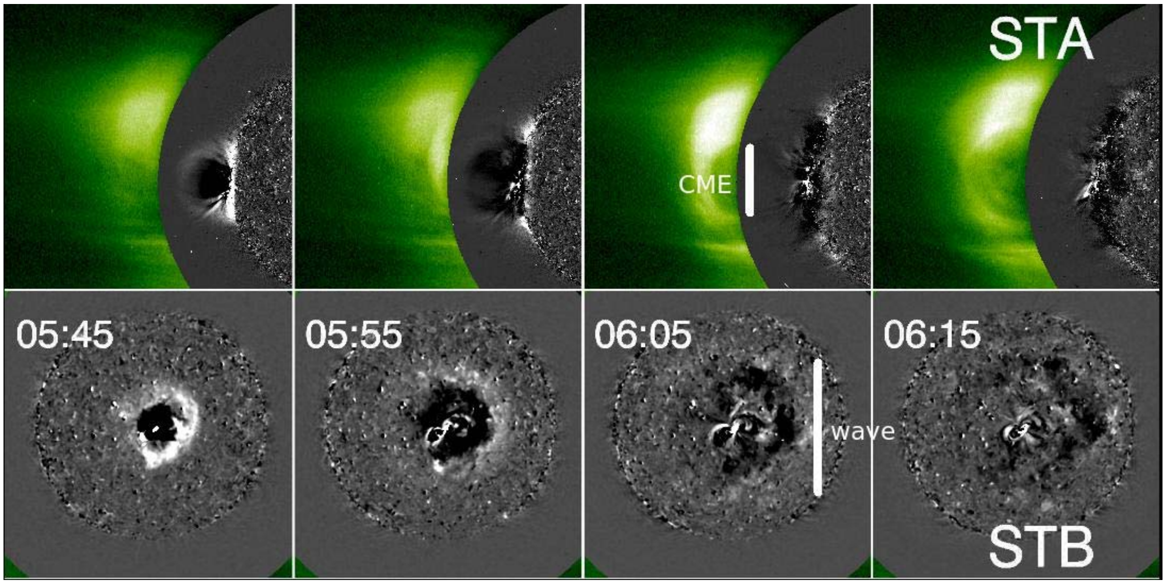
Problem 1 - The astronomers want to know the distance that the CME is from Earth, which is the length of the segment EC. The also want to know the approach angle, mSEC. Use either a scaled construction (easy: using compass, protractor and millimeter ruler) or geometric calculation (difficult: using trigonometric identities) to determine EC from the available data.

Givens from satellite orbits:

SB = SA = SE = 150 million km AE = 136 million km BE = 122 million km
mASE = 54 degrees mBSE = 48 degrees
mEAS = 63 degrees mEBS = 66 degrees mAEB = 129 degrees

Find the measures of all of the angles and segment lengths in thee above diagram rounded to the nearest integer.

Problem 2 - If the CME was traveling at 2 million km/hour, how long did it take to reach the distance indicated by the length of segment SC?



A solar tsunami that occurred in February 13, 2009 has recently been identified in the data from NASA's STEREO satellites. It was spotted rushing across the Sun's surface. STEREO recorded the wave from two positions separated by 90 degrees, giving researchers a spectacular view of the event. Satellite A (STA) provided a side-view of the explosion, called a Coronal Mass Ejection (CME), while Satellite B (STB) viewed the explosion from directly above. The technical name is "fast-mode magnetohydrodynamic wave" – or "MHD wave" for short. The one STEREO saw raced outward at 560,000 mph (250 km/s) packing as much energy as 2,400 megatons of TNT.

Problem 1 - In the lower strip of images, the sun's disk is defined by the mottled circular area, which has a physical radius of 696,000 kilometers. Use a millimeter ruler to determine the scale of these images in kilometers/mm.

Problem 2 - The white circular ring defines the outer edge of the expanding MHD wave. How many kilometers did the ring expand between 05:45 and 06:15? (Note '05:45' means 5:45 o'clock Universal Time).

Problem 3 - From your answers to Problem 1 and 2, what was the approximate speed of this MHD wave in kilometers/sec?

Problem 4 - Kinetic Energy is defined by the equation K.E. = $1/2\ m\ V^2$ where m is the mass of the object in kilograms, and V is its speed in meters/sec. Suppose the mass of the CME was about 1 million metric tons, use your answer to Problem 3 to calculate the K.E., which will be in units of Joules.

Problem 5 - If 1 kiloton of TNT has the explosive energy of 4.1×10^{12} Joules, how many megatons of TNT does the kinetic energy of the tsunami represent?

This image shows the large galaxy, NGC-6872, interacting with a smaller galaxy, IC-4970, located just above the center of NGC-6872. These galaxies are located in the southern constellation Pavo, and about 300 million light years from the Sun. From tip to tip, NGC-6872 measures about 700,000 light years, making it nearly 3 times as big as the Milky Way. The image is a composite made by NASA's Spitzer and Chandra satellites, and a ground-based telescope. Although NGC-6872 is dramatically bigger, IC-4970 is the real 'player' in this collision. It contains a massive black hole that is emitting energy as it absorbs interstellar gas and dust. This material has been gravitationally ripped from the larger galaxy. The X-ray power, alone, is about 450 million times the sun's total light output!

When black holes 'digest' gas, stars and other forms of matter that enter their event horizons, the energy that is produced as the matter falls in can be converted into heat in the orbiting 'accretion disk' that surrounds a black hole. The amount of heat energy that is emitted by the gas in this disk each second (power) can be detected at great distances as a brilliant source of light or other forms of electromagnetic radiation. The formula that approximately relates the rate of in-falling matter into a super-massive black hole (**R** in solar masses per year), to the emitted power (**L** in multiples of the sun's power) is given by $L = 1.1 \times 10^{12}\, R$ solar luminosities. (Note 1 solar mass = 2×10^{33} grams, and 1 solar luminosity = 4×10^{33} ergs/sec).

Problem 1 - What is the minimum accretion rate that is needed to account for the x-ray power of the black hole in the core of IC-4970?

Problem 2 - How much mass would have to be accreted in order for the supermassive black hole to have the same power as an average quasar with a luminosity of about 2 trillion times the luminosity of our sun?

Problem 3 - The supermassive black hole in the center of the Milky Way has an estimated output equal to 2,500 suns. About how fast is it accreting matter?

$$B = B_0 10^{-0.4m}$$

Since the time of the Greek astronomer Hipparchus (190BC), astronomers have used a 'magnitude' scale to indicate the brightness of stars. A star of the First Magnitude (+1.0) is brighter than a star of the Second Magnitude (+2.0) and so on, which means that, the more positive a star's magnitude is, the fainter is the star! Although modern astronomers would have preferred a more conventional scale, we are basically stuck with this ancient convention.

On the stellar magnitude scale, a difference of 5 magnitudes is exactly a brightness difference of 100 times. This magnitude scale, **m**, is related to a physical brightness scale, **B**, using the formula shown to the left.

The star field image is from the Hubble Space Telescope Exoplanet Survey and shows a range of stellar brightnesses spanning a magnitude range from about +13 to +17

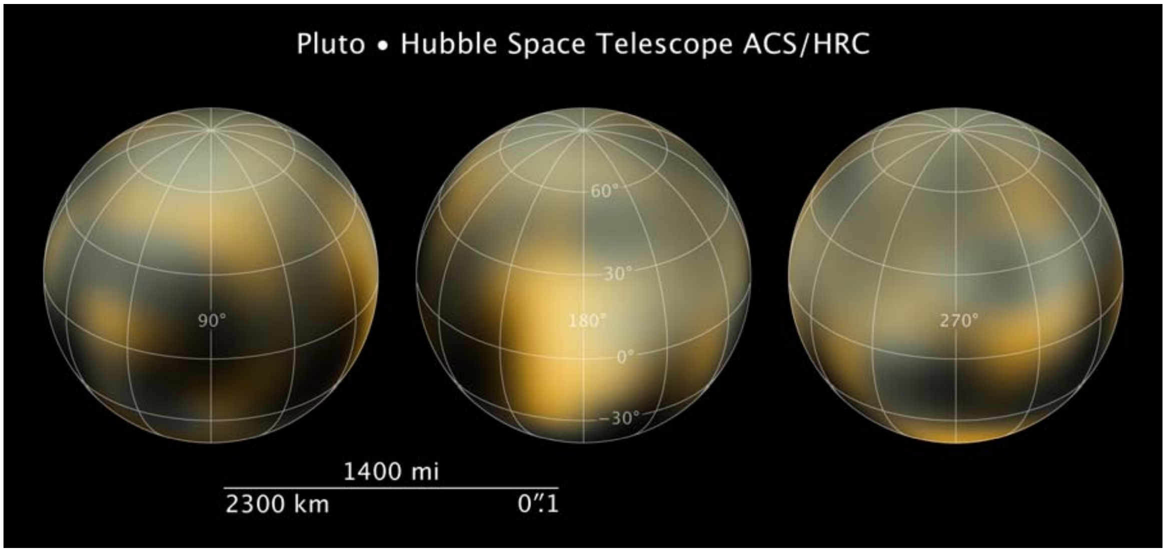
Problem 1 - In this formula, to what magnitude does a brightness of B_0 correspond?

Problem 2 - The sun has a magnitude of -26.5, and the faintest star detected by the Hubble Space Telescope has a magnitude of +28.5 A) What is the magnitude difference between these two objects? B) By what factor to they differ in brightness?

Problem 3 - An astronomer has a digital camera that can accommodate a brightness level change of about 10 million from the brightest to the faintest object that can be imaged without 'saturating' the camera. What magnitude difference does this range correspond to?

Problem 4 - NASA's WISE infrared sky survey satellite will detect stars at a wavelength of 4.6 microns (called M-band) to a brightness limit of 160 microJanskys. If B0 is 180 Janskys at this wavelength, and for the visual magnitude scale (called V-band) B0 = 3,781 Janskys. What will be the equivalent magnitude limit of the WISE survey at 4.6 microns, and in the visual band?

Seeing a Dwarf Planet Clearly: Pluto



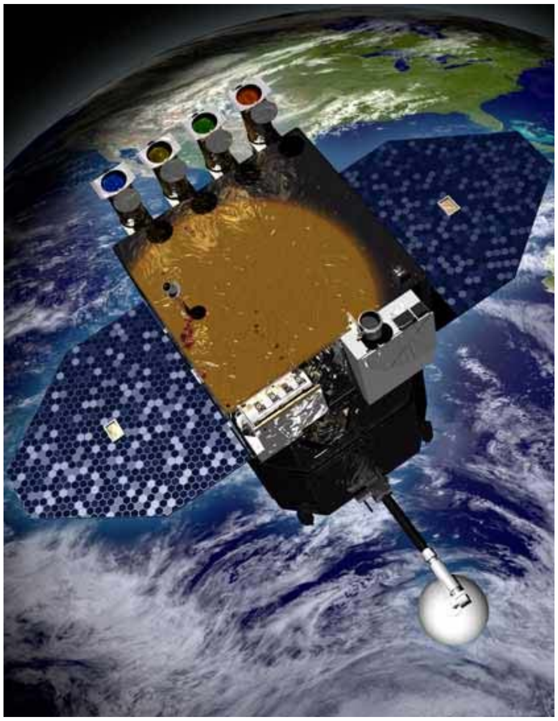
Recent Hubble Space Telescope studies of Pluto have confirmed that its atmosphere is undergoing considerable change, despite its frigid temperatures. The images, created at the very limits of Hubble's ability to see small details (sometimes called a telescope's resolving power), show enigmatic light and dark regions that are probably organic compounds (dark areas) and methane or water-ice deposits (light areas). Since these photos are all that we are likely to get until NASA's New Horizons spacecraft arrives in 2015, let's see what we can learn from the image!

Problem 1 - Using a millimeter ruler, what is the scale of the Hubble image in kilometers/millimeter?

Problem 2 - What is the largest feature you can see on any of the three images, in kilometers, and how large is this compared to a familiar earth feature or landmark such as a state in the United States?

Problem 3 - The satellite of Pluto, called Charon, has been used to determine the total mass of Pluto. The mass determined was about 1.3 x 10^{22} kilograms. From clues in the image, calculate the volume of Pluto and determine the average density of Pluto. How does it compare to solid-rock (3000 kg/m^3), water-ice (917 kg/m^3)?

Inquiry: Can you create a model of Pluto that matches its average density and predicts what percentage of rock and ice may be present?



The 15 instruments on NASA's latest solar observatory will usher in a new era of solar observation by providing scientists with 'High Definition'-quality viewing of the solar surface in nearly a dozen different wavelength bands.

One of the biggest challenges is how to handle all the data that the satellite will return to Earth, every hour of the day, for years at a time! It is no wonder that the design and construction of this data handling network has taken nearly 10 years to put together! To make sense of the rest of this story, here are some units and prefixes you need to recall (1 byte = 8 bits):

Kilo = 1 thousand
Mega = 1 million
Giga = 1 billion
Tera = 1 trillion
Peta = 1,000 trillion
Exa = 1 million trillion

Problem 1 - In 1982 an IBM PC desktop computer came equipped with a 25 megabyte hard drive (HD) and cost $6,000. In 2010, a $500 desktop comes equipped with a 2.5 gigabyte hard drive. By what factor do current hard drives have more storage space than older models?

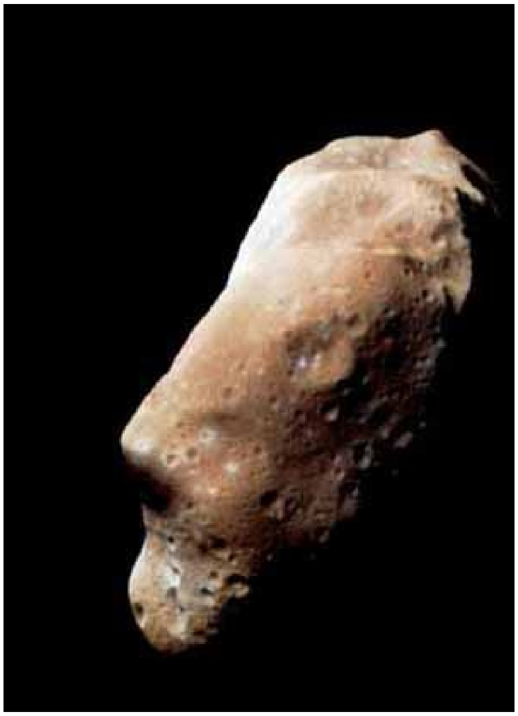
Problem 2 - A 2.5 gigabyte hard drive is used to store music from iTunes. If one typical 4-minute, uncompressed, MPEG-4 song occupies 8 megbytes, about A) How many uncompressed songs can be stored on the HD? B) How many hours of music can be stored on the HD? (Note: music is actually stored in a compressed format so typically several thousand songs can be stored on a large HD)

Problem 3 - How long will it take to download 2 gigabytes of music from the iTunes store A) Using an old-style 1980's telephone modem with a bit rate of 56,000 bits/sec? B)With a modern fiber-optic cable with a bit rate of 16 megabits/sec?

Problem 4 - The SDO satellite's AIA cameras will generate 67 megabits/sec of data as they take 4096x4096-pixel images every 3/4 of a second. The other two instruments, the HMI and the EVE, will generate 62 megabits/sec of data. The satellite itself will also generate 20 megabits/sec of 'housekeeping' information to report on the health of the satellite. If a single DVD can store 5 gigabytes of information, how many DVDs-worth of data will be generated by the SDO: A) Each day? B) Each year?

Problem 5 - How many petabytes of data will SDO generate during its planned 5-year mission?

Problem 6 - It has been estimated that the total amount of audio, image and video information generated by all humans during the last million years through 2009 is about 50 exabytes including all spoken words (5 exabytes). How many DVDs does this equal?



Asteroid Gaspara

Astronomers studying the asteroid 24-Themis detected water-ice and carbon-based organic compounds on the surface of the asteroid.

NASA detects, tracks and characterizes asteroids and comets passing close to Earth using both ground and space-based telescopes.

NASA is particularly interested in asteroids with water ice because this resource could be used to create fuel for interplanetary spacecraft.

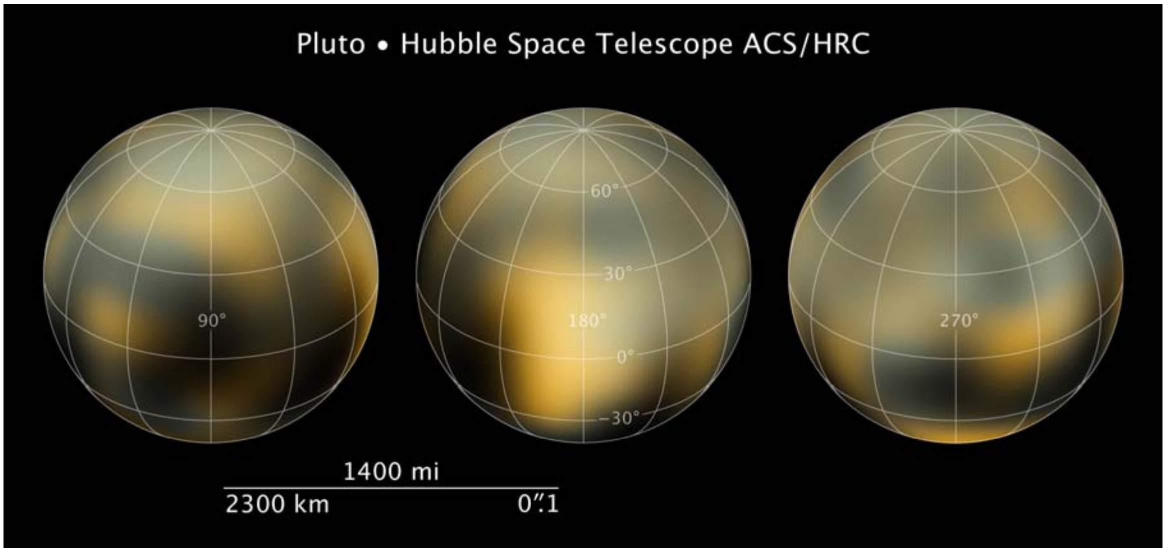
On October 7, 2009, the presence of water ice was confirmed on the surface of this asteroid using NASA's Infrared Telescope Facility. The surface of the asteroid appears completely covered in ice. As this ice layer is sublimated (goes directly from solid to gaseous state) it may be getting replenished by a reservoir of ice under the surface. The orbit of the asteroid varies from 2.7 AU to 3.5 AU (where 1 AU is the 150 million km distance from Earth to the sun) so it is located within the asteroid belt. The asteroid is 200 km in diameter, has a mass of 1.1×10^{19} kg, and a density of 2,800 kg/m^3 so it is mostly rocky material similar in density to Earth's.

By measuring the spectrum of infrared sunlight reflected by the object, the NASA researchers found the spectrum consistent with frozen water and determined that 24 Themis is coated with a thin film of ice. The asteroid is estimated to lose about 1 meter of ice each year, so there must be a sub-surface reservoir to constantly replace the evaporating ice.

Problem 1 – Assume that the asteroid has a diameter of 200 km. How many kilograms of water ice are present in a layer 1-meter thick covering the entire surface, if the density of ice is 1,000 km/meter^3? (Hint: Volume = Surface area x thickness)

Problem 2 – Suppose that only 1% by volume of the 1-meter-thick 'dirty' surface layer is actually water-ice and that it evaporates 1 meter per year, what is the rate of water loss in kg/sec?

The Changing Atmosphere of Pluto



Recent Hubble Space Telescope studies of Pluto have confirmed that its atmosphere is undergoing considerable change, despite its frigid temperatures. Let's see how this is possible!

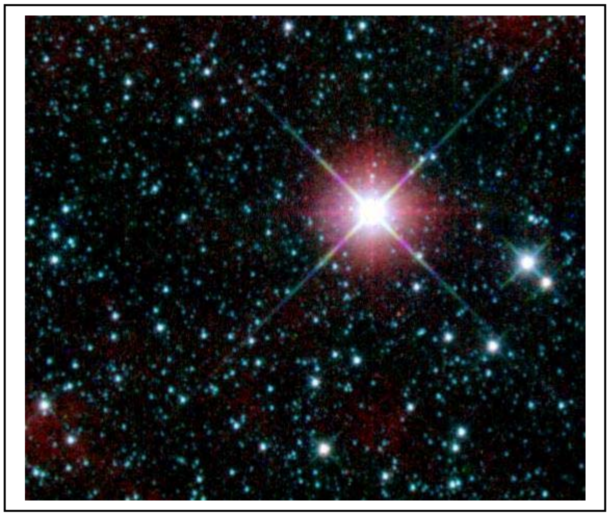
Problem 1 - The equation for the orbit of Pluto can be approximated by the formula $2433600=1521x^2+1600y^2$. Determine from this equation, expressed in Standard Form, A) the semi-major axis, a; B) the semi-minor axis, b; C) the ellipticity of the orbit, e; D) the longest distance from a focus called the aphelion; E) the shortest distance from a focus, called the perihelion. (Note: All units will be in terms of Astronomical Units. 1 AU = distance from the Earth to the Sun = 1.5×10^{11} meters).

Problem 2 - The temperature of the methane atmosphere of Pluto is given by the formula

$$T(R) = \left(\frac{L(1-A)}{16\pi\sigma R^2}\right)^{\frac{1}{4}} \text{ degrees Kelvin (K)}$$

where L is the luminosity of the sun ($L=4 \times 10^{26}$ watts); σ is a constant with a value of 5.67×10^{-8}, R is the distance from the sun to Pluto in meters; and A is the albedo of Pluto. The albedo of Pluto, the ability of its surface to reflect light, is about A = 0.6. From this information, what is the predicted temperature of Pluto at A) perihelion? B) aphelion?

Problem 3 - If the thickness, H, of the atmosphere in kilometers is given by H(T) = 1.2 T with T being the average temperature in degrees K, can you describe what happens to the atmosphere of Pluto between aphelion and perihelion?



There are many situations in astrophysics when two distinct functions are multiplied together to form a new function.

If there are N light bulbs, each with a brightness of W watts, then the total brightness, T of all these bulbs is just N x W. For N=3 bulbs and W = 100 watts we have T = 300 watts.

Suppose N(m) tells us the number of stars in an area of the sky with a brightness of m. Let a second function, S(m), represent the number of watts per square meter at the Earth that a star with a brightness of m produces. Then N(m)S(m) will be the total number of watts/meter^2 produced by the stars in the sample that have a brightness of m.

NASA's, Wide-Field Infrared Survey Explorer (WISE) satellite is surveying the sky to catalog stars visible at a wavelength of 3.5 microns in the infrared spectrum. If the differential star count function $N(m)=0.000005\ m^{+7.0}$ stars, and the star brightness function is defined by $S(m)= 350\ 10^{-0.4m}$ Janskys. Use this information to answer the following problems:

Problem 1 - Graph the functions Log(N(m)) and Log(S(m)) as individual histograms over the domain m:[+6, +16] for integer values of m.

Problem 2 - Graph the product of these functions N(m)S(m) over the domain m:[+6, +16] for integer values of m.

Problem 3 - What is the sum, T, of N(m)S(m) for each integer value of m in the domain m:[+6, +16], and how does this sum relate to the area under the curve for N(m)S(m)?

Problem 4 - What is the integral of N(m)S(m) from m=+6 to m= +16? You do not need to evaluate it!

The Wide-field Infrared Survey Experiment (WISE) recently took its first photo of a test field in the constellation Carina to check out its instruments.

WISE is based on a 40-cm (16-inch) telescope designed to detect radiation at four wavelengths: 3.4, 4.6, 12 and 22 microns. The telescope is kept cold using solid hydrogen at a temperature of -438 F (12 Kelvin), and will be able to function for about 10 months in space until the hydrogen evaporates. In this time, WISE will take over a million pictures of the whole sky, revealing hundreds of millions of stars, galaxies, asteroids and other objects that shine brightly in infrared light.

The image to the left, measures 47arcminutes x 23 arcminutes (1 arcminute is 1/60 of one angular degree). It is a portion of the WISE 'First Light' image near the bright star V482 Carinae seen to the right.

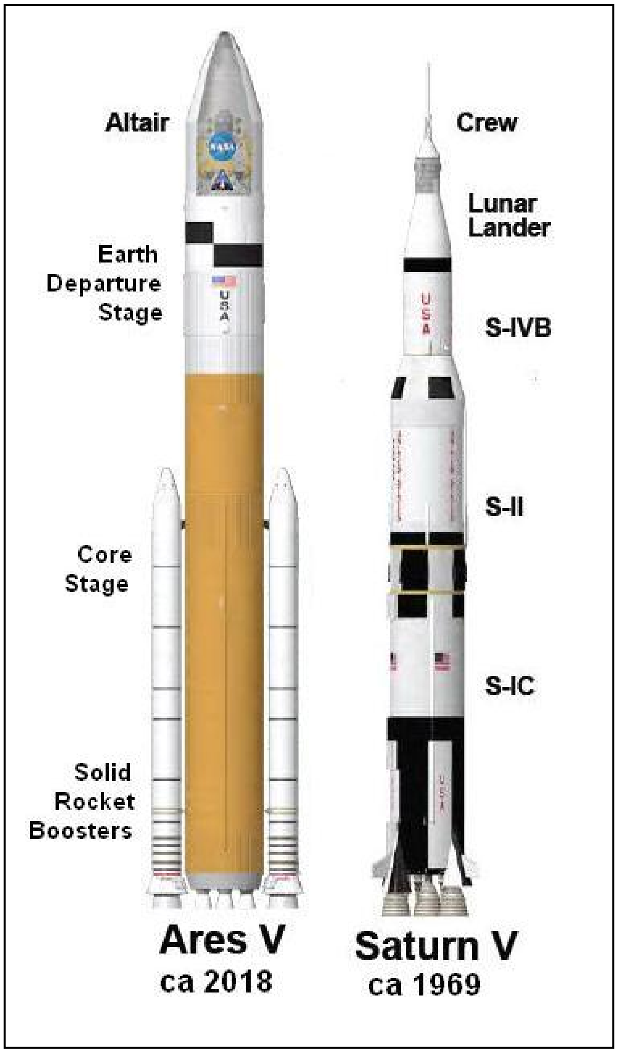
Astronomers not only study individual stars in a field like this, but also count the stars in each brightness interval in order to mathematically model how stars are distributed in the Milky Way. Such models help us understand the shape and history of our galaxy.

Problem 1 - The bright star seen in this field is V482 Carinae. It has a stellar brightness of +6.0. If the area of this field has dimensions of 0.8 degrees x 0.4 degrees, how many stars as bright as V482 are present, on average, A) per square degree of the sky? B) Across the entire sky if its area is 41,253 deg^2?

Problem 2 - The function A(m) represents the total number of stars per deg^2 counted in the stellar brightness (called 'magnitude') bin from m-1/2 to m+1/2 centered on m. The function has the units of 'stars/deg^2/magnitude'. If $A(m)= 3m^{+3.5}$, how many stars would be counted, on average, in: A) a field that is the size of the full moon, (area = 0.19 deg^2) at a magnitude of m=+12.0? B) the same field and with a magnitude bin +2.3 in size?

Problem 3 - Near the wavelength of 3.4 microns being explored by WISE, an astronomer estimated from the Spitzer Infrared Observatory 'First Light' survey that at this wavelength, $A(m) = 2.4 \times 10^{-6}\ m^{+7.4}$ stars/magnitude/deg^2. Suppose that the faintest star detectable by WISE has a magnitude of m = +15. Using the method of integration, how many total number of stars would WISE be able to detect in a field equal to the WISE survey area of 0.64 deg^2, and that are fainter than V482 Carina?

The Ares-V Cargo Rocket



The Ares-V rocket, now being developed by NASA, will weigh 3,700 tons at lift-off, and be able to ferry 75 tons of supplies, equipment and up to 4 astronauts to the moon. As a multi-purpose launch vehicle, it will also be able to launch complex, and very heavy, scientific payloads to Mars and beyond. To do this, the rockets on the Core Stage and Solid Rocket Boosters (SRBs) deliver a combined thrust of 47 million Newtons (11 million pounds). For the rocket, let's define:

T(t) = thrust at time-t
m(t) = mass at time-t
a(t) = acceleration at time-t

so that:

$$a(t) = \frac{T(t)}{m(t)}$$

The launch takes 200 seconds. Suppose that over the time interval [0,200], T(t) and m(t) are approximately given as follows:

$$T(x) = 8x^3 - 16x^2 - x^4 + 47$$

$$m(x) = 35 - x^2$$

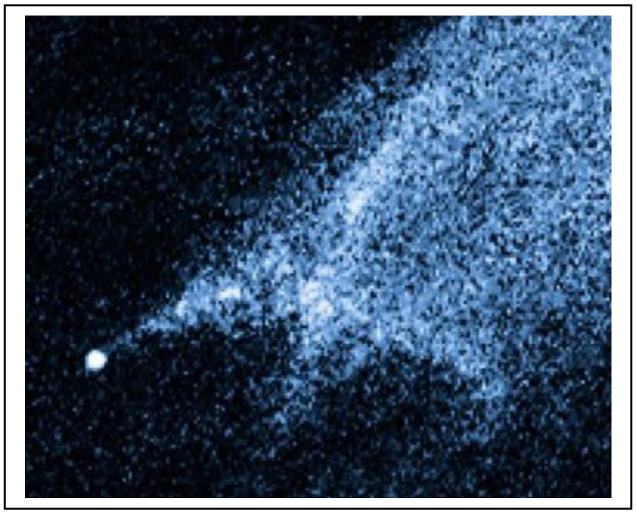
where t = 40 x

Problem 1 - Graph the thrust curve T(x), and the mass curve m(x) and find all minima, maxima inflection points in the interval [0,5]. You may use a graphing calculator, or Excel spreadsheet, or differential calculus.

Problem 2 - Graph the acceleration curve a(x) and find all maxima, minima, inflection points in the interval [0,5]. You may use a graphing calculator, or Excel spreadsheet, or differential calculus.

Problem 3 - For what value of x will the acceleration of the rocket be at its absolute maximum in the interval [0,5]? How many seconds will this be after launch? You may use a graphing calculator, or Excel spreadsheet, or differential calculus.

Hubble Spies Colliding Asteroids!



It doesn't look like much, but this picture taken by the Hubble Space Telescope on January 25, 2010 shows all that remains of two asteroids that collided! The object, called P/2010 A2, was discovered in the asteroid belt 290 million kilometers from the sun, by the Lincoln Near-Earth Asteroid Research sky survey on January 6, 2010.

Hubble shows the main nucleus of P/2010 A2, about 150 meters in diameter, lies outside its own halo of dust. This led scientists to the interpretation that it is the result of a collision.

How often do asteroids collide in the asteroid belt? The collision time can be estimated by using the formula:

$$T = \frac{1}{NAV}$$

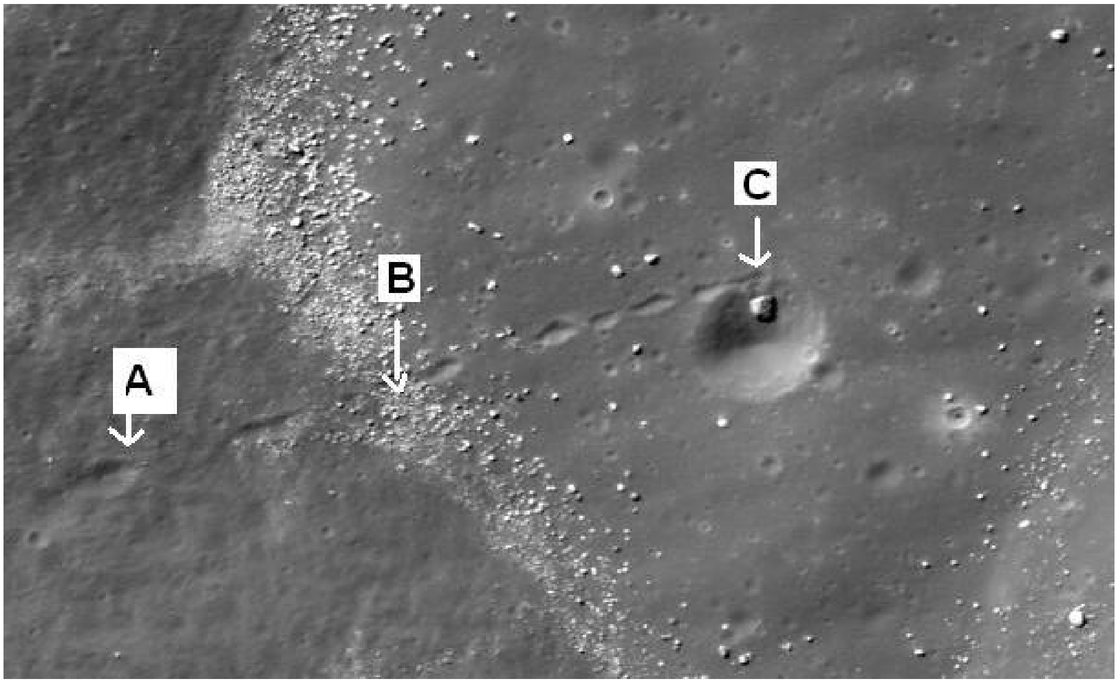
Where N is the number of bodies per cubic kilometer, v is the speed of the bodies relative to each other in kilometers/sec and A is the cross-sectional area of the body in square-kilometers. The answer, T, will be in the average number of seconds between collisions.

Estimating A: Assume that the bodies are spherical and 100 meters in diameter .What will be A, the area of a cross-section through the body?

Estimating the asteroid speed V: At the orbit of the asteroids, they travel once around the sun in about 3 years. What is the average speed of the asteroid in kilometers/sec at a distance of 290 million kilometers?

Estimating the density of asteroids N: - This quantity is the number of asteroids in the asteroid belt, divided by the volume of the belt in cubic kilometers. A) Assume that the asteroid belt is a thin disk 1 million kilometers thick, with an inner radius of 1.6 AU and an outer radius of 2.5 AU. If 1 AU = 150 million kilometers, want is the volume? B) Based on telescopic observations, an estimate for the number of asteroids in the belt that are larger than 100 meters across is about 30 billion. From this information, and your volume estimate, what is the average density of asteroids in the asteroid belt?

Estimating the collision time T: From the formula, A) what do your estimates for N, V and A imply for the average time between collisions in years? B) What are the uncertainties in your estimate?



This image (LROC MAC M122597190L), taken by the Lunar Reconnaissance Orbiter shows a boulder that has rolled and skipped down hill from the left-hand edge of the image to a 'hole-in-one' location in a small crater. The width of the image is 510 meters. To two significant figure accuracy in your answers:

Problem 1 – Mark those portions of the path where the boulder must have A) rolled and B) skipped, in order to cover the distance.

Problem 2 – What is the scale of this image in meters/km?

Problem 3 – Assuming that the boulder is roughly a sphere in shape with a density of D=3000 kg/m^3, what is A) The diameter of the boulder? B) The mass of the boulder in tons?

Problem 4 – How far did the boulder skip and roll from A) Point A to B? B) From Point B to C?

Problem 5 – Suppose that the crater wall slope from Point A to B is 60 degrees to the horizontal, and the settled regolith from B to C has a 'repose' slope of 30 degrees to the horizontal. What is the total distance traveled A) Horizontally? B) Vertically?

Problem 6 – If the average vertical speed from Point A to B is 7 m/s and from Point B to C is 13 m/s, how long did the boulder take to travel from Point A to Point C?

Useful Internet Resources

<u>Space Math @ NASA</u>

http://spacemath.gsfc.nasa.gov

<u>Practical Uses of Math and Science (PUMAS)</u>

http://pumas.gsfc.nasa.gov

<u>Teach Space Science</u>

http://www.teachspacescience.org

<u>Space Weather Action Center</u>

http://sunearthday.nasa.gov/swac

<u>THEMIS Classroom guides on Magnetism</u>

http://ds9.ssl.berkeley.edu/themis/classroom.html

<u>The Stanford Solar Center</u>

http://solar-center.stanford.edu/solar-math/

<u>A Math Refresher</u>

http://istp.gsfc.nasa.gov/stargaze/Smath.htm

A note from the Author:

June, 2010

Hi again!

Here is another collection of 'fun' problems based on NASA space missions across the solar system and the universe. Thanks to the relentless march of current events, the problems range from the spectacular launch of the Ares 1X in the Fall of 2009, to the devastating crisis of the BP oil leak in the Spring of 2010.

Mathematics underlies many of the dramatic stories from the Space Age, but unless you know how to look beyond the verbiage of press releases, it is hard to see the connections between scientific ideas and discoveries, and the mathematics that give them life and certainty. My goal, with Space Math @ NASA is to try to show how exciting ideas in science connect with mathematics. As an astronomer, it is second-nature to think of celestial events and objects in mathematical terms, revealing their inner-beauty. I hope that by working through many of the problems in this book you and your students will better appreciate the many subtle interconnections that underlie the physical world.

This has been a very complex year for NASA as it launches its last Shuttle missions in 2010. With only one more launch to go in November, I am also a bit worried about what lies in store for us in the future. Our astronauts will have to travel to launch facilities in Russia to be ferried up to the International Space Station for the next few years. It may seem like an international embarrassment to us, but in fact it demonstrates that we are now willing to join the international community in a shared vision for space exploration among manned missions. The successful launch of the SpaceX Falcon-9 rocket in June also underscores the fact that our commercial space systems are now up to speed in taking the next step into the manned space arena.

President Obama made an historic decision in his vision for NASA. The new Vision that was offered will open up NASA to using domestic, commercial launch systems to ferry astronauts to the ISS, and prolong the ISS operation through 2020. In the vision offered by the previous administration, which was never funded to succeed, ISS was scheduled for de-orbit in ca 2016 only a few years after achieving full operation as a science research facility following 1 years of construction.

The historic launch of Falcon-9 will be followed in the next few years by the return of our astronauts to the ISS using our own launch vehicles again, but this time at nearly 10-times lower cost that the Space Shuttle. The commercialization of launch systems in the USA will also mean that many non-astronaut sightseers will now have access to space, with thousands of people already having paid for the first tickets!

The next 10 years will be a very exciting period for space travel!

Sincerely,
Dr. Sten Odenwald
NASA Astronomer
sten.f.odenwald@nasa.gov

National Aeronautics and Space Administration

Space Math @ NASA
Goddard Spaceflight Center
Greenbelt, Maryland 20771
spacemath.gsfc.nasa.gov

www.nasa.gov