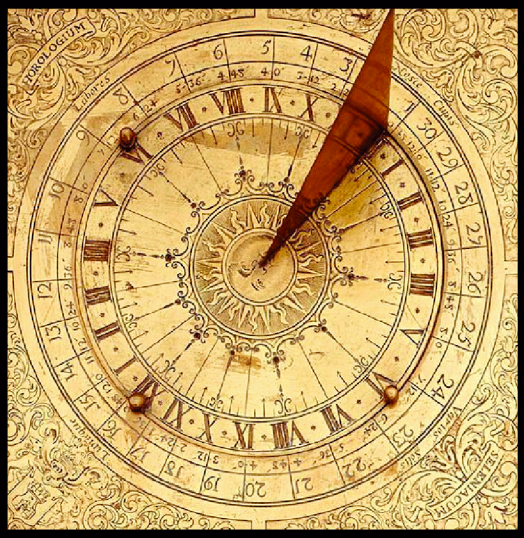
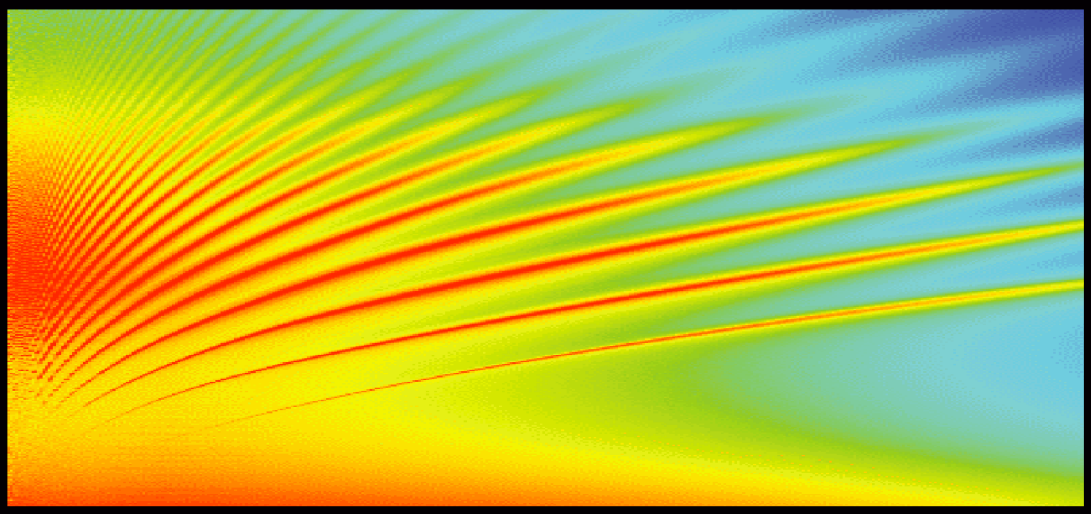
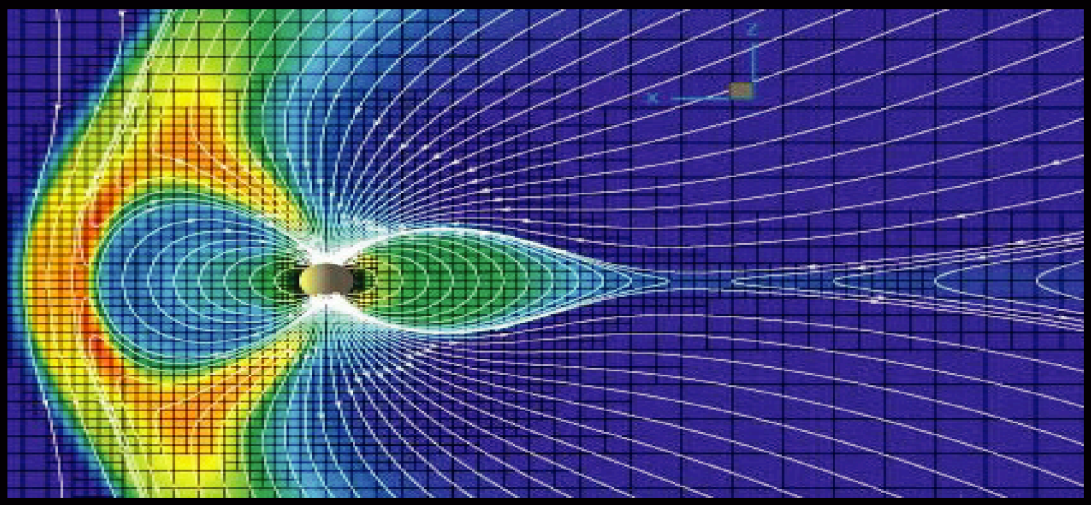
National Aeronautics and Space Administration







Space Math - I

This collection of activities is based on a weekly series of space science problems distributed to thousands of teachers during 2004-2005 school year. They were intended as extra-credit problems for students looking for additional challenges in the math and physical science curriculum in grades 7 through 9. The problems were designed to be authentic glimpses of modern engineering issues that come up in designing satellites to work in space, and to provide insight into the basic phenomena of the Sun-Earth system, specifically 'Space Weather'. The problems were designed to be 'one-pagers' with the student work sheet (with the top line for the student's name) and a Teacher's Guide and Answer Key as a second page. This compact form was deemed very popular by participating teachers.

This booklet was created by the NASA, IMAGE satellite program's Education and Public Outreach Project (POETRY).

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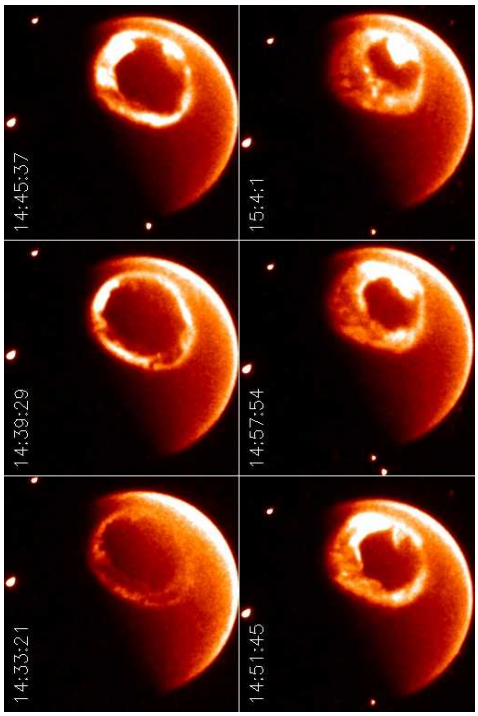

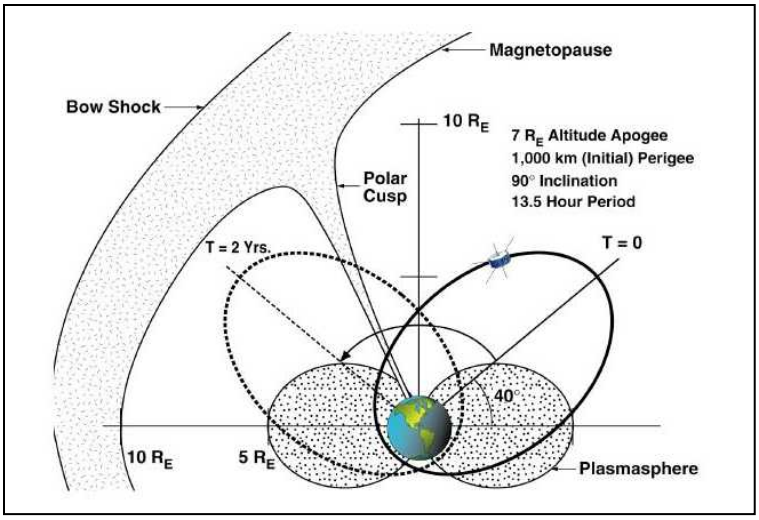

A series of images (left) of the Northern Lights from space taken by the IMAGE satellite. The satellite orbits Earth in an elliptical path (above), which takes it into many different regions of Earth's environment in space.

For more weekly classroom activities about the Sun-Earth system visit the IMAGE website,
http://image.gsfc.nasa.gov/poetry/weekly/weekly.html
Add your email address to our mailing list by contacting Dr. Sten Odenwald at
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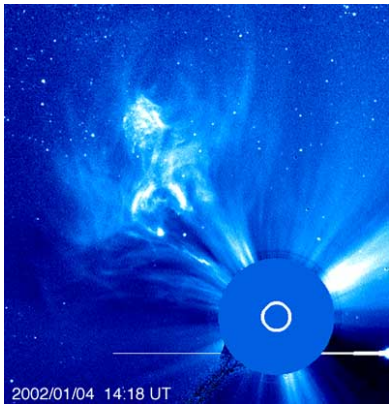
Background

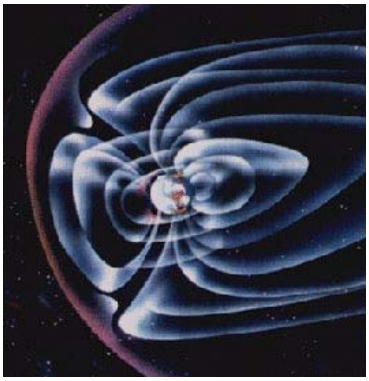
We live next to a very stormy star, the Sun, but you would hardly notice anything unusual most of the time. Its constant sunshine hides spectacular changes. Only the dazzling glow of the Northern Lights suggests that invisible forces are clashing in space. These forces may cause all kinds of problems for us, and our expensive technology. It doesn't take long for 'solar storms' to get here, either. X-rays from flares make the trip to Earth in just under 9 minutes, while the fastest moving plasma can make the journey in only 17 hours. Solar flares disrupt Earth's ionosphere and can cause satellite malfunctions, while the plasma clouds can change Earth's magnetic field. This leads to the displays of the aurora that humans have marveled at for thousands of years. Aurora light up the sky with billions of watts of power and cover millions of square kilometers. Why does all this happen? (*Photo- Auroral curtain by Jan Curtis)*

Space Weather



It has to do with Earth's magnetic field and how it is disturbed by solar storms and the solar wind. The wind carries its own magnetic field with it, and travels at speeds of millions of kilometers per hour. Scientists keep track of this interplanetary storminess using numbers that follow its ups and downs just like meteorologists follow a storm's speed, pressure and humidity. Periods of increased and decreased solar activity come and go every 11 years. Solar flares also have their own story to tell just like flashes of lightning in a bad storm. (*Photo - Coronal Mass Ejection seen by SOHO satellite)*

Models and Forecasting



Scientists have to keep track of many different kinds of phenomena in the universe, both big and small. That's why they have invented a way to write very big and very small numbers using 'scientific notation. They also have to master how to think in three-dimensions and how to use mathematical models. Once they find the right models, they can use them to make better predictions of when the next solar storm will arrive here at Earth, and what it will do when it gets here! *(Sketch of Earth's magnetic field)*

The following table connects the activities in this booklet to topics commonly covered in Grade 7, 8 and 9 pre-algebra and algebra textbooks. The specific national math and science education standards (NSF 'Project 2061') targeted by this product are:

Grade 6-8

Most of what goes on in the universe involves some form of energy being transformed into another form
Graphs can show a variety of possible relationships between two variables
A system can include processes as well as things
Locate information in reference books, and computer data bases
Understand writing that incorporates circle charts, bar graphs line graphs, tables, diagrams and symbols

Grade 9-12

Find answers to problems by substituting numerical values in simple algebraic formulas.
Use tables, charts and graphs in making arguments and claims in oral and written presentations.
Distances and angles inconvenient to measure directly can be found by using scale drawings.

Topic	Problem Number in the Book																			
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Reading to be Informed	X	X	X				X	X			X		X	X				X	X	X
Scale drawings													X							
Triangle Properties											X									
Speed Distance, time												X			X					
Solving for X										X										
Equations and substitution																X	X	X	X	X
Calculating Areas of polygons							X													
Positive & negative numbers									X											
Decimal math.							X													
Time calculations	X	X												X	X					
Percentages				X										X						
Venn Diagrams				X																
Pie Charts					X															
Graph Analysis						X		X				X		X						
Open Inquiry						X														
Probability				X																

On August 28, 1859 a massive solar storm caused spectacular aurora seen all over the globe. It was reported in all the major newspapers, poems were written about it, and famous artists painted its shapes and forms. It also caused severe problems with telegraph networks at the time, which lasted for many hours world-wide. Although scientists gave detailed reports of the changing forms of this vivid display, many ordinary citizens offered their own impressions of this event too. Below are two of these descriptions seen from two different locations.

Galveston, Texas:

August 28 as early as twilight closed, the northern sky was reddish, and at times lighter than other portions of the heavens. At 7:30 PM a few streamers showed themselves. Soon the whole sky from Ursa Major to the zodiac in the east was occupied by the streams or spiral columns that rose from the horizon. Spread over the same extent was an exquisite roseate tint which faded and returned. Stately columns of light reaching up about 45 degrees above the horizon moved westward. There were frequent flashes of lightning along the whole extent of the aurora. At 9:00 PM the whole of the streaking had faded leaving only a sort of twilight over the northern sky."

London, England.

"At 0:15 AM on August 28th the auroral light in the north assumed the form of a luminous arch, similar to daybreak, and in the southwest there was an intense glare of red covering a very large extent of the sky. At 00:20 AM streamers appeared; at 00:25 AM the streamers rose to the zenith and were tinged with crimson at their summits. At 00:45 AM frequent coruscations appeared in the aurora. At 01:20 AM the arch which had partially faded began to reform and the body of the light was very strong but not bright enough to read newspaper print. At 1:30 AM the light had begun to fade. By 2:00 AM the aurora was very indistinct."

A common problem scientists face when organizing observations from different laces around the world is that observers like to note when things happened by their local time. Scientists simplify these accounts by converting them into Universal Time., which is the local time in Greenwich, England also called Greenwich Mean Time (GMT). To make time calculations easier, UT is expressed in the 24-hour clock format so that 11:00 AM is written as 11:00, but times after noon are written, for example, as 1:00 PM is written as 13:00, and 10:00 PM is written as 22:00. Since London is very close to Greenwich, the times mentioned in the London account above are already in Universal Time and only need to be converted to the 24-hour format. For Galveston, Texas, its time is 5 hours behind UT so that to get the equivalent UT for Galveston, first convert the Galveston times to the 24-hour format, then add 5 hours.

Question 1 - From these two descriptions, can you extract the specific points of each narrative. What are their similarities?

Question 2 - From the sequences of events in each description, can you create a timeline for the aurora display that fits the most details?

Question 3 - Why was the aurora observed to reach closer to zenith in London than in Galveston?

The aurora (see back page of this book) is a spectacular, visual experience for every human that has watched it. Although scientists want to study it to learn about its secrets, other observers prefer to simply experience the phenomenon for its beauty and awesome mystery. Here is a vivid description of an aurora observed on August 28, 1859 by Captain Stanard from Cleveland, Ohio.

"At 9:00 PM, a belt began to rise up in the north, and as the convex edge attained a height of about 40 degrees above the horizon, it began to shoot out long, attenuated bright rays close together, moving slowly to the west and reaching to the zenith. Near the convex edge they were of a bright yellow, changing as they shot up to orange, and near the zenith to a bright red, the middle and lower ends remaining yellow and orange. As the fiery points of the rays shot into the broad belt overhead, which had still remained like a belt of luminous fog, the whole thing was changed in an instant into bright red color. The color deepening as it neared the eastern horizon, to a bright crimson, and at the western end near the star Arcturus, into a bright scarlet, gradually growing fainter in the zenith, and increasing in brightness nearer the horizon. After 15 minutes it resolved itself into converging rays that came from the zenith."

" At 9:45 PM, A double arch formed from two narrow ribbons of light 15 degrees wide running from Canes Venatici to the southern edge of Perseus. The bright star Capella shining through the narrow black space between them. Ten minutes later, bright rays suddenly shot up in quick successive flashes from the lower through the upper arch, reaching nearly to the zenith, and moving slowly to the west until they reached the constellation Corona Borealis, lighting up the northwestern sky with yellow, orange and red. There commenced a sudden flashing of horizontal wavy bands from the upper arch towards the zenith."

From the information in this description, try to recreate what Mr. Stanard saw for one particular moment as he watched the aurora dance across the sky. Use colored pencils, crayons, watercolors or other media to render a view of the aurora based on this description.

In 1999, Dr. Sten Odenwald wrote a book called 'The 23rd Cycle: Learning to live with a stormy star" that described all of the ways that severe space weather events, called solar storms, can affect our satellite technology, our electrical power, and even the health of astronauts and airline passengers and crew.

Read the excerpt from his book, and answer the following questions based on the information in the excerpt.

Question 1: What topic is this part of the book describing?

Question 2: Describe how a solar storm is involved with this topic?

Question 3: What does the article identify as a possible risk for passengers?

Here is a short list of your radiation exposure at ground level each year, in terms of a unit of radiation dosage called the milliRem.

Radon gas in your basement ……160 milliRems per year
The ground under your feet…… 60
Nuclear reprocessing plants…… 40
Cosmic rays at sea level…… 38
Cosmic rays from high altitude city (Denver)….130
Medical imaging…… 30
Food and water…….. 25

Question 4: If you add up the different exposures in the list above, what is your total dosage each year if you were living at sea level? Living at high altitudes in Denver?

Question 5: According to the information in the article, how much extra radiation exposure would a passenger receive in a 10-hour flight at 35,000 feet?

Question 6: How much extra radiation will an airline crew member receive during 900 hours per year of flying?

Question 7: Do you think the radiation risk from solar storms is a significant one? Present the evidence that demonstrates the size of the health risk, and the evidence that suggests that the health risk is minimal.

Airline Travel and Solar Storms

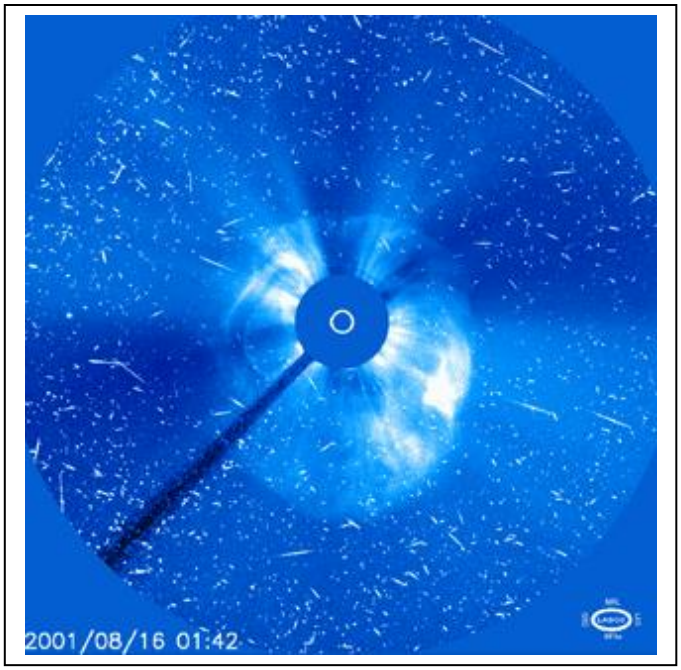
Jet airliners fly at altitudes above 35,000 feet which is certainly not enough to get them into space, but it is more than enough to subject the pilots and stewardesses to some respectable doses when looked at over the course of their careers, and thousands of flights. A trip on a jet plane is often taken in a party-like atmosphere with passengers confident that, barring any unexpected accidents and food problems, they will return to Earth safely and with no lasting physical affects. But depending on what the Sun is doing, a solar storm can produce enough radiation to equal a significant fraction of a chest X-ray's dosage even at typical passenger altitudes of 35,000 feet. Airline pilots and flight attendants can spend over 900 hours in the air every year, which makes them a very big target for cosmic rays and anything else our Sun feels like adding to this mix. According to a report by the Department of Transportation, the highest dosages occur on international flights passing close to the poles where the Earth's magnetic field concentrates the particles responsible for the dosages.

Although the dosage you receive on a single such flight per year is very small, about one milliRem per hour, frequent fliers that amass over 100,000 miles per year would accumulate nearly 500 millirems each year. Airline crews who spend 900 hours in the air would absorb even higher doses, especially on polar routes. For this population, their lifetime cancer rate would be 23 cancers per 100 people. By comparison, the typical cancer rate for ground dwellers is about 22 cancers per 100. But the impact does not end with the airline crew. The federally recommended limit for pregnant women is 500 millirems per year. Even at these levels, about four extra cases of mental retardation would appear on average per 100,000 women stewardesses if they are exposed between weeks 8 to 15 in the gestation cycle. This is a time when few women realize they are pregnant, and when critical stages in neural system formation are taking place in the fetus.

Matthew H. Finucane, air safety and health director of the Association of Flight Attendants in Washington DC, has claimed that these exposure rates are alarming, and demands that the FAA to do something about it. One solution is to monitor the cabin radiation exposure and establish OSHA guidelines for it. If possible, he also wants to set up a system to warn crews of unusually intense bursts of cosmic radiation, or solar storm activity during a flight. Meanwhile, the European Aviation Agency is contemplating going even further. They want to issue standard dosimetry badges to all airline personnel so that their annual exposures can be rigorously monitored. This is a very provocative step to take, because it could have a rather chilling effect on airline passengers. It might also raise questions at the ticket counter that have never been dealt with before*, 'Excuse me, can you give me a flight from Miami to Stockholm that will give me less than one chest X-ray extra dosage?'* How will the traveler process this new information, given our general nervousness over simple diagnostic X-rays?

Consider this: during September 29, 1989, for example, a powerful X-ray flare caused passengers on high-flying Concord airliners to receive dosages equal to two chest X-rays per hour. At the end of the flight, each passenger had silently received hundreds of additional millirems added to their regular background doses. Still, these occasional dosages the average person receives while flying, compared to the dosages we might accumulate once we land at another geographic location, are rather inconsequential over a lifetime. Compared to the quality of life that we gain in exchange for the minor radiation exposure we risk, most people will grudgingly admit the transaction is a bargain. Statisticians who work with insurance companies often think in terms of the number of days lost to your life expectancy from a variety of causes. On this scale, smoking 20 cigarettes a day costs you 2200 days; being overweight by 15% costs you 730 days; and an additional 300 millirem per year over the natural background dose (about 250 milliRem) reduces your life expectancy by 15 days. "

[Dr. Sten Odenwald, 'The 23rd Cycle' Columbia University Press, 2000]

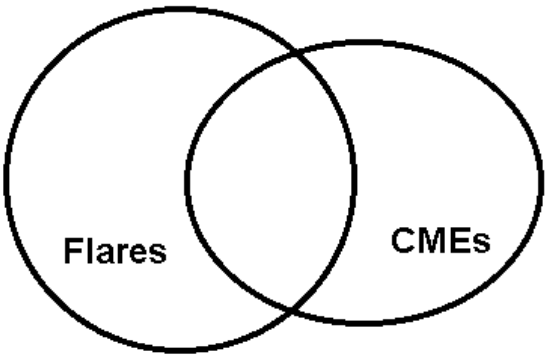


Solar flares are violent releases of energy from the sun that last 10 to 20 minutes and produce intense flashes of x-rays, which travel at the speed of light to Earth. Coronal mass ejections (CMEs) are enormous releases of matter (plasma) from the sun that travel at nearly a million miles per hour to Earth. When a CME is directed towards Earth it is called a **halo CME** because images from the SOHO satellite like the one to the left, look as though the sun is surrounded by a halo of glowing gas. We know that halo CMEs cause the Northern Lights because of the way that they affect Earth's magnetic field. The question we want to answer is, 'Do flares cause CMEs to happen, or vice versa?

A scientist named Dr. C. A. Flair is trying to decide if there is a relationship between CMEs and flares by studying how many of these events occur, and how often CMEs and flares coincide. He has created the following list:

Solar Flares	**22**
Halo CME	**12**
Both Flares and Halo CMEs	**7**

The scientist decides to analyze the results. His first step is to construct a Venn Diagram to display the data. Place the data in the correct locations.



Question 1 - What is the total number of individual events involved in this sample?

Question 2 - What does the overlapping part of the diagram represent?

Question 3 - Based on this data, what is the probability of a flare occurring?

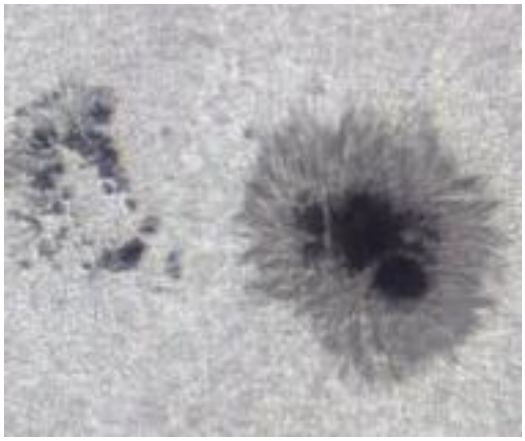
Question 4 - What is the probability of a CME Halo occurring?

Question 5 - What fraction of the time do flares and a CMEs occur at the same time?

Question 6 - In your own words, what would be your answer to the question?

Sunspots are some of the most interesting, and longest studied, phenomena on the sun's surface. The table below shows the area of a sunspot. In comparison, the surface area of Earth is '169' units on the sunspot scale. The table also shows the brightest flare seen from the vicinity of the sunspot s beginning November 6, 2004. Flares are ranked by their brightness 'C', 'M' and 'X' with M-class flares being 10x more luminous that C-class flares, and X-class flares being 10x brighter than M-class flares.

Date	Spot #	Area	Flare
Nov 6	#696	820	M
Nov 7	#696	910	M
Nov 8	#696	650	X
Nov 10	#696	730	M
Nov 11	#696	470	X
Dec 2	#708	130	M
Dec 3	#708	150	M
Dec 9	#709	20	C
Dec 29	#713	150	M
Dec 30	#715	260	M
Dec 31	#715	350	M
Jan 1	#715	220	M
Jan 2	#715	180	X
Jan 4	#715	130	C
Jan 10	#719	100	M
Jan 14	#718	160	C
Jan 15	#720	1540	M
Jan 16	#720	1620	X
Jan 17	#720	1630	M
Jan 18	#720	1460	X
Jan 19	#720	1400	M



During the 75 day time period covered by this table, there were a total of (720-696=) 24 catalogued sunspots. The table shows only those catalogued sunspots that were active in producing flares during this time. Sunspot areas are in terms of millionths of the solar hemisphere area, so '1630' means 0.163% of the Sun's face. Earth's area = 169 millionths by comparison!

Sunspot and solar flare data from NOAA SWN data archive at http://www.sec.noaa.gov/Data/index.html

Question 1: Construct a pie chart for the X, M and C-class flare data. During this 75-day period, what percentage of flares are X-class?

Question 2: What percentage of sunspots produce X-class flares?

Question 3: What percentage of sunspots did not produce any flares during this time?

Question 4: What seems to be the minimum size for a sunspot that produces an X-class flare? An M-class flare? A C-class flare?

Question 5: If the area of Earth is '169' in the sunspot units used in the above tables, what are the maximum and minimum size of the sunspots compared to the area of Earth?

Satellite technology is everywhere! Right now, there are over 1587 working satellites orbiting Earth. They represent over $160 billion in assets to the world's economy. In the United States alone, satellites and the many services they provide produce over $225 billion every year. But satellites do not work forever. Typically they have to be replaced every 10 to 15 years as new services are created, and better technology is developed. Satellites in the lowest orbits, called Low Earth Orbit (LEO) orbit between 300 to 1000 kilometers above the ground. Because Earth's atmosphere extends hundreds of kilometers into space, LEO satellites eventually experience enough frictional drag from the atmosphere that at altitudes below 300 km, they fall back to Earth and burn up. The table below gives the number of LEO satellites that re-entered Earth's atmosphere, and the average sunspot number, for each year since 1969.

Year	Sunspots	Satellites
2004	43	19
2003	66	31
2002	109	38
2001	123	41
2000	124	37
1999	96	25
1998	62	30
1997	20	21
1996	8	22
1995	18	20
1994	31	17
1993	54	28
1992	93	41
1991	144	40
1990	145	30
1989	162	45
1988	101	33
1987	29	13
1986	11	16
1985	16	17
1984	43	14
1983	65	28
1982	115	19
1981	146	32
1980	149	41
1979	145	42
1978	87	33
1977	26	18
1976	12	16
1975	14	15
1974	32	21
1973	37	14
1972	67	12
1971	66	19
1970	107	25
1969	105	26

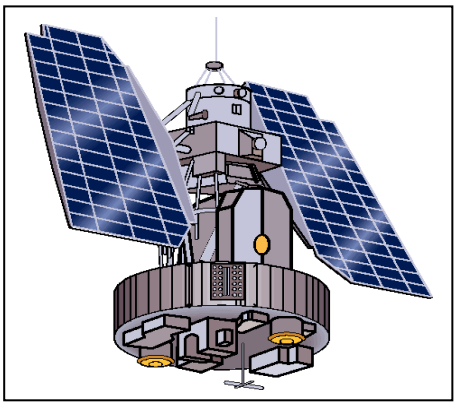


Figure: A typical weather satellite

Question 1: On the same graph, plot the number of sunspots and decayed satellites (vertical axis) for each year (horizontal axis). During what years did the peaks in the sunspots occur?

Question 2: When did the peaks in the satellite re-entries occur?

Question 3: Is there a correlation between the two sets of data?

Question 4: If you are a satellite operator, should you be concerned about the sunspot cycle?

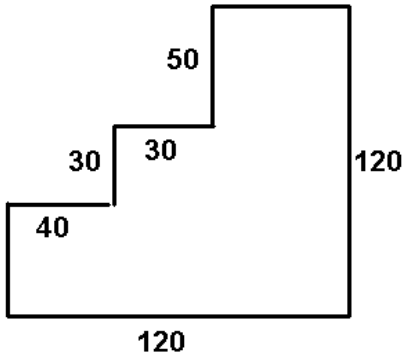
Question 5: Do some research on the topic of how the sun affects Earth. Can you come up with at least two ways that the sun could affect a satellite's orbit?

Question 6: Can you list some different ways that you rely on satellites, or satellite technology, during a typical week?

The outside surface of the NASA, IMAGE satellite is covered with solar cells (black) to collect sunlight and generate electricity for its many instruments and systems.

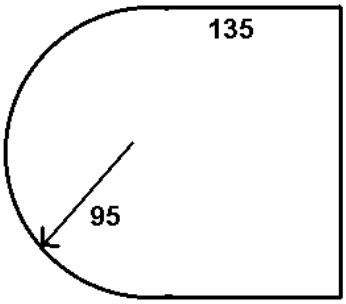
Solar Electricity.

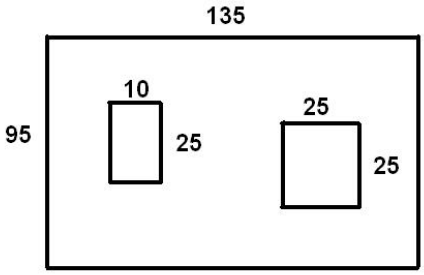
Most satellites depend on sunlight converted into electricity and stored in batteries, to run their instruments. Typical research satellites operate with only 200 to 800 watts of electricity generated by sunlight and 'solar cells'. Solar cells can be attached directly to the outer surface of a satellite, or can be found on 'solar panels' that the satellite deploys after it reaches its orbit. Engineers decide how many solar cells are needed by finding out how much power the satellite needs to operate, and then figuring out what the satellite's surface area is. If the satellite is not big enough, additional solar panels may be needed to supply the electricity. In this exercise, you will calculate the perimeter of the satellite area in centimeters, and the area, in square centimeters, of several kinds of satellite surfaces, and decide if there is enough available surface to supply the electrical needs of the satellite. All measurements in the diagrams are in centimeters. Some dimensions are missing.



Question 1 – What is the perimeter? The area? The solar cells can provide 0.03 watts per square centimeter, and the satellite needs 257 watts. Is there enough surface area in the figure to the left to meet the electrical needs?

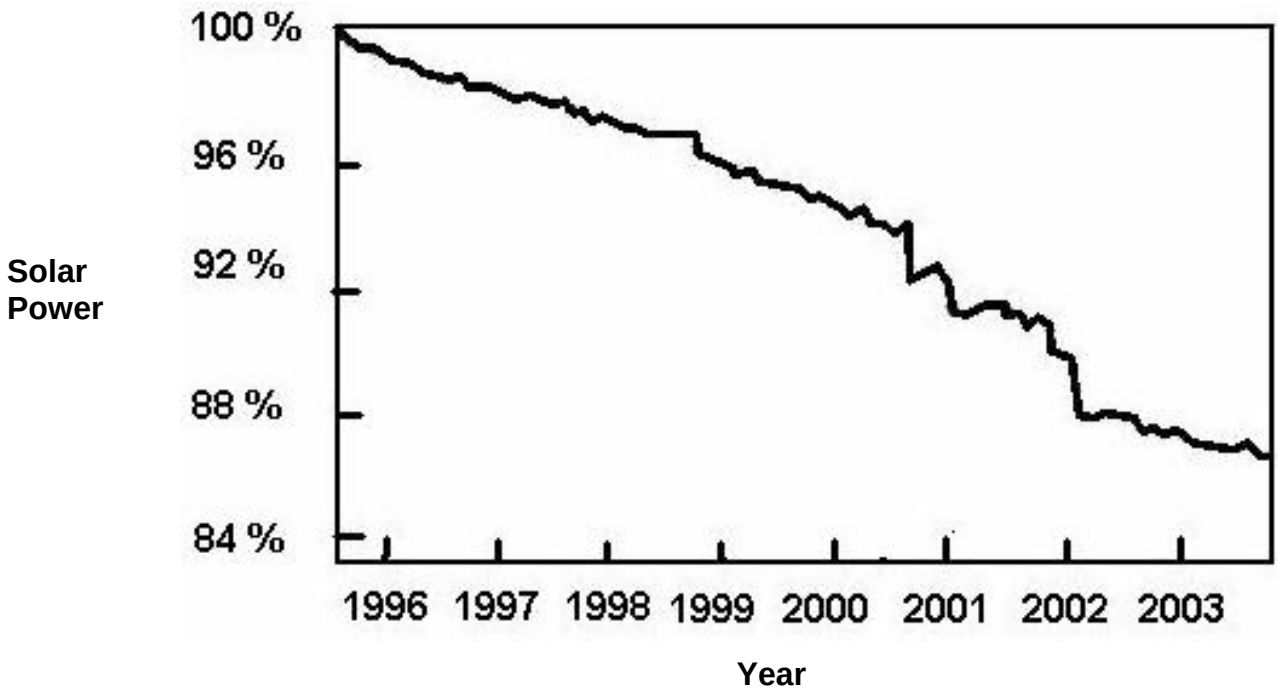
Question 2 - What is the perimeter? The area? The solar cells can provide 0.03 watts per square centimeter, and the satellite needs 957 watts. Is there enough surface area in the figure to the right to meet the electrical needs?





Question 3 - What is the outside perimeter? The area with the two holes removed? The solar cells can provide 0.03 watts per square centimeter, and the satellite needs 759 watts. Is there enough surface area in the figure to the left to meet the electrical needs?

Soon after the first satellites were placed into orbit, engineers began to notice that the electrial power needed to operate the satellites was slowly declining. Very careful studies of the way that solar cells worked in space soon came up with a culprit: Cosmic rays! Cosmic rays are very high-speed ions, protons and electrons that travel through space. Some are produced by the sun during solar flares, while others come from space beyond the solar systems itself. Cosmic rays are deflected by strong magnetic fields. Fortunately, Earth has a very strong magnetic field that shields us from most of these cosmic rays, but enough get through that they collide with solar panels and solar cells on satellites orbiting Earth. Over time, these fast-moving particles cause changes in solar cells, making them less able to generate the electricity they are supposed to when sunlight falls on them. Thanks to decades of study, engineers can make very accurate models of these cosmic rays effects and incorporate them into the design of satellite power systems. Below is an actual graph prepared by Dr. Paul Brekke, the Project Scientist for the Solar Heliospheric Observatory. It shows how fast the satellite's solar arrays have changed their power output since 1995.



Question 1: By how much has the SOHO satellite power declined by 2002?

Question 2: Satellites often remain usable for 15 years. By what percentage will the electricity from the SOHO solar panels have declined by that time?

Question 3: The instruments on a satellite require 850 watts of power to operate. If a scientist wants her instrument to continue working for 15 years, how large should the satellite power supply be at the time of launch?

Question 4: The surface of a satellite has an area of 1000 square centimeters. The solar cells can generate about 0.03 watts per square centimeter. A) What is the total power available to the satellite by covering its surface with solar cells? B) If the satellite is to last 15 years, what is the maximum power that the instruments can draw and still work after 15 years?

Scientists have to use mathematics to analyze data, and to build models of the phenomena they are studying. Can you solve this crossword puzzle using the mathematical clues provided? When you're finished, color the blank squares with your favorite aurora colors!

													L						
										d					m				
	z			J			o												
										e									
	v														s				
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							h			r									
	b											k				T			
c	y						p												
							w												

Extra Credit Problem

Word Bank:

+4	proton	- 14	RPI	+6	ionosphere	-1	plasma
-16	FUV	11	photons	+27	spectrum	-27	beams
+3	radiation	0	flares	-10	field	-7	magnetism
-6	earth	-8	solar	-13	envelope	-2	current
-3	aurora	- 23	cone	+2	corona	10	humanity
+1	rays	13	oval	-11	Plasmasphere	-26	echos
-9	polarity	+23	STP	-17	CMEs	+26	UT
+19	TOD	9	ions	-19	sunspot	+25	ENA
-21	FAC	-5	IMAGE	-4	neutron	+8	electrons
+14	watt	17	UV	+7	crown	+5	magnetosphere

The numerical answer to each problem below will tell you which word to use in the word bank to fill in the row or column with the indicated letter.

Down:

l) - 2 (- 5 + 3)

m) - 4 (3 – 1)

e) + 2 (-6 + 3) + 4

n) - 3 (- 5 + 2) + (- 4 + 3)

g) + 5 – (- 8 + 2)

o) - 4 (7 – 3) + (8 – 3)

p) + 1 – 1 + 3 + 4 – 5 – 8

r) - 3 (- 12 + 9)

T) + 2 – 3 + 5 – 10 + 2

u) + 4 (- 5 + 3) – (- 7 + 6)

v) + 3 (- 8 + 6) - 3

x) + 4 (8 – 3) – 3 (+ 6 – 8 + 12)

y) + 2 (+ 8 – 5 + 10) – (+ 10 + 5 – 2)

J) - 2 (- 3 + 2 – 5 – 6) + 2 – 2 (+ 26)

Across:

a) + 2 (- 3 + 8) – 5

b) + 3 (+ 2 + 1) – 2 (- 3 + 9)

c) - 15 (- 2 + 3) + 3 (- 2 + 9)

d) - 1 + 1 – 1 + 1 – 1 + 1

e) - 6 + (+ 11 – 3)

f) - 3 (- 8 + 11) + 4

g) (+ 1 – 1) + (+ 1 – 2)

h) + 4 (+ 5 – 2) – 3 (+ 3)

k) + 5 (+ 8 – 5) – 4 (- 6 + 8)

s) - 1 + 1 – 1 + 1 + 1 – 1 + 1

w) + 3 (- 2 + 8) + (- 12 + 4)

z) - 5 + 2 (- 8 + 4)

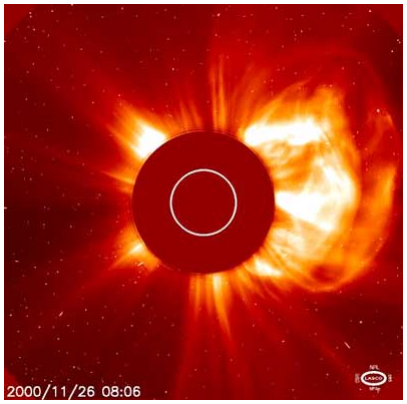
Name _____

A Bird's-Eye Look at the Sun-Earth System

Solar flares are powerful releases of energy in the tangled A_**magnetic fields** of the sun, thousands of kilometers above certain sunspots. Within a few minutes, these B_____ attempt to reconnect themselves into simpler shapes, releasing their stored energy. This energy heats the local C_____ to millions of degrees, producing tremendous blasts of x-ray and gamma ray energy. Within 8.5 minutes, this D_____ energy can reach Earth and disrupt the ionosphere. The E_____ is the region of our atmosphere where radio waves can be reflected back to the ground so that we can send radio programs and messages around the world. But during F_____, radio broadcasting can be shut down for hours.

Scientists eventually discovered that, in addition to flares, discharges of plasma from the sun could also occur. With satellite sensors in space, they eventually caught sight of how the sun could from time to time release billions of tons of plasma traveling at millions of miles per hour. When these magnetized plasma clouds wash across Earth's magnetic field – called the G_____, these clouds can dump huge amounts of matter and energy into Earth's environment. H_____ and magnetic storms are often the consequence of these I_____.

The entire Earth-Sun system follows a complicated give-and-take when severe solar J_____ occur. Earth counteracts this onslaught of plasma and energy by a complex series of adjustments only available to it because it has a powerful magnetic field. With the magnetic field, most of the solar storm energy is diverted. What little enters the magnetosphere eventually finds its way into circulating K_____ which return some, but not all of the magnetospheric plasma back to the solar wind – a constant stream of matter that leaves the solar surface. Solar storms cause Earth's magnetic field to be pulled into the shape of a L_____. The distant M_____ can snap like pulled taffy as it attempts to relieve the magnetic stresses building up in the system. The released energy causes currents of N_____ to flow into the upper atmosphere where they collide with atoms of oxygen and nitrogen to produce the spectacular displays of the Aurora Borealis and O_____.



Coronal Mass Ejection (CME) seen by SOHO satellite.

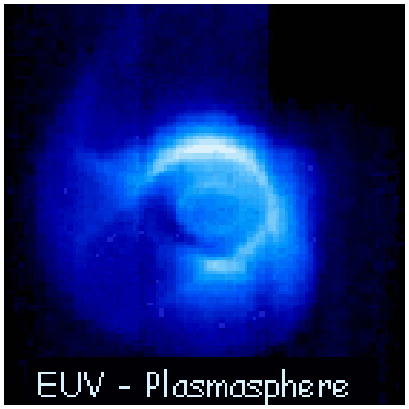


IMAGE sees Earth's upper atmosphere – the Plasmasphere.

-4 = Ring current	-9 = ionosphere	4 = electromagnetic
0 = Magnetotail	-3 = solar flares	10 = oxygen
3 = Plasma	8 = corona	1 = Coronal Mass Ejection
-7 = Sunspot	-1 = magnetosphere	-11 = stratosphere
2 = Aurora	11 = atmosphere	12 = plasmasphere
-10 = Ozone layer	**6 = magnetic fields**	5 = Aurora Australis
-6 = Comet	9 = sphere	-8 = photosphere
7 = Fields	-2 = storms	-5 = charged particles

Extra Credit Problem

Solve these equations for X to find the number in the Word Bank. Write the word on the indicated lettered line in the essay, then answer the questions below.

A $(x - 2) - 4 = 0$ **Answer: x= 6. From Word Bank 6 = 'magnetic fields'**

B $2(3-x) + 8 = 0$

C $8x - 2(2x - 4) + 4(4 - 3x) = 0$

D $-3x + 4 = 5x - 28$

E $-15 - 2x = 3(x + 10)$

F $4x - 4 - (3x - 4) + 3 = 0$

G $-(2x - 3) - (8 + 3x) = 0$

H $-(4x - 18) + 4(4 - 2x) - 2(3x-1) = 0$

I $-(3-4x) + 2(5-6x) - 6 = 3(2x - 5) - (x-3)$

J $-(2x + 8) + 6x = -16$

K $3(2x + 10) + 3x + 6 = 0$

L $-x + 3(2x - 5) + 3x = 2x - 51$

M $-x + 5 + (3x - 2) + 6 = 3(x + 3) - 2x$

N $-6(x+5) = 0$

O $5(x + 5) + 10(x - 3) = 70$

Question 1: Do the particles that cause aurora come from Earth's environment or directly from the Sun?

Question 2: What is a Coronal Mass Ejection?

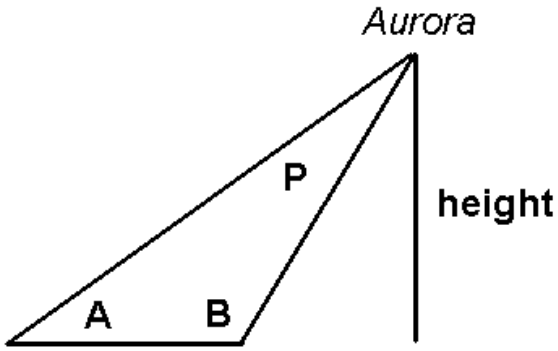
Question 3: What roles do magnetic fields play in causing disturbances on the sun and Earth?

Question 4: Where does the energy come from to cause solar storms and aurora?

Question 5: How does Earth's magnetic field prevent solar storms from reaching the atmosphere?

Image courtesy Tim Tomljanovich, http://www.nsaclub.org/photos/aurora/

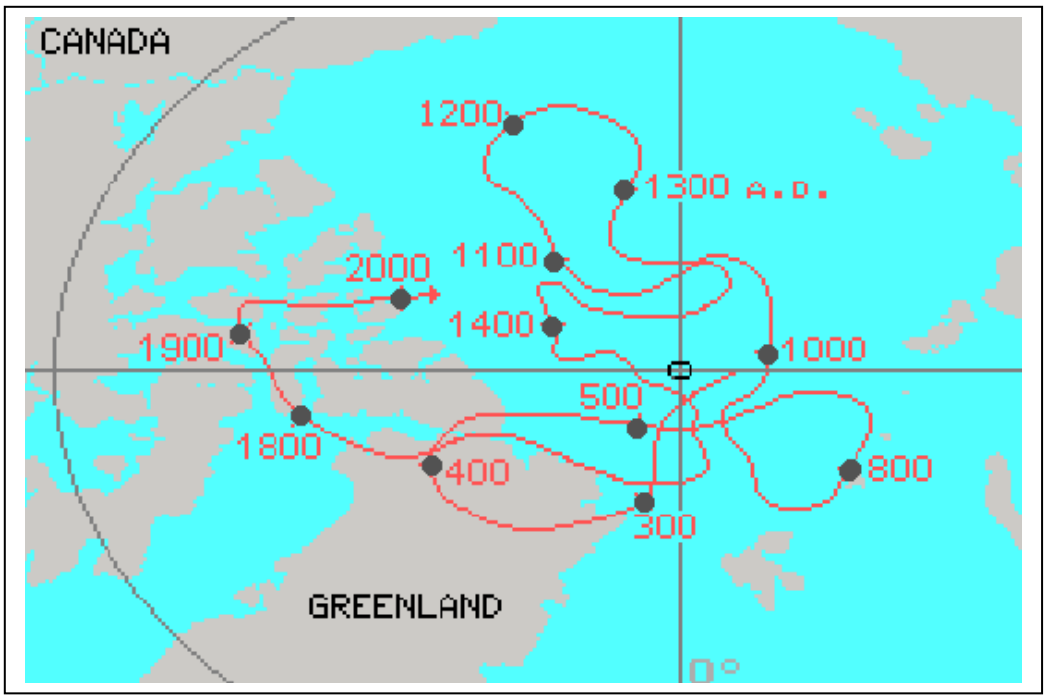
For thousands of years, people living at northern latitudes had no idea how high up the Aurora Borealis was located. Before the advent of photography in the 1880's, auroral observers tried to determine the height of aurora by the method of triangulation. One of the earliest of these measurements was made by the French scientist Jean-Jacques d'Ortous de Mairan between 1731 and 1751. From two stations 20 km apart, observers measured the angles A and B between the ground and a specific spot on an aurora. From the geometry of the triangle, they estimated that aurora's height was between 650 - 1,000 km above the ground. More precise measurements yielded estimates from 70 to 200 kilometers.



Question 1 - Suppose that two observers were located 30 kilometers apart. Observer A measured an angle of 53 degrees and Observer B measured an angle of 114 degrees. By making a scaled drawing of this triangle, what was the height of the auroral feature they were studying?

Question 2 - Use a protractor to measure the vertex angle, P. What happens to the measurement of angle P if you decrease the 'baseline' distance between the observers to 5 kilometers?

Question 3 – What would the measurements of the two angles be if the aurora were located over a spot half-way between the two observers?



The Earth's magnetic North Pole moves around quite a bit, especially if you plot its position during the last 1700 years! The scale of the above plot is approximately 380 km per centimeter. Use this scale and a piece of string and the dates for each position, to answer the following questions:

Question 1: What is the total distance that the North Magnetic Pole wandered from 300 AD to 2000 AD?

Question 2: What is the shortest distance that the pole wandered in a 100-year period?

Question 3: What is the longest distance that the pole wandered in a 100-year period?

Question 4: What is the average speed of the wander from 300 AD to 2000 AD?

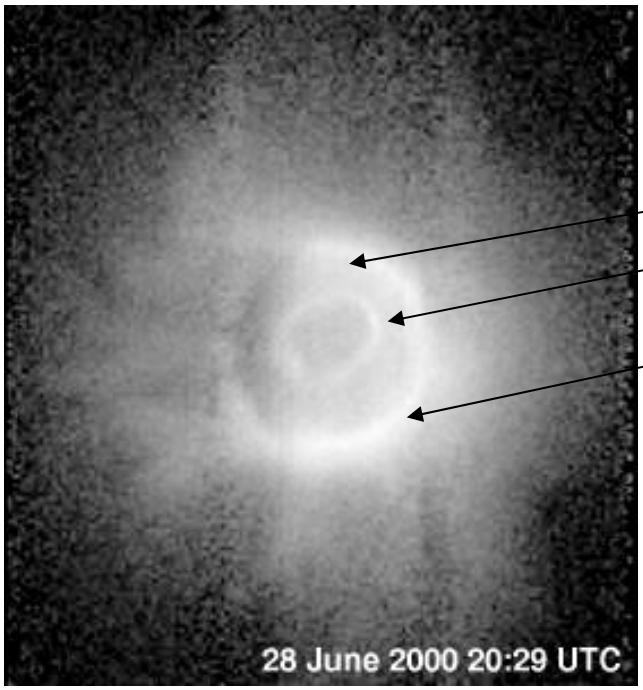
Question 5: What is the fastest speed it traveled in a 100-year period?

Question 6: Is the speed of the wander during the last 100 years unusual compared to the average speed or to the fastest speed?

The Plasmasphere.

Most people, when asked, would say that the atmosphere of Earth probably comes to an end a few hundred kilometers above the surface of Earth. In fact, our atmosphere has been detected more than 10,000 kilometers above the surface. This region of very dilute gas is called the Plasmasphere because the atoms are often ionized by the very harsh ultraviolet light from the sun. This kind of gas is called a plasma. Wherever the ultraviolet light reaches the plasmasphere's gas, the gas glows and can be photographed with special equipment.

The image below was taken by the IMAGE EUV instrument on June 28, 2002 at 4:09 PM (EDT). It shows the plasmasphere as revealed by the glow of ionized helium atoms at a wavelength of 121.6 nanometers (1216 Angstroms). Additional 'still' images and movies can be retrieved at the IMAGE satellite's EUV instrument website at the University of Arizona (http://euv.lpl.arizona.edu/euv/)



The image was taken above the North Pole. Helium atoms excited by the ultraviolet light from the sun give off a special wavelength of light that the IMAGE instrument can detect.

The round disk in the center is Earth.

The smaller, irregular oval-shaped ring is the ring of aurora (the auroral oval).

The larger ring is the glow of helium atoms in the dense part of the plasmasphere nearest Earth.

With the center of the Earth disk as the origin, use a millimeter ruler and a compass to answer the following questions:

Question 1 - Is the gas in the plasmasphere lumpy or smooth? Use the picture above to give examples that support your answer.

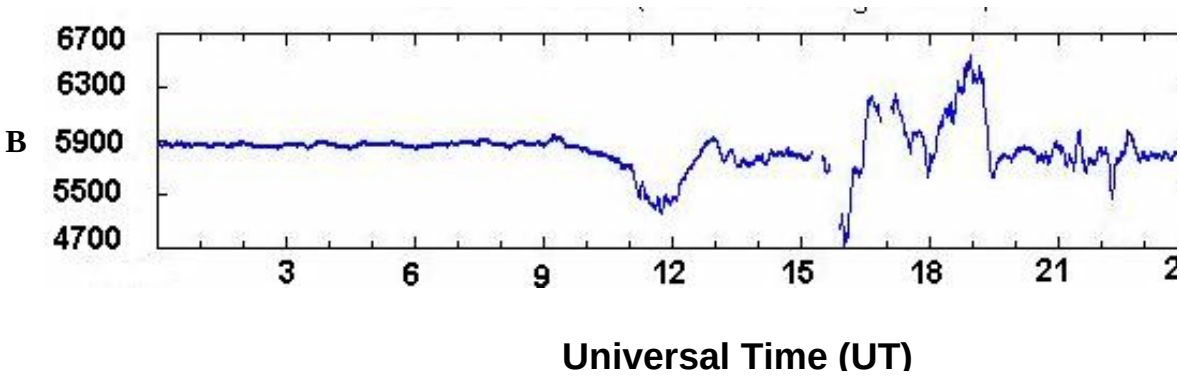
Question 2 - If the radius of Earth is 6,378 kilometers, what is the maximum and minimum range for the plasmasphere altitude above Earth's surface?

Question 3 - Using the scale of this image calculated from Question 2, draw circles that represent the orbits of the International Space Station (altitude 400 km), the Global Positioning Satellite system (altitude 20,200 km) and GEO communications satellites (altitude 35,900 km). Are they inside or outside the plasmasphere?

Magnetic Storms are disturbances in Earth's magnetic field that can be detected from the ground using sensitive instruments called magnetometers. Dozens of these instruments located at 'Magnetic Observatories' around the world keep track of these disturbances. The graph below shows these changes during a 24-hour period on October 24, 2003.

The vertical axis in the plot gives the magnitude, B, of this magnetic change on Earth that is in the East to West direction. The strength of a magnet can be described in terms of a unit called a Tesla. On this plot, the vertical axis gives the magnetic strength in units of nano-Teslas (nT). One nano-Tesla is one billionth of a Tesla (so 1 billion nano-Teslas = 1 Tesla) . We can see that on this day, Earth's magnetic field varied between 4700 and 6700 nano-Teslas.

The horizontal axis is the time measured in Universal Time (UT). When scientists study events that change in time, they often use Universal Time, which is also known as Greenwich Mean Time. All scientific measurements are referred to this standard of time keeping to avoid problems converting from one time zone to another. Universal Time or 'UT' follows a 24-hour clock, so that 6:00 PM is written as 18:00 and 1:00 PM is written as 13:00. Use the time calculator at http://www.indiana.edu/~animal/fun/conversions/worldtime.html to convert your local time to Universal Time!



Question 1 – If a magnetometer measured a magnetic field of 137,000 nT, how many Teslas would that correspond to?

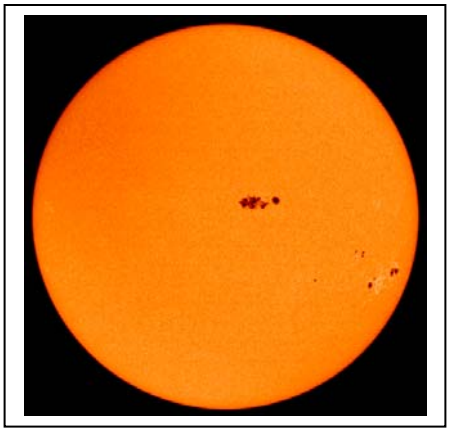
Question 2 – What is the corresponding Universal Time for 9:45 PM?

Question 3 - From the above graph, at what time was the magnetic storm most severe in terms of the absolute magnitude of its change in B?

Question 4 – At what time did the storm episode begin and end?

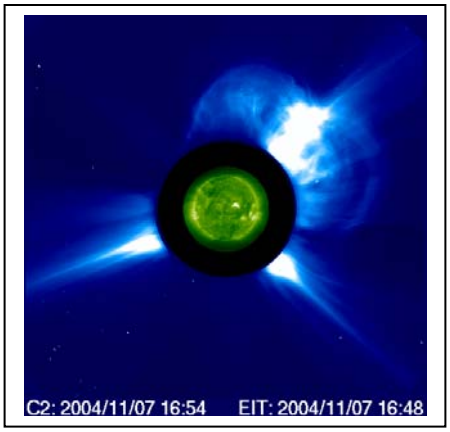
Question 5 – As a percentage of 5900 nT, what was the largest change in the magnetic field?

Question 6 - How long did the magnetic storm last?



November 6 - 9, 2004 were very active days for solar and auroral events. A major sunspot group, AR 0696, with a complex magnetic field produced several Coronal Mass Ejections and flares during this time. There was an X2.0-class solar flare on November 7, 16:06 UT and a CME ejection. On November 9 and 12:00 UT, the beginnings of a major geomagnetic storm started. The NOAA Space Weather Bulletin announced that:

The Geomagnetic field is expected to be at unsettled to major storm levels on 09 November due to the arrival of a CME associated with the X2.0 flare observed on 07 November. Unsettled to minor storm levels are expected on 10 November. Quiet to active levels are expected on 11 November.



Many observers as far south as Texas and Oklahoma reported seeing beautiful aurora on Sunday night from an earlier CME/flare combination on Saturday, November 6th. In the space provided below, calculate the speed of the CME as it traveled to Earth between November 7th - 9th assuming that the distance to Earth is 93 million miles, or 147 million kilometers.

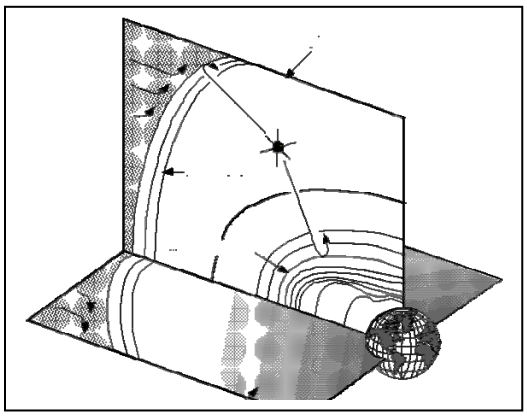
Question 1 - How long did it take for the CME to arrive?

Question 2 - What is the speed of the CME in miles per hour?

Question 3 - What is the speed of the CME in kilometers per hour?

Question 4 - What is the speed of the CME in miles per second?

Question 5 - What is the speed of the CME in kilometers per second?



The IMAGE spacecraft (shown as the 'star' in the figure) contains an instrument called the Radio Plasma Imager (RPI). This instrument sends out a powerful pulse of radio energy (the fish hook-shaped lines) at frequencies from 3,000 to 3 million cycles per second (Hertz). When the echos from these pulses are later received by the instrument, they can be analyzed to find the location and density of the plasma that reflected them.

Clouds of plasma in space have an interesting property: When radio waves reflect off of them, the radio frequency of the reflected signal depends on the density of the cloud! The formula that relates the reflection frequency, F, to the density, N, is given by

$$F = 9000 \times (N)^{1/2}$$

The unit of frequency is Hertz (cycles per second) and the unit for the density of the cloud is electrons per cubic centimeter.

In this problem, you will use some of the same methods and equations that IMAGE scientists use, to study the properties of plasma clouds near Earth. Although the properties of these clouds, and their locations, have been 'made-up' for this problem, your analysis of them will be similar to the methods employed by IMAGE scientists using real data.

With the formula above, solve for the density, N, and complete the table entries below.

Location	Direction (degrees)	Distance in Earth Radii (Re)	Reflection Frequency (Hertz)	Density (electrons per cc)
1	300	1.0	284,000	995
2	315	2.5	201,000	
3	350	6.5	12,600	
4	45	4.5	20,100	
5	60	3.9	25,500	
6	90	4.1	28,500	
7	120	4.0	25,500	
8	135	5.5	20,100	
9	215	7.2	12,600	
10	230	3.5	220,000	
11	270	1.2	348,000	

Applications of the Pythagorean Theorem - Magnetism.

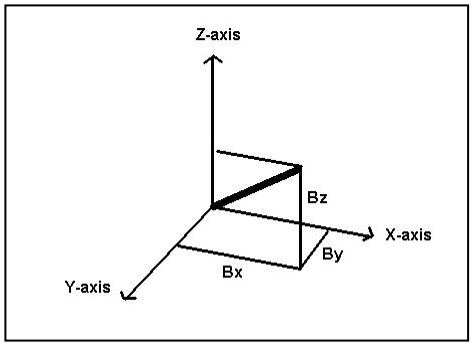
Unlike temperature, magnetism requires three numbers to define the strength of its field in space. Scientists call magnetism a **Vector** quantity because it is defined by both its magnitude at a point in space, and its direction at that point, given by the coordinate (X, Y, Z). The Pythagorean Theorem is used to calculate the magnitude (or total strength) of the magnetic field from the separate Bx, By and Bz quantities that make up its description as a field in 3-dimensional space. To find the Bx, By and Bz components of Earth's magnetic field (in units of nanoTeslas, nT) where you live, visit the International Geomagnetic Reference Field Model (Part 2 Form)

http://nssdc.gsfc.nasa.gov/space/model/models/igrf.html

Enter the year (2004) and the requested geographic latitude, longitude (in degrees, minutes and seconds – D M S entries in table) and elevation (Use 0.0 km for table). You can find the geographic coordinates for a specific location at

http://geonames.usgs.gov

Follow 'Query GNIS" to the input form. Select 'Civil' for a town name.



The Pythagorean Theorem in 3-dimensions is →

$$D = \sqrt{x^2 + y^2 + z^2}$$

Use the Pythagorean Theorem to fill-in the last column of the table

City	Longitude D	Longitude M	Longitude S	Latitude D	Latitude M	Latitude S	Bx (nT)	By (nT)	Bz (nT)	Total B (nT)
Chicago	87	54	55	41	50	05	26600	1234	48620	**55434**
Boston	71	05	00	42	18	00	25251	2234	46676	
Miami	80	32	00	25	37	00	36274	0.2	28396	
Hollywood	118	20	00	34	01	00	32161	-2684	39236	
Bangor	68	47	15	44	49	56	23437	2600	48244	
Kansas City	94	43	37	39	07	06	28846	365	46535	
Sioux Falls	96	43	48	43	32	48	25602	283	50988	
Spokane	117	22	00	47	37	00	22977	-3263	53054	
Provo	103	52	06	43	10	02	26045	-875	50767	
Anchorage	149	15	02	61	10	00	16377	-3572	53739	
Honolulu	154	53	24	19	33	15	32644	1402	14594	
Sedona	111	47	35	34	50	38	31818	-1978	41379	

Question 1 - What cities have the highest and lowest magnetic field (**B**) strengths?

Question 2 - What is the average **B** value of Earth's magnetic field for all locations?

Question 3 – Some adults think that Sedona Arizona has special 'powers'. How does the magnetism at this location compare to other locations in the table?

Question 4: Plot the **By** values on a map. What pattern do you see?

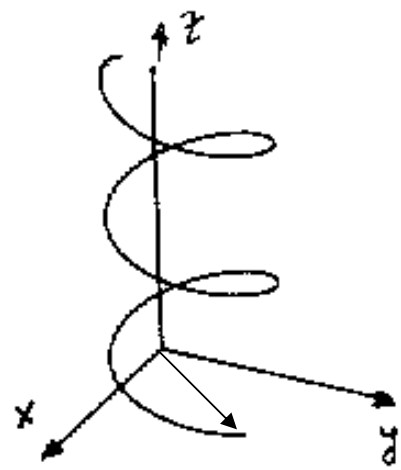
Magnetic forces and Particle Motion.

Magnetic forces are more complicated than gravity in several important ways. Unlike gravity, but similar to the ordinary Coulomb force between charged particles, magnetic forces depend on the degree to which a particle is charged. Also unlike gravity, magnetic forces possess a quality called 'polarity'. All magnets have both a north and a south 'pole'. Because of the property of polarity, the motion of charged particles in a magnetic field is more complicated than the motion under gravitational forces alone.

One common motion is for a charged particle to move in a spiral path along a line of magnetic force. As the particle moves along the field, it also executes a circular 'orbit' around the line of force, so its path resembles a helix. The spiral path can be thought of as a circular path with a radius, R, which moves at a constant speed along the line of force. Adding up the 'circular' and 'linear' motions of the particle gives you a spiral path like an unwound spring or 'Slinky' toy.

In this exercise, we will calculate the radius of such a spiral path and see how it depends on both the strength of the magnetic field, and the speed of the charged particle as it orbits the line of magnetic force. The formula for the radius of the spiral (shown by the arrow in the figure below) is given by:

$$q V B = \frac{m V^2}{R}$$

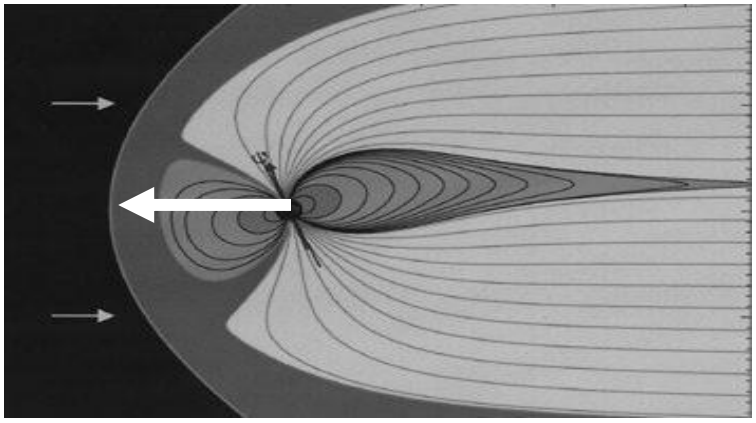


where q is the charge on the particle (in Coulombs), V is the speed of the particle as it orbits the line of force (in meters/s), B is the magnetic field strength (in Teslas), m is the mass of the particle (in kilograms), and R is the radius of the particle's orbit (in meters).

Question 1 – What is the relationship for R after solving and simplifying this equation?

Question 2 – An electron with a charge **q** = 1.6×10^{-19} Coulombs and a mass **m** = 9.1×10^{-31} kilograms is traveling at **V** = 1.0×10^{9} meters/sec in a magnetic field with a strength **B** = 0.00005 Tesslas. What is the radius of its spiral orbit in meters?

Question 3 - If an oxygen ion has twice the electron's charge, and 29400 times an electron's mass, what will its spiral radius be for the same values of **B** and **V** in Question 1?



When the solar wind flows past Earth, it pushes on Earth's magnetic field and compresses it. There is a point in space called the magnetopause where the pressure of the solar wind balances the outward pressure of Earth's magnetic field. The distance from the Earth, R, (white arrow in drawing) where these two pressures are balanced is given by the equation:

$$R^6 = \frac{0.72}{8\pi D V^2}$$

In this equation, D is the density in grams per cubic centimeter (cc) of the gas (solar wind, etc) that collides with Earth's magnetic field, and V is the speed of this gas in centimeters per second. Let's do an example to see how this equation works!

The solar wind has a typical speed of 450 km/s or equivalently V = 4.5 x 10^7 cm/s. To find the density of the solar wind in grams/cc we have to do a two-step calculation. The wind usually has a particle density of about 5 particles/cc, and since these particles are typically protons (each with a mass of 1.6 x 10^{-24} gm) the density is then 5 x (1.6 x 10^{-24} gm)/cc so that D = 1.28 x 10^{-23} gm/cc.

We substitute D and V into the equation and get R^6 = 1105242.6. so that R = $(1105242.6)^{1/6}$. To solve this, we use a calculator with a key labeled $\mathbf{Y^x}$ First type '1105242.6' and hit the '**Enter**' key. Then type '0.1666' (which equals 1/6) and press the $\mathbf{Y^x}$ key. In this case the answer will be '10.16' and it represents the value of R in multiples of the radius of Earth (6378 kilometers). Scientists simplify the mathematical calculation by using the radius of Earth as their unit of distance, but if you want to convert 10.16 Earth radii to kilometers, just multiply it by '6378 km' which is the radius of Earth to get 64,800 kilometers. That is the distance from the center of Earth to the magnetopause where the magnetic pressure is equal to the solar wind pressure for the selected speed and density. These will change significantly during a 'solar storm'.

Now lets apply this example to finding the magnetopause distance for some of the storms that have encountered Earth in the last five years. Complete the table below, rounding the answer to three significant figures:

Storm	Date	Day Of Year	Density (particle/cc)	Speed (km/s)	R (km)
1	11/20/2003	324	49.1	630	
2	10/29/2003	302	10.6	2125	
3	11/06/2001	310	15.5	670	
4	3/31/2001	90	70.6	783	
5	7/15/2000	197	4.5	958	

Question: The fastest speed for a solar storm 'cloud' is 1500 km/s. What must the density be in order that the magnetopause is pushed into the orbits of the geosynchronous communication satellites at 6.6 Re?

Kinetic Energy and Voltage.

When matter moves, it possesses a quantity of energy called Kinetic Energy. This energy can be described by a simple mathematical formula. When matter that carries a charge moves in an electric field, it also carries kinetic energy. By comparing the two mathematical descriptions of kinetic energy, we can relate how the speed of a particle, **V**, changes as its mass, **m**, charge, **q**, or the electrical voltage, **E**, it moves in, is changed. Scientists often measure a charged particle's kinetic energy in terms of its voltage. In this exercise, we will look at the energies of various types of charged particle systems and determine the average speed of the particle. The formula that relates the kinetic energy of a particle to its voltage is given below. The left side is the definition for kinetic energy based on its charge and the voltage, **E**. The right side is the definition of kinetic energy based on the particles mass and speed.

$$qE = \frac{1}{2} m V^2$$

Question 1 – What is the formula for the speed of the particle, **V**, after solving the equation for V, and simplifying?

Question 2 – An electron with a charge **q** = 1.6×10^{-19} Coulombs and a mass **m** = 9.1×10^{-31} kilograms and has an energy **E** = 1000 Volts. What is the speed of the particle in meters per second?

Question 3 - If an oxygen ion has twice the charge of an electron, and 29400 times an electron's mass, what will its speed be for the same amount of energy E = 1,000 Volts?

Useful Internet Resources

The human and technological impacts of solar storms and space weather:
http://www.solarstorms.org

Newspaper accounts of aurora and technology impacts from 1800-2001:
http://www.solarstorms.org/SRefHistory.html

Space weather and satellite failures
http://www.solarstorms.org/Ssatellites.html

NOAA space weather forecasting center
http://www.noaa.sec.gov/SWN

Space weather summaries and daily updates:
http://www.spaceweather.com

NASA Student Observation Network –Tracking a Solar Storm
http://son.nasa.gov/tass/index.htm

Archive of NASA TV programs about space weather for grades 6-10
http://www.solarstorms.org/STV.html

Movies and animations about space weather
http://www.solarstorms.org/SMovies.html

Frequently Asked Questions about space weather
http://www.solarstorms.org/SFAQs.html

Additional classroom activities
http://image.gsfc.nasa.gov/poetry/activities.html

Exploring Space Science Mathematics pre-algebra problem book
http://image.gsfc.nasa.gov/MathDocs/spacemath.html

Exploring Earth's Magnetic Field primer
http://image.gsfc.nasa.gov/poetry/magnetism/magnetism.html

IMAGE, Student's Guide to Sun-Earth Science topics
http://image.gsfc.nasa.gov/poetry/educator/students.html

The IMAGE, Soda Bottle Magnetometer
http://image.gsfc.nasa.gov/poetry/workbook/magnet.html

The Mysterious Van Allen Radiation Belts
http://radbelts.gsfc.nasa.gov/outreach/outreach.html

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