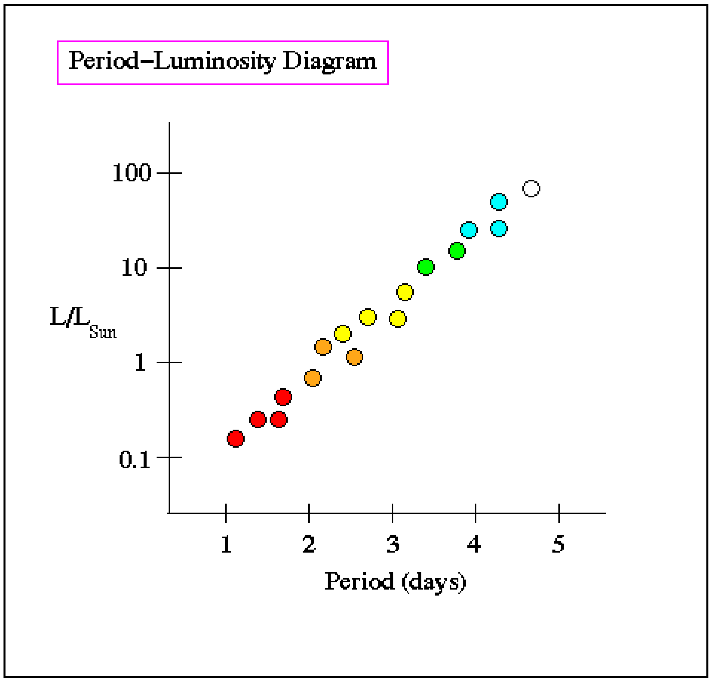
Inverse and joint variation



One of the first things that scientists do with data is to graph it in various ways to see if a pattern emerges.

If two variables are selected that lead to a smooth curve, the variables can be shown to lead to 'correlated' behavior that can either represent a direct, or inverse, variation. The specific shape of the curve indicates the exponent.

The example to the left shows that for Cepheid variable stars, the Log of the star's luminosity, L, is proportional to its period because the slope of the curve is 'fit' by a linear equation.

Problem 1 – The radius of a black hole, R, is proportional to its mass, M, and inversely proportional to the square of the speed of light, c. If the constant of proportionality is twice Newton's constant of gravity, G, what is the mathematical equation for the black hole radius?

Problem 2 – The luminosity, L, of a star is proportional to the square of its radius, R, and proportional to its surface temperature, T, to the fourth power. What is the equation for L if the proportionality constant is C?

Problem 3 – The thickness of a planetary atmosphere, H, is proportional to temperature, T, and inversely proportional to the product of its molecular mass, m, and the local acceleration of gravity, g. What is the equation for H if the constant of proportionality is k?

Problem 4 – The temperature of a planet, T, to the fourth power is proportional to the luminosity, L, of the star that it orbits, and inversely proportional to the square of its distance from its star, D. If the proportionality constant is (1-A) where A is a constant indicating the reflectivity of the planet, what is the equation for the temperature?

Problem 1 – The radius of a black hole, R, is proportional to its mass, M, and inversely proportional to the square of the speed of light, c. If the constant of proportionality is twice Newton's constant of gravity, G, what is the equation for the black hole radius?

Answer: $R = \frac{2GM}{c^2}$

Problem 2 – The luminosity, L, of a star is proportional to the square of its radius, R, and proportional to its surface temperature, T, to the fourth power. What is the equation for L if the proportionality constant is C?

Answer: $L = CR^2T^4$

Problem 3 – The thickness of a planetary atmosphere, H, is proportional to temperature, T, and inversely proportional to the product of its molecular mass, m, and the local acceleration of gravity, g. What is the equation for H if the constant of proportionality is k?

Answer: $H = k\frac{T}{mg}$

Problem 4 – The temperature of a planet, T, to the fourth power is proportional to the luminosity, L, of the star that it orbits, and inversely proportional to the square of its distance from its star, D. If the proportionality constant is (1-A) where A is a constant indicating the reflectivity of the planet, what is the equation for the temperature?

Answer: $T^4 = \frac{(1-A)L}{D^2}$

Since 1800, scientists have measured the average sea level using a variety of independent methods including hundreds of tide gauges and satellite data. The table below gives the average sea level change since 1910 based upon research published by the International Commission on Global Climate Change (2007)

Year	Height (cm)	Year	Height (cm)
1910	+1	1960	+11
1920	+2	1970	+12
1930	+4	1980	+13
1940	+5	1990	+14
1950	+7	2000	+18

Problem 1 – State whether there is a direct correlation, an inverse correlation or no correlation between the year and the average sea height change since 1910.

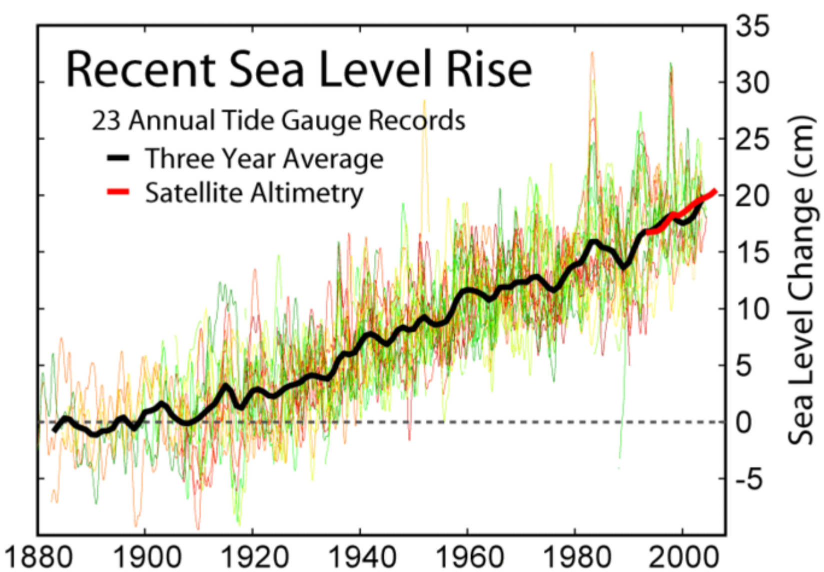
Problem 2 – What is the mathematical formula that models the general behavior of the data in this table?

Problem 3 - Assuming that the underlying physical conditions remain the same, and your model holds true, for a doubling of the period since 1910, what does your model predict for the sea level change in A) 2020? B) 2050? C) 2100?

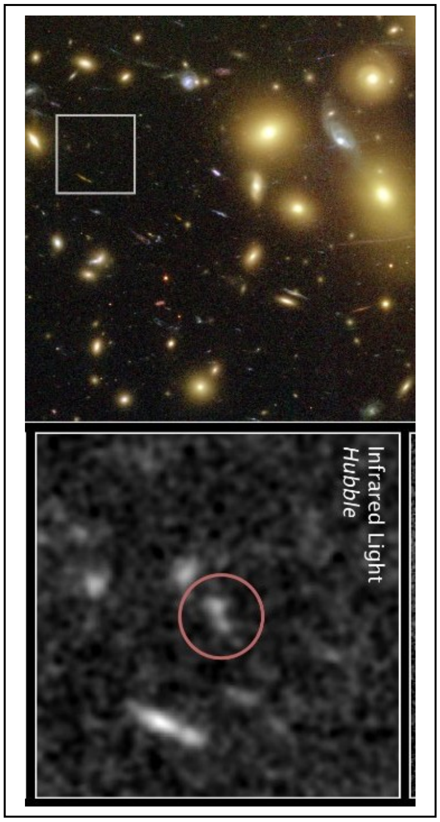
Problem 1 – State whether there is a direct correlation, an inverse correlation or no correlation between the year and the average sea height change since 1880.
Answer: **Students may graph the data to visually determine the trend. From the table, they should notice that, as the year increases, so too does the sea level change, indicating a direct, rather than inverse, correlation.**

Problem 2 – What is the mathematical formula that models the general behavior of the data in this table?
Answer:
Students may use a variety of methods for 'fitting' the data. Since a direct correlation is of the form H = cY where H is the height in centimeters and Y is the number of years since 1910, selecting a point in the middle of the range, (1950, +7), leads to +7 = c (1950-1910) so c = 7/40 =0.175 and so **H = 0.175Y**.

Problem 3 - Assuming that the underlying physical conditions remain the same, and your model holds true, for a doubling of the period since 1880, what does your model predict for the sea level change in A) 2020? B) 2050? C) 2100?
Answer: A) H(2020) = 0.175(2020-1910) = **+19 centimeters**;
B) H(2050) = 0.175(2050-1910) = **+24 centimeters;**
C) H(2100) = 0.175(2100-1910) = **+33 centimeters;**



Actual sea level change data from ICGCC report.



In 2008, the Hubble and Spitzer Space Telescopes identified one of the farthest galaxies from our Milky Way. It is so distant that light has taken over 12.9 billion years to reach Earth, showing us what this galaxy looked like 12.9 billion years ago. The actual distance to this galaxy today is approximately given by the following formula:

$$D(z) = \frac{877}{(1+z)}\left(\frac{z}{8} + \frac{7}{8} - \frac{7}{8}\sqrt{\frac{z}{4}+1}\right)$$

The distance, **D**, is defined in units of billions of light years so that 'D = 10' means 10 billion light years. The quantity **z** is called the redshift of the galaxy. It can easily be measured by analyzing the light from the galaxy and is defined by the domain Z:[0,+infinity]

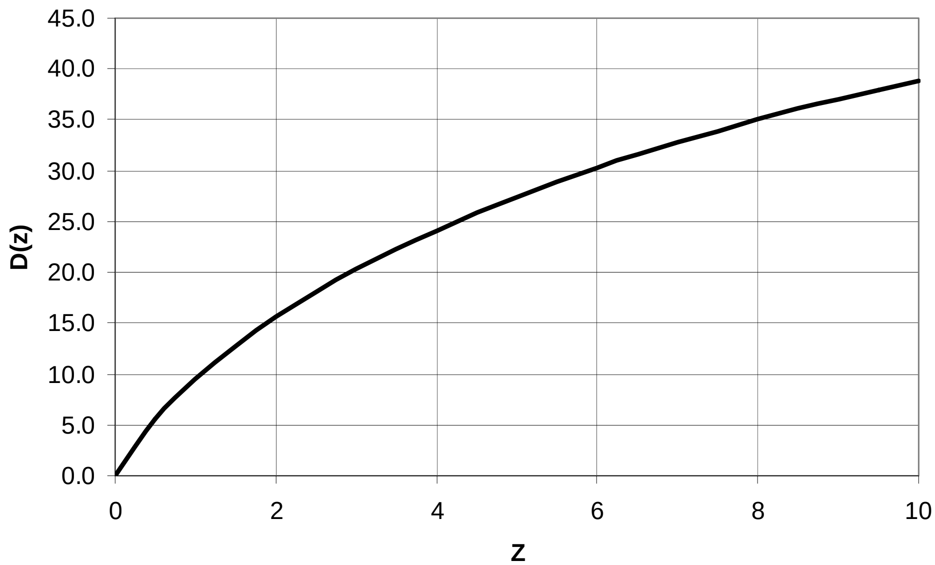
Top image shows where the galaxy A1689-zD1 was found. Bottom image shows the galaxy at the center of the field studied by the Hubble Space Telescope.

Problem 1 – Graph the function D(z) over the domain Z: [0,+10].

Problem 2 - The most distant known galaxy identified in 2008 is called A1689-zD1. The data from the Hubble Space Telescope indicated a redshift of z = 7.6. How far from Earth is this galaxy in terms of billions of light years?

Problem 3 - An astronomer is studying two pairs of galaxies, A,B, and C, D, located in the Hubble Deep Field. He measures the redshifts to each galaxy and determines that Z(A) = 1.5, Z(B) = 2.5 and that Z(C) = 5.5 and Z(D) = 6.5. Which pair of galaxies have members that appear to be the closest together in actual distance, D?

Problem 1 – Graph the function D(z) over the domain Z: [0,+10].



Problem 2 - The most distant known galaxy identified in 2008 is called A1689-zD1. The data from the Hubble Space Telescope indicated a redshift of z = 7.6. How far from Earth is this galaxy in terms of billions of light years?

Answer:

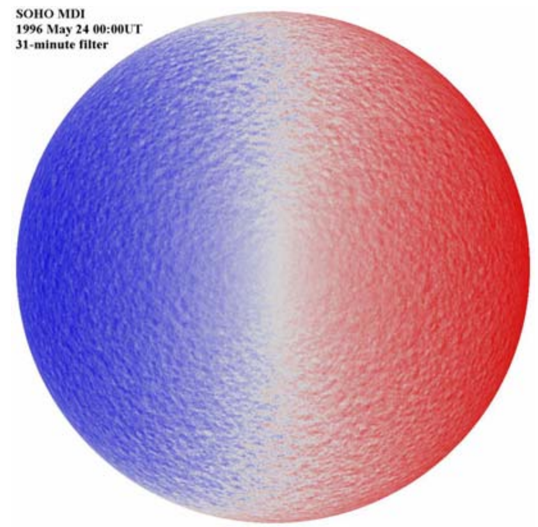
$D(7.6) = 877\ [7.6/8 + 7/8 - 7/8(7.6/4 + 1)^{1/2})]/(1+7.6)$
$= 877\ [0.95 + 0.875 - 0.875(1.70)]/8.6$
$= 877\ (0.335)/8.6$
= **34.2 billion light years**.

Problem 3 - An astronomer is studying two pairs of galaxies, A,B, and C, D, located in the Hubble Deep Field. He measures the redshifts to each galaxy and determines that Z(A) = 1.5, Z(B) = 1.6 and that Z(C) = 7.5 and Z(D) = 7.6. Which pair of galaxies have members that appear to be the closest together in actual distance, D?

Answer: D(A) = 12.8 billion light years
D(B) = 13.4 billion light years
D(C) = 33.9 billion light years
D(D) = 34.2 billion light years

The pair of galaxies **C and D are closer together** (34.2 – 33.9 = 0.3 billion light years) than galaxies A and B which are 13.4 – 12.8 = 0.6 billion light years apart.

Note: The equation for D(z) does not include additional cosmological factors that will change the answers downward by about 10%.



The rotating sun causes a Doppler Effect. Atoms emitting light on the left half of the sun (east) are 'blue-shifted' as the sun rotates from east to west. (Courtesy NASA /SOHO/ GONG)

When a source of light moves relative to an observer, the frequency of the light waves will be increased if the movement is towards the observer, or decreased if the motion is away from the observer. This phenomenon is called the Doppler Effect, and it is given by the formula:

$$f = f_s \frac{c}{c+V}$$

where V is the speed of the source, fs is the normal frequency of the light seen by the observer when V=0, and f is the 'shifted' frequency of the light from the moving source as seen by the observer. C = 3.0×10^8 meters/sec is the speed of light.

Problem 1 – The Sun rotates at a speed of 2 kilometers/sec at the equator. If you are observing the light from hydrogen atoms at a frequency of fs=4.57108×10^{14} Hertz, A) About what would be the frequency, f, of the blue-shifted eastern edge of the sun? B) What would be the difference in megaHertz between f and fs?

Problem 2 – In 2005, astronomers completed a study of the pulsar B1508+55 which is a rapidly spinning neutron star left over from a supernova explosion. They measured a speed for this dead star of 1,080 km/sec. If they had been observing a spectral line at a frequency of fs=1.4×10^9 Hertz, what would the frequency of this line have been if the neutron star were moving directly away from Earth?

Problem 3 – An astronomer is using a radio telescope to determine the Doppler speed of several interstellar clouds. He uses the light from the J=2-1 transition of the carbon monoxide molecule at a known frequency of fs=230 gigaHertz, and by measuring the same molecule line in a distant cloud he wants to calculate its speed v. A) What is the function f(v)? B) Graph g(v)=f(v)-fs in megaHertz over the interval 10 km/sec < v < 200 km/sec. C) A cloud is measured with a Doppler shift of g(v) = 80.0 megaHertz, what is the speed of the cloud in kilometers/sec?

Problem 1 – Answer: A) $f = 4.57108 \times 10^{14}$ Hertz $(3.0\text{x}10^{8}) / (2.0 + 3.0\text{x}10^{8})$

$f = 4.57108 \times 10^{14}(0.999999993)$

$\mathbf{f = 4.57107997 \times 10^{14}}$ **Hertz**

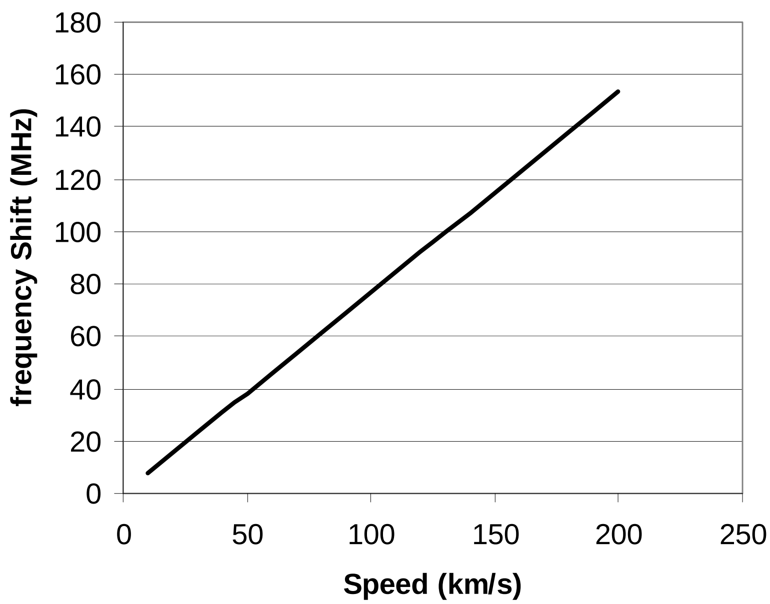
B) $F\text{-}Fs = 4.57107997 \times 10^{14} - 4.57108 \times 10^{14} = 3{,}048{,}000$ Hertz.

So the frequency of the hydrogen light would appear at **3.048 megaHertz** higher than the normal frequency for this light.

Problem 2 – Answer: $f = 1.4\text{x}10^{9}$ Hertz x $(1 + 1{,}080/3.0\text{x}10^{5})$ = $\mathbf{1.405 \times 10^{9}}$ **Hertz**.

Problem 3 – A) What is the function f(v)?
Answer: $f(v) = 2.3\text{x}10^{9}\ (1 + v/300000)$

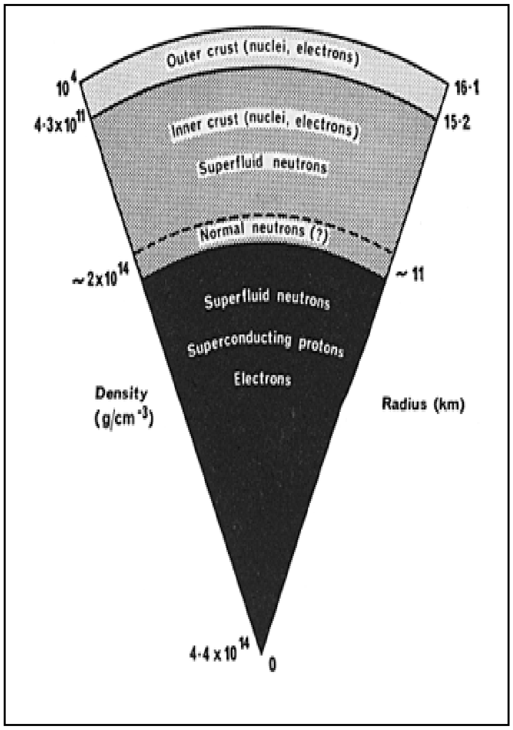
B) Graph g(v)=f(v)-fs in megaHertz over the interval 10 km/sec < v < 1,000 km/sec.
Answer: g(v) = 230 (v/300) megaHertz. or **g(v) = 0.767 v megaHertz**



C) A cloud is measured with g(v) = 80.0 megaHertz, what is the speed of the cloud in kilometers/sec?

Answer: From the graph, g(80) = 104 km/sec and by calculation, 80 = 0.767v so v = 80/.767 = **104 km/sec**.

Graphing General Rational Functions 9.3.1



A neutron star is the dense, compressed remnants of a massive star that went supernova. The matter has become so compressed that it exceeds the density of an average atomic nucleus. A cubic centimeter of its mass would equal 100 million tons!

Under this extreme compression, ordinary nucleons (protons and neutrons) interact with each other like a gas of particles. Each nucleon carries a specific amount of kinetic energy given by the approximate formula:

$$W(k) = 20k^2 - k^3\left(\frac{40-k^3}{1+3k}\right) \text{ MeV}$$

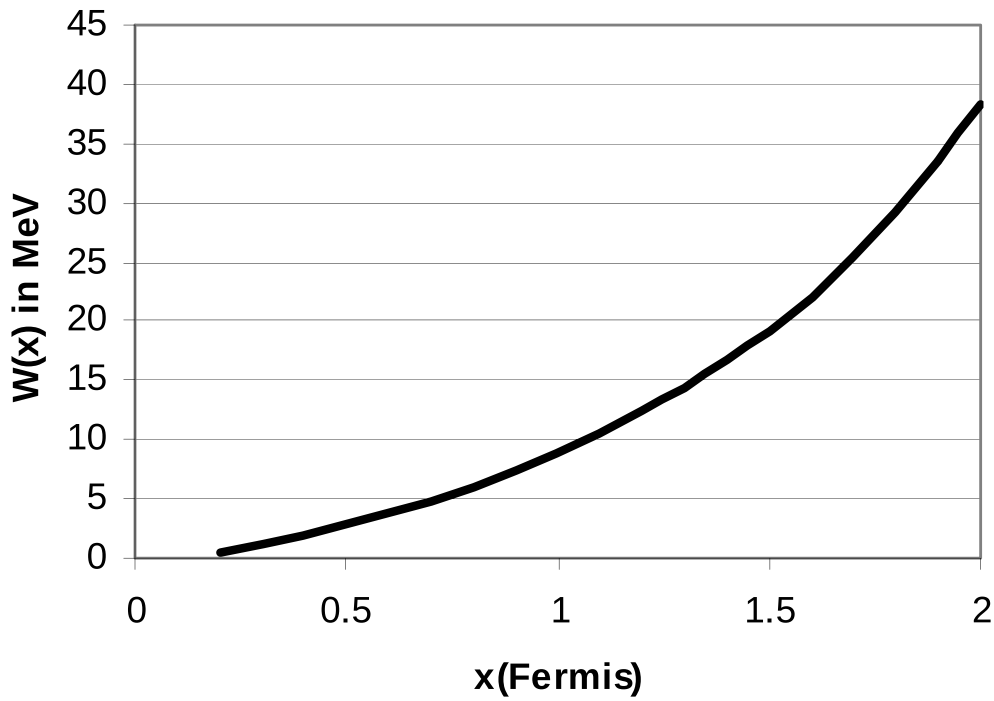
where k is the average distance between the nucleons in units of Fermis (1 Fermi = 10^{-15} meters). The units of kinetic energy are million electron-volts (MeV) where 1 MeV = 1.6×10^{-13} Joules. (Note: On this scale, the energy equivalent to the mass of 1 electron equals 0.511 MeV.)

Problem 1 – Graph the function W(x) over the domain x:[0,2.0]

Problem 2 - As the amount of compression increases and the nucleons are crowded closer and closer together, what happens to the individual particle energies in this nuclear 'gas'?

Problem 3 – The average separation of nucleons in a dense neutron star is about 0.002 Fermis. The average separation of the protons and neutrons in a nucleus of uranium is about 1.3 Fermis. What is the ratio of the energy of the uranium nucleons to the neutron star nucleons?

Problem 1 – Graph the function W(x) over the domain x:[0,2.0]



Problem 2 – Answer: As nucleons are compressed to higher densities, their energies become lower. This is the opposite effect that you get if you compress ordinary gas made from atoms, which become more energetic (hotter) as the gas is compressed to smaller volumes!

Problem 3 – Answer: Neutron star matter:

$W(0.002) = 20\ (0.002)^2 - (0.002)^3\ (40 - (0.002)^3/(1+3(0.002)))$ MeV
$W(0.002) = 0.00008$ MeV

Uranium nucleus:

$W(1.3) = 20\ (1.3)^2 - (1.3)^3\ (40 - (1.3)^3/(1+3(1.3)))$ MeV
$W(1.3) = 14.4$ MeV

Ratio: Uranium/Neutron Star = 14.4/0.00008 = **180,000 times**.

So, the nucleons inside compressed neutron star matter are nearly 200,000 times lower in energy. In other words, the more you compress a nucleon gas, the 'cooler' it gets!

The corona of the sun is easily seen during a total solar eclipse, such as the one in the image to the left photographed by John Walker in 2001.

The corona is produced by the hot outer atmosphere of the Sun and consists of atoms emitting light. Astronomers have developed mathematical models of the density of the corona that produces the same intensity as the real corona at different distances from the center of the sun. One such formula is as follows:

$$N(R) = \frac{10^8}{R^6}\left(1 + \frac{2}{R^{10}}\right)$$

where N is the number of atoms per cubic centimeter, and r is the distance from the sun in units of the solar radius (1 unit = 670,000 km; so that 'r = 2' means 2 x 670,000 km etc).

Problem 1 – For a model that spans the domain from 1 < R < 10, what is the corresponding range of N(R)?

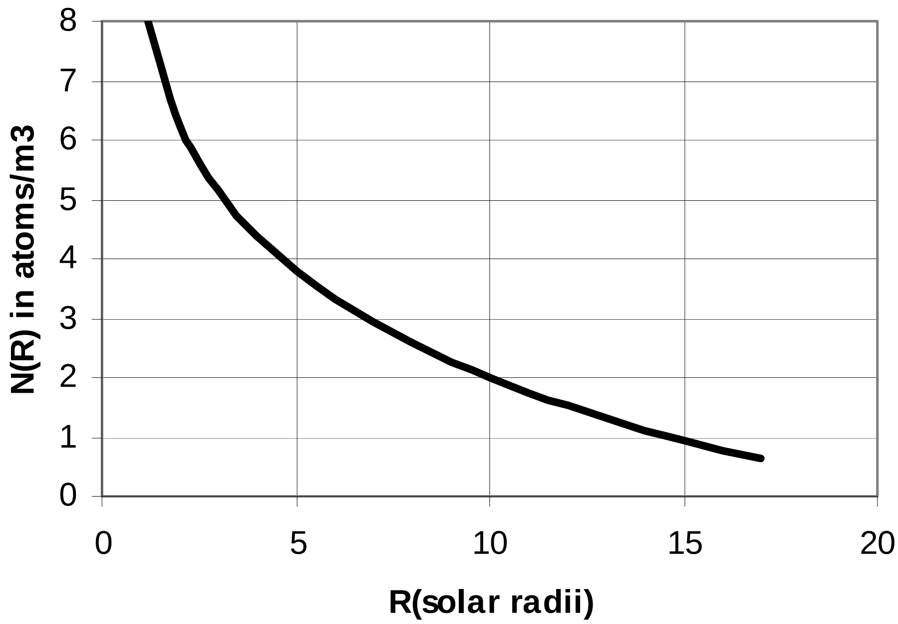
Problem 2 – Although it is easy to graph functions when the ranges are small, when the ranges are very large, as in the case of N(R), it is far easier to plot the logarithm of the function value N(R). This leads to a more readable graph. Graph the mathematical model for the solar coronal density over the domain r:[1.0, 10.0], however, use the graphing method of plotting log10(N(R)) vs R rather than N(R) vs R.

Problem 3 – Using the photograph as a clue, over what density range does the brightest portion of the corona correspond?

Problem 1 – For a model that spans the domain from 1 < R < 10, what is the corresponding range of N(R)?

Answer: N(1) = 3.0 x 10^8 atoms/cm^3 and N(10) = 100 atoms/cm^3, so the range is N:[300,000,000 , 100]

Problem 2 – Answer:

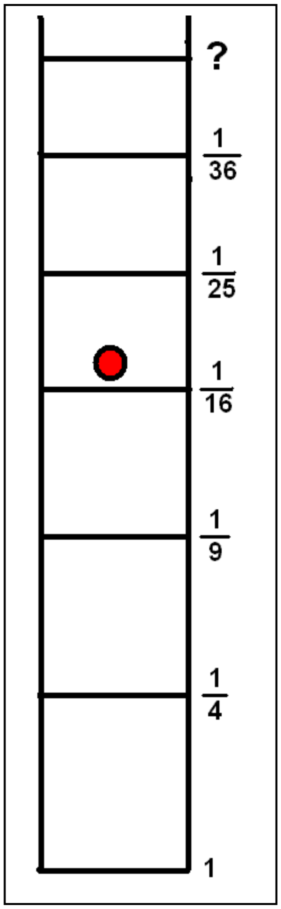


Problem 3 – Using the photograph as a clue, over what density range does the brightest portion of the corona correspond?

Answer: The photograph shows the bright 'white' zone extends from radial distances between r = 1.0 and r = 2.0 from the center of the sun. From the function, N(R), this corresponds to a density range

N(1) = 3.0 x 10^8 atoms/cm^3 to N(2) = 1.6 x 10^6 atoms/cm^3.

Adding and Subtracting Complex Fractions

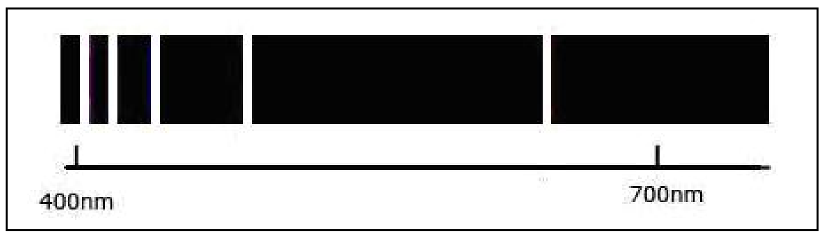


The single electron inside a hydrogen atom can exist in many different energy states. The lowest energy an electron can have is called the Ground State: this is the bottom rung on the ladder marked with an energy of '1'.

The electron must obey the Ladder Rule. This rule says that the electron can gain or lose only the exact amount of energy defined by the various ladder intervals.

For example, if it is located on the third rung of the ladder marked with an energy of '1/9', and it loses enough energy to reach the Ground State, it has to lose exactly 1 - 1/9 = 8/9 units of energy.

The energy that the electron loses is exactly equal to the energy of the light that it emits. This causes the spectrum of the atom to have a very interesting 'bar code' pattern when it is sorted by wavelength like a rainbow. The 'red line' is at a wavelength of 656 nanometers and is caused by an electron jumping from Energy Level 3 to Energy Level 2 on the ladder.



To answer these questions, use the Energy Fractions in the above ladder, and write your answer as the simplest fraction. Do not use a calculator or work with decimals because these answers will be less-exact than if you leave them in fraction form!

Problem 1 - To make the red line in the spectrum, how much energy did the electron have to lose on the energy ladder?

Problem 2 - How much energy will the electron have to gain (+) or lose (-) in making the jumps between the indicated rungs:
A) Level-2 to Level-5
B) Level-3 to Level-1
C) Level-6 to Level-4
D) Level-4 to Level-6
E) Level-2 to Level-4
F) Level-5 to Level-1
G) Level-6 to Level-5

Problem 3 - From the energy of the rungs in the hydrogen ladder, use the pattern of the energy levels (1, 1/4, 1/9, 1/16, 1/25, …) to predict the energy of the electron jumping from A) the 10th rung to the 7th rung; B) the rung M to the lower rung N.

Problem 4 - If an energy difference of '1' on the ladder equals an energy of 14 electron-Volts, in simplest fractional form, how many electron-Volts does the electron lose in jumping from Level-6 to Level-4?

Answer Key

Problem 1 - Answer: The information in the figure says that the electron jumped from Level-3 to Level-2. From the energy ladder, this equals a difference of 1/9 - 1/4. The common denominator is ' 4 x 9 = 36' so by cross-multiplying, the fractions become 4/36 - 9/36 and the difference is -5/36 .Because the answer is negative, the electron has to **lose 5/36** of a unit of energy to make the jump.

Problem 2 - How much energy will the electron have to gain (+) or lose (-) in making the jumps between the indicated rungs:
A) Level-2 to Level-5 = 1/4 - 1/25 = (25 - 4)/100 = **+21/100** so it has to GAIN energy.
B) Level-3 to Level-1 = 1/9 - 1 = 1/9 - 9/9 = **-8/9** so it has to LOSE energy.

C) Level-6 to Level-4 = 1/36 - 1/16 = **-5/144** so it has to LOSE energy
Two ways to solve:
First: (16 - 36) / (16x36) = -20 / 576 then simplify to get - 5 / 144
Second: Find Least Common Multiple
36: 36, 72, 108, **144**, 180, …
16: 16, 32, 48, 64, 80, 96, 112, 128, **144**, 160, …
LCM = 144, then
1/36 - 1/16 = 4/144 - 9/144 = **-5/144**

D) Level-4 to Level-6 = 1/16 - 1/36 = **+5/144** so it has to Gain energy.
E) Level-2 to Level-4 = 1/4 - 1/16 = 4/16 - 1/16 **= +3/16** so it has to GAIN energy
F) Level-5 to Level-1 = 1/25 - 1 = 1/25 - 25/25 = **-24/25** so it has to LOSE energy
G) Level-6 to Level-5 = 1/36 - 1/25 = (25 - 36)/ 900 = **- 11/900** so it has to LOSE energy

Problem 3 - Answer: Students should be able to see the pattern from the series progression such that the energy is the reciprocal of the square of the ladder rung number.

Level 2 Energy = $1/(2)^2 = 1/4$
Level 5 Energy = $1/(5)^2 = 1/25$

A) the 10th rung to the 7th rung: Energy = 1/100 - 1/49 = (49 - 100)/4900 = **-51/4900**.

B) the rung M to the lower rung N. Energy = $1/M^2 - 1/N^2$

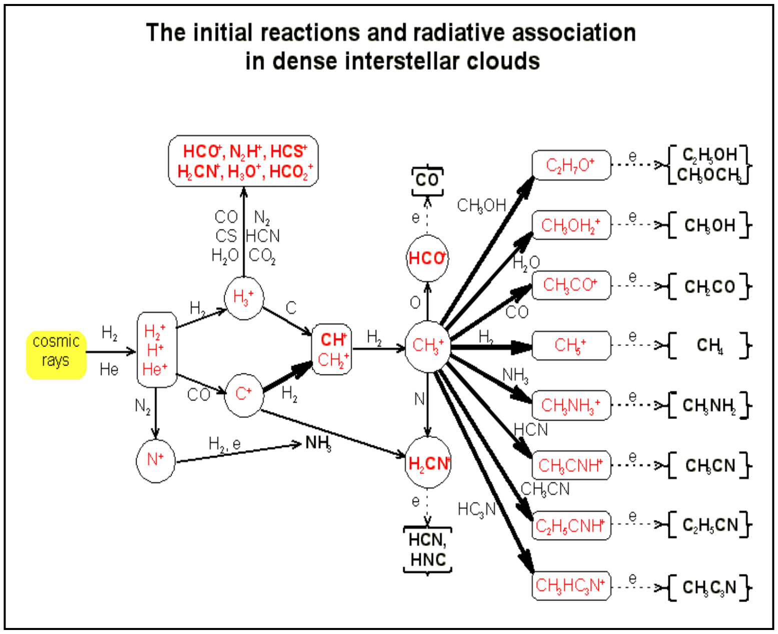
Problem 4 - If an energy difference of '1' on the ladder equals an energy of 13.6 electron-Volts, in simplest fractional form how many electron-Volts does the electron lose in jumping from Level-6 to Level-4?

Answer; The energy difference would be 1/36 - 1/16 = **-5/144** energy units.

Since an energy difference of 1.0 equals 14 electron-Volts, by setting up a ratio we have:

$$\frac{5/144 \text{ Units}}{1 \text{ Unit}} = \frac{X}{14 \text{ eV}} \quad \text{so} \quad X = 14 \times (5/144) = \frac{5 \times 2 \times 7}{2 \times 72} = \mathbf{\frac{35}{72}\ eV}$$

Adding and Subtracting Complex Fractions 9.5.2



Because molecules and atoms come in 'integer' packages, the rations of various molecules or atoms in a compound are often expressible in simple fractions. Adding compounds together can often lead to interesting mixtures in which the proportions of the various molecules involve mixed numbers.

The figure shows some of the ways in which molecules are synthesized in interstellar clouds. (Courtesy D. Smith and P. Spanel, "Ions in the Terrestrial Atmosphere and in Interstellar Clouds", *Mass Spectrometry Reviews*, v.14, pp. 255-278.)

In the problems below, do not use a calculator and state all answers as simple fractions or integers.

Problem 1 - <u>What makes your car go</u>: When 2 molecules of gasoline (ethane) are combined with 7 molecules of oxygen you get 4 molecules of carbon dioxide and 6 molecules of water.

A) What is the ratio of ethane molecules to water molecules?
B) What is the ratio of oxygen molecules to carbon dioxide molecules?
C) If you wanted to 'burn' 50 molecules of ethane, how many molecules of water result?
D) If you wanted to create 50 molecules of carbon dioxide, how many ethane molecules would you have to burn?

Problem 2 - <u>How plants create glucose from air and water:</u> Six molecules of carbon dioxide combine with 6 molecules of water to create one molecule of glucose and 6 molecules of oxygen.

A) What is the ratio of glucose molecules to water molecules?
B) What is the ratio of oxygen molecules to both carbon dioxide and water molecules?
C) If you wanted to create 120 glucose molecules, how many water molecules are needed?
D) If you had 100 molecules of carbon dioxide, what is the largest number of glucose molecules you could produce?

Problem 1 - What makes your car go: When 2 molecules of gasoline (ethane) are combined with 7 molecules of oxygen you get 4 molecules of carbon dioxide and 6 molecules of water.

A) In this reaction, 2 molecules of ethane yield 6 molecules of water, so the ratio is 2/6 or 1/3.
B) 7 oxygen molecules and 4 carbon dioxide molecules yield the ratio **7/4**

C) The reaction says that 2 molecules of ethane burn to make 6 molecules of water. If you start with 50 molecules of ethane, then you have the proportion:

$$\frac{\text{50 ethane}}{\text{2 ethane}} = \frac{\text{x-water}}{\text{6 -water}} \quad \text{so} \quad X = 25 \times 6 = \textbf{150 water molecules.}$$

D) Use the proportion:

$$\frac{\text{50 Carbon Dioxide}}{\text{4 carbon dioxide}} = \frac{\text{X ethane}}{\text{2 ethane}} \quad \text{so} \quad X = 2 \times (50/4) = \textbf{25 molecules ethane}$$

Problem 2 - How plants create glucose from air and water: Six molecules of carbon dioxide combine with 6 molecules of water to create one molecule of glucose and 6 molecules of oxygen.
A) What is the ratio of glucose molecules to water molecules?
B) What is the ratio of oxygen molecules to both carbon dioxide and water molecules?
C) If you wanted to create 120 glucose molecules, how many water molecules are needed?
D) If you had 100 molecules of carbon dioxide, what is the largest number of glucose molecules you could produce?

A) Glucose molecules /water molecules = **1 / 6**

B) Oxygen molecules / (carbon dioxide + water) = 6 / (6 + 6) = 6/12 = **1/2**

C)

$$\frac{\text{120 glucose}}{\text{1 glucose}} = \frac{\text{X water}}{\text{6 water}} \quad \text{so} \quad X = 6 \times 120 = \textbf{720 water molecules}$$

D)

$$\frac{\text{100 carbon dioxide}}{\text{6 carbon dioxide}} = \frac{\text{X glucose}}{\text{1 glucose}} \quad \text{so} \quad X = 100/6 \text{ molecules.}$$

The problem asks for the largest number that can be made, so we cannot include fractions in the answer. We need to find the largest multiple of '6' that does not exceed '100'. This is 96 so that 6 x 16 = 96. That means we can get no more than **16 glucose molecules** by starting with 100 carbon dioxide molecules. (Note that 100/6 = 16.666 so '16' is the largest integer).

Adding and Subtracting Complex Fractions

5 B	6 C	7 N	8 O	9 F	10 Ne
13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
81 Ti	82 Pb	83 Bi	84 Po	85 At	86 Rn

The Atomic Number, **Z**, of an element is the number of protons within the nucleus of the element's atom. This leads to some interesting arithmetic!

A portion of the Periodic Table of the elements is shown to the left with the symbols and atomic numbers for each element indicated in each square.

Problem 1 - Which element has an atomic number that is 5 1/3 larger than Carbon (C)?

Problem 2 - Which element has an atomic number that is 5 2/5 of Neon (Ne)?

Problem 3 - Which element has an atomic number that is 8/9 of Krypton (Kr)?

Problem 4 - Which element has an atomic number that is 2/5 of Astatine (At)?

Problem 5 - Which element has an atomic number that is 5 1/8 of Sulfur (S)?

Problem 6 - Which element has an atomic number that is 3 2/3 of Fluorine (F)?

Problem 7 - Which element in the table has an atomic number that is both an even multiple of the atomic number of carbon, an even multiple of the element magnesium (Mg) which has an atomic number of 12, and has an atomic number less than Iodine (I)?

Answer Key

Problem 1 - Which element has an atomic number that is 5 1/3 larger than Carbon (C)? **Answer: Carbon = 6 so the element is 6 x 5 1/3 = 6 x 16/3 = 96/3 = 32 so Z=32 and the element symbol is Ge (Germanium).**

Problem 2 - Which element has an atomic number that is 5 2/5 of Neon (Ne)? **Answer: Neon = 10, so 10 x 5 2/5 = 10 x 27/5 = 270/5 = 54, so Z=54 and the element is Xe (Xenon).**

Problem 3 - Which element has an atomic number that is 8/9 of Krypton (Kr)? **Answer: Krypton=36 so 36 x 8/9 = 288/9 = 32, so Z=32 and the element is Ge (Germanium).**

Problem 4 - Which element has an atomic number that is 2/5 of Astatine (At)? **Answer; Astatine=85 so 85 x 2/5 = 170/5 = 34, so Z=34 and the element is Se (Selenium).**

Problem 5 - Which element has an atomic number that is 5 1/8 of Sulfur (S)? **Answer; Sulfur = 16 so 16 x 5 1/8 = 16 x 41/8 = 82, so Z=82 and the element is Lead (Pb).**

Problem 6 - Which element has an atomic number that is 3 2/3 of Fluorine (F)? **Answer: Fluorine = 9 so 9 x 3 2/3 = 9 x 11/3 = 99/3 = 33, so Z=33 and the element is As (Arsenic).**

Problem 7 - Which element in the table has an atomic number that is both an even multiple of the atomic number of carbon, an even multiple of the element magnesium (Mg) which has an atomic number of 12, and has an atomic number less than Iodine (I)?

Answer: The first relationship gives the possibilities: 6, 18, 36, 54. The second clue gives the possibilities 36 and 84. The third clue says Z has to be less than I = 53, so the element must have Z = 36, which is Krypton.

Adding and Subtracting Complex Fractions

Our Milky Way galaxy is not alone in the universe, but has many neighbors.

The distances between galaxies in the universe are so large that astronomers use the unit 'megaparsec' (mpc) to describe distances.

One *mpc* is about 3 1/4 million light years.

Hubble picture of a Ring Galaxy (AM 0644 741) at a distance of 92 mpc.

Problem 1 - The Andromeda Galaxy is 3/4 mpc from the Milky Way, while the Sombrero Galaxy is 12 mpc from the Milky Way. How much further is the Sombrero Galaxy from the Milky Way?

Problem 2 -The Pinwheel Galaxy is 3 4/5 mpc from the Milky Way. How far is it from the Sombrero Galaxy?

Problem 3 - The Virgo Galaxy Cluster is 19 mpc from the Milky Way. About how far is it from the Pinwheel Galaxy?

Problem 4 - The galaxy Messier 81 is located 3 1/5 mpc from the Milky Way. How far is it from the Andromeda Galaxy?

Problem 5 - The galaxy Centaurus-A is 4 2/5 mpc from the Milky Way. How far is it from the Andromeda Galaxy?

Problem 6 - The galaxy Messier 63 is located about 4 1/5 mpc from the Milky Way. How far is it from the Pinwheel galaxy?

Problem 7 - The galaxy NGC-55 is located 2 1/3 mpc from the Milky Way. How far is it from the Andromeda galaxy?

Problem 8 - In the previous problems, which galaxy is 2 1/15 mpc further from the Milky Way than NGC-55?

Extra for Experts: How far, in light years, is the Virgo Galaxy Cluster from the Milky Way?

Answer Key

Problem 1 - The Andromeda Galaxy is 3/4 mpc from the Milky Way, while the Sombrero Galaxy is 12 mpc from the Milky Way. How much further is the Sombrero Galaxy from the Milky Way? Answer: 12 mpc - 3/4 mpc = **11 1/4 mpc**

Problem 2 -The Pinwheel Galaxy is 3 4/5 mpc from the Milky Way. How far is it from the Sombrero Galaxy? Answer: 12 mpc - 3 4/5 mpc = **8 1/5 mpc**

Problem 3 - The Virgo Galaxy Cluster is 19 mpc from the Milky Way. About how far is it from the Pinwheel Galaxy? Answer: 19 mpc - 3 4/5 mpc = **15 1/5 mpc.**

Problem 4 - The galaxy Messier 81 is located 3 1/5 mpc from the Milky Way. How far is it from the Andromeda Galaxy? Answer: 3 1/5 mpc - 3/4 mpc = 16/5 mpc - 3/4 mpc = 64/20 mpc - 15/20 mpc = 49/20 mpc = **2 9/20 mpc.**

Problem 5 - The galaxy Centaurus-A is 4 2/5 mpc from the Milky Way. How far is it from the Andromeda Galaxy? Answer: 4 2/5 mpc - 3/4 mpc = 88/5 mpc - 15/20 mpc = 73/20 mpc = **3 13/20 mpc**

Problem 6 - The galaxy Messier 63 is located about 4 1/5 mpc from the Milky Way. How far is it from the Pinwheel galaxy? Answer: 4 1/5 mpc - 3 4/5 mpc = 21/5 mpc - 19/5 mpc = **2/5 mpc.**

Problem 7 - The galaxy NGC-55 is located 2 1/3 mpc from the Milky Way. How far is it from the Andromeda galaxy? Answer: 2 1/3 mpc - 3/4 mpc = 7/3 mpc - 3/4 mpc = 28/12 mpc - 9/12 mpc = 19/12 mpc = **1 7/12 mpc.**

Problem 8 - In the previous problems, which galaxy is 2 1/15 mpc further from the Milky Way than NGC-55? Answer; NGC-55 is located 2 1/3 mpc from the Milky Way, so the mystery galaxy is located 2 1/3 mpc + 2 1/15 mpc = 7/3 mpc + 31/15 mpc = 35/15 mpc + 31/15 mpc = 66/15 mpc = 4 6/15 mpc or **4 2/5 mpc.** This is the distance to the Centaurus-A galaxy.

Extra for Experts: How far, in light years, is the Virgo Galaxy Cluster from the Milky Way?
The distance is 19 megaparsecs, but 1 parsec equals 3 1/4 light years, so the distance to the Virgo Cluster is

19 million parsecs x (3 1/4 lightyears/parsec) = 19 x 3 1/4 = 19 x 12/4 = 228/4 **= 57 million light years**