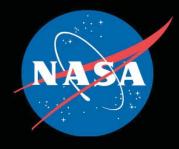
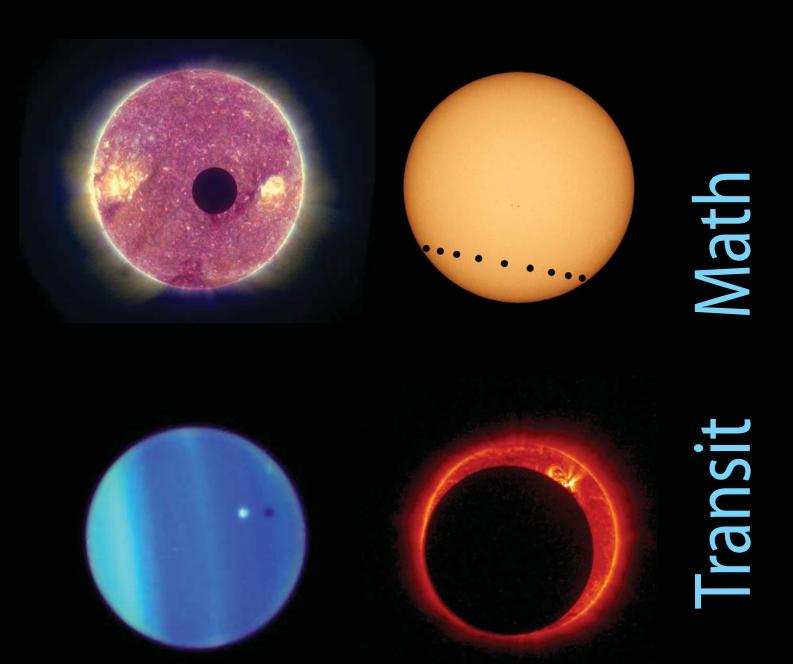
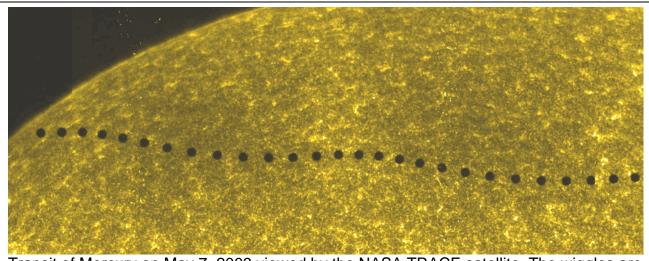
# National Aeronautics and Space Administration





# Transit Math



Transit of Mercury on May 7, 2003 viewed by the NASA TRACE satellite. The wiggles are caused by the satellite's polar orbit around Earth which causes the perspective to change in a north-south direction.

# Mathematical problems featuring transit applications

### Dr. Sten Odenwald NASA / ADNET Corp. Sten.F.Odenwald@nasa.gov

This collection of activities is based on a weekly series of space science problems intended for students looking for additional challenges in the math and physical science curriculum in grades 6 through 12. The problems were created to be authentic glimpses of modern science and engineering issues, often involving actual research data.

The problems were designed to be 'one-pagers' with a detailed Answer Key as a second page. This compact form was deemed very popular by participating teachers.

#### Acknowledgments:

We would like to thank the many teachers that support Space Math @ NASA for their careful reading of this resource for accuracy, and for many valuable comments.

Ms. Elaine Lewis, coordinator of NASA's Sun-Earth Day Transit of Venus event in 2012, was instrumental in writing the 'How to use this resource' essay to help teachers better integrate the problems into classroom situations.

### **Image Credits:**

Front Cover Top) Transit of Venus, June 8, 2004 as seen by NASA TRACE satellite; Middle) NASA, Sun-Earth Day Transit of Venus poster for 2004; Bottom) Transit of Mercury as seen by NASA SOHO satellite in 2006. Back Cover: Transit of Venus, 2004, courtesy Fred Espenak; Transit of the Moon across sun, STEREO, February 25, 2007; Exoplanet Transit, courtesy Lynette Cook; Transit of Phobos across sun, NASA Opportunity Rover; Transit of Ariel across Uranus, by Hubble Space Telescope

For more weekly classroom activities about astronomy and space visit the Space Math@ NASA website, http://spacemath.gsfc.nasa.gov

Created under an Education and Public Outreach grant, NNH08CD59C administered through the NASA Science Mission Directorate.



### **Additional Titles in this Series**

**Black Holes** (2008) 11 Problems An introduction to the basic properties of black holes using elementary algebra and geometry. Students calculate black hole sizes from their mass, time and space distortion, and explore the impact that black holes have upon their surroundings.

**Image Scaling** (2008) 11 Problems Students work with a number of NASA photographs of planets, stars and galaxies to determine the scales of the images, and to examine the sizes of various features within the photographs using simple ratios and proportions.

**Radiation** (2007) 19 Problems An introduction to radiation measurement, dosimetry and how your lifestyle affects how much radiation your body absorbs.

**Solar Math** (2008) 15 Problems Exploring solar storms and solar structure using simple math activities. Calculating speeds of solar flares from photographs, and investigating solar magnetism.

**Lunar Math** (2008) 17 Problems An exploration of the moon using NASA photographs and scaling activities. Mathematical modeling of the lunar interior, and problems involving estimating its total mass and the mass of its atmosphere.

**Magnetic Math** (2009) 37 Problems Six hands-on exercises, plus 37 math problems, allow students to explore magnetism and magnetic fields, both through drawing and geometric construction, and by using simple algebra to quantitatively examine magnetic forces, energy, and magnetic field lines and their mathematical structure.

**Earth Math** (2009) 46 Problems Students explore the simple mathematics behind global climate change through analyzing graphical data, data from NASA satellites, and by performing simple calculations of carbon usage using home electric bills and national and international energy consumption.

**Electromagnetic Math** (Draft:2010) 84 Problems Students explore the simple mathematics behind light and other forms of electromagnetic energy including the properties of waves, wavelength, frequency, the Doppler shift, and the various ways that astronomers image the universe across the electromagnetic spectrum to learn more about the properties of matter and its movement.

**Space Weather Math** (Draft:2010) 96 Problems Students explore the way in which the sun interacts with Earth to produce space weather, and the ways in which astronomers study solar storms to predict when adverse conditions may pose a hazard for satellites and human operation in space. Six appendices and an extensive provide a rich 150-year context for why space whether is an important issue.

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# Alignment with Standards

#### AAAS Project:2061 Benchmarks

**(9-12)** - Mathematical modeling aids in technological design by simulating how a proposed system might behave.

2B/H1 ---- Mathematics provides a precise language to describe objects and events and the relationships among them. In addition, mathematics provides tools for solving problems, analyzing data, and making logical arguments.

2B/H3 ----- Much of the work of mathematicians involves a modeling cycle, consisting of three steps: (1) using abstractions to represent things or ideas, (2) manipulating the abstractions according to some logical rules, and (3) checking how well the results match the original things or ideas. The actual thinking need not follow this order. 2C/H2

Topic								Ρ	ro	bl	en	n		N	un	nb		S													
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Linear Equations																											X	X			
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# **Mathematics Topic Matrix**

Торіс											Ρ	rn	bl	٥n	n		N	un	h		'C										
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Inquiry																															
Technology, rulers					X																										
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Time, distance,				X		X	X	X	X	X	Χ	Х	X																		
speed Areas and																															
volumes																															
Scale drawings				X	X	X		X																							
Geometry		Χ	X	X	X	X	X	X	X	X	Χ	Х	X																		
Probability, odds		_		_		_		_	_	_	_																		_		
Scientific Notation																															
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Fractions																															
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Pie Graphs																															
Linear Equations																															
Rates & Slopes																															
Solving for X							X																								
Evaluating Fns	X					X	X																								
Modeling	X	Χ	Х																												
Trigonometry																															
Logarithms																															
Calculus																															
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# Mathematics Topic Matrix (Cont'd)

### How to Use this Resource

The Introduction of **Transit Math** is an excellent resource for teachers in grades 5-8. The introduction clearly explains eclipses, transits and occultations. The identification activity is a great way for students to test their own knowledge prior to reading the introduction section and again after reading the section. High School students would need to read the introduction as a way to re-visit the content.

Ms. Smith took advantage of the coming Transit of Venus to make a history, science and mathematics connection for her middle school students. She divided her students into groups based on the time frames in the selected readings from **Transit Math**:

- Ancient History to 1882 AD,
- The Modern Era of Transit Observations,
- Transit of Venus December 4, 1639,
- Transit of Venus June 5, 1761,
- Transit of Venus June 3, 1769,
- Distance to the Sun,
- Transit of Venus December 8, 1874,
- Transit of Venus December 6, 1882.

Each team will present a summary report from their historical selection, the students can also use the resources included from the Library of Congress to enhance their presentation. The students will develop a flier or news article (there are articles from the past newspapers in Appendix C to serve as a guide) to advertise the coming Transit of Venus on June 5-6, 2012. In their media presentation, the students should include what a transit is and why the Transit of Venus is a rare event. **Transit Math** provides math challenges for grades 5-12, Ms Smith selects the ones that work for the level of her middle school students and challenges them, as they learn about the transit and prepare for the final transit of Venus in our lifetime. Ms. Smith plans to use the geometric concept of parallax so her students can understand how scientists determine distances to the planets.

If you missed it in 2004 you have 1 last chance! NASA's *Sun-Earth Day* will host a live webcast from Mauna Kea Hawaii, to view the entire event For details visit their website at (<u>http://sunearthday.nasa.gov</u>

# What are eclipses, transits and occultations?

Although planets, stars and other celestial bodies move through space in complicated ways, space is so vast that rarely do such bodies collide. However, when you watch these movements from a distant vantage point, sometimes it looks as though collisions occur because of your perspective.

<u>Eclipses.</u> One of the most common 'collisions' that you have probably heard about is called an eclipse. To create an eclipse, from your vantage point the angular sizes of two bodies have to be exactly equal. An example is the diameter of the sun and moon during a total solar eclipse.



Total solar eclipses have been watched by humans for thousands of years. Mostly with a sense of dread and fear. who would not be a little nervous to see the sun suddenly disappear into a black 'hole' as though eaten by some evil space creature! Even today, total solar eclipses are highly anticipated, and frequently get millions of people to travel to just he right spots on Earth to see the 5-minute spectacle. The most dramatic 'totalities' occur when the sun's outer atmosphere, the corona, is active.



If the diameters are just slightly different and the moon is slightly smaller in angular size than the sun as viewed from Earth, you can still see a small part of the sun's surface even when both the sun and moon are exactly 'on top' of each other. This is called an annular eclipse. Even though barely 5% of the solar surface remains exposed, the solar surface is so bright that you may not even notice the slight diminution of sunlight when it happens.

For thousands of years, astronomers have been able to predict when solar eclipses will happen. The table below shows the solar eclipses that will be visible from Earth through 2016.

Date	Туре	Date	Туре
July 11, 2010	Total	August 21, 2017	Total
May 20, 2012	Annular	July 2, 2019	Total
November 13, 2012	Total	December 26, 2019	Annular
May 10, 2013	Annular	June 21, 2020	Annular
April 29, 2014	Annular	December 14, 2020	Total
March 9, 2016	Total	June 10, 2021	Annular
September 1, 2016	Annular	December 4, 2021	Total

#### **Upcoming Solar Eclipses**

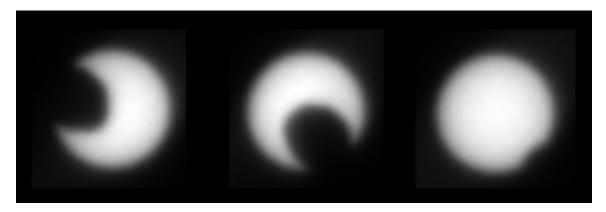
<u>Transits.</u> When the nearby body has an angular size <u>much smaller</u> than the more distant body, the encounter seen from your vantage point is called a transit. There are many more transits to be seen each year than solar eclipses. The most common transits seen each month occur when the satellites of Jupiter or Saturn pass across the apparent face of their planet as viewed from Earth.



This image was taken by the Cassini spacecraft and shows the satellite lo passing across the face of Jupiter. The satellite can be seen at the center of the image, and its black shadow is to the right. As seen from the cloud-tops of Jupiter, the angular diameter of the sun is the same as the angular diameter of the satellite lo, so under the shadow of lo, an observer on Jupiter would see a total solar eclipse.

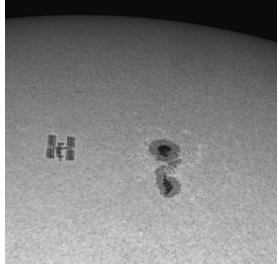
As seen from Earth, we are watching the small disk of Io pass across the much larger disk of Jupiter, so this would be called a transit!

Sometimes we can see transits from an entirely different perspective than Earth. For example in the sequence of three images below, the satellite of Mars, Phobos was photographed by the Mars Rover Opportunity, passing across the face of the sun as viewed from the surface of Mars on March 13, 2004.



There are also occasions when Mercury and Venus pass across the face of the sun as viewed from Earth, although these events are rare. The sequence of images below was taken by the SOHO spacecraft of the transit of Mercury on November 8, 2006.





Amateur photographers have also been very ingenious in catching transits of man-made objects across the face of the sun and moon. The image to the left shows the International Space Station in orbit around Earth as it passed across the face of the sun, which contained two sunspots: each of which are the same physical size as Earth!

Photographer John Stetson had to carefully prepare for this event by knowing exactly the path of the ISS and the sun through the sky. On March 3, 2010, the transit took less than one second so hundreds of consecutive

photographs had to be taken by a camera in order to capture just a few with the ISS in the field of view!

The table below gives the transits of Mercury and Venus during the 21st Century. Notice that the Transits of Venus happen in June (also December) while the transits of Mercury happen in November and May.

Table of upcoming planetally transits										
Transit	Date									
Mercury	May 7, 2003									
Venus	June 8, 2004									
Mercury	November 8, 2006									
Venus	June 5, 2012									
Mercury	May 9, 2016									
Mercury	November 11, 2019									
Mercury	November 13, 2032									
Mercury	November 7, 2039									
Mercury	May 7, 2049									
Mercury	November 9, 2052									
Mercury	May 10, 2062									
Mercury	November 11, 2065									

#### Table of upcoming planetary transits

<u>Occultations.</u> A phenomenon related to a transit is called an occultation. These occur when one body, such as the moon or a planet, passes in front of a more distant star or other object of interest such as an asteroid or even another planet! The moon frequently 'occults' bright stars as it moves across the sky.



On a number of occasions, the moon passes in front of a more distant planet such as Mars, Jupiter or Saturn. As viewed from Earth, on occasion, the satellites of Jupiter can occult each other. This is called a mutual event!

The spectacular image to the left shows the occultation of Saturn by the Moon on March 2, 2007 captured by photographer Pete Lawrence using Celestron-14 telescope. <u>pete.lawrence@digitalsky.org.uk</u>

The table below gives a list of upcoming planetary occultations of interest.

Event	Date								
Mercury occults Theta Ophiuchi	December 4, 2015								
Venus occults Regulus	October 1, 2044								
Mercury occults Alpha Libra	November 10, 2052								
Venus occults Jupiter	November 22, 2065								
Mercury occults Neptune	July 15, 2067								
Mercury occults Mars	August 11, 2079								
Mercury occults Jupiter	October 27, 2088								
Mercury occults Jupiter	April 7, 2094								

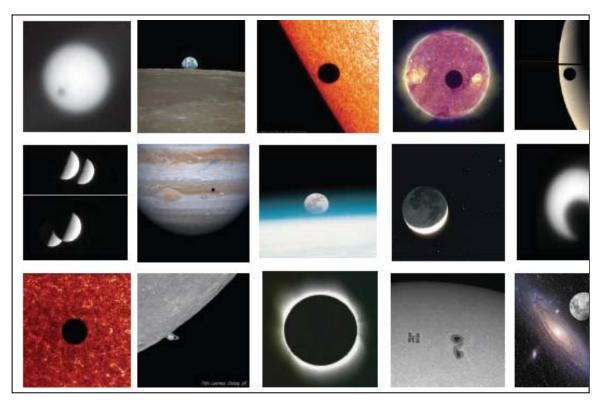
#### Table of planetary occultations

Lunar occultations are much more common. The moon constantly passes in front of thousands of stars every hour. Less frequently it passes in front of planets or asteroids. A short list of the brighter bodies is provided below for 2012:

Event	Date										
Moon occults Mercury	November 14										
Moon occults Pluto	March 16										
Moon occults Spica	July 25, August 21, Sept 18, Oct 15.										

#### Lunar occultations for 2012

# Eclipses, Transits and Occultations



The images above show a variety of transits, eclipses and occultations. The images are labeled from left to right as (Top Row) A, B, C, D, E; (Middle Row) F, G, H, I, J, (Bottom Row), K, L, M, N, O. Using the definitions of these three astronomical events, identify which images go along with each of the three types of events. One example is shown below.

- A) Deimos and the Sun \_\_\_\_\_
- B) Moon and Earth
- C) Sun and Mercury \_\_\_\_\_
- D) Sun and Moon <u>Transit</u>
- E) Rhea and Saturn \_\_\_\_\_
- F) Rhea and Dione
- G) Jupiter and Io
- H) Earth and Moon \_\_\_\_\_

- I) Moon and Star Cluster \_\_\_\_\_
- J) Sun and Phobos \_\_\_\_\_
- K) Sun and Venus
- L) Moon and Saturn
- M) Sun and Moon
- N) Sun and Space Station \_\_\_\_\_
- O) Moon and galaxy \_\_\_\_\_

- A) **<u>Transit</u>** of Deimos across the Sun seen by Opportunity Rover
- B) Occultation of Earth by the Moon seen by Apollo-8 astronauts
- C) Transit of Mercury across sun seen by TRACE?
- D) Transit of Moon across Sun seen by STEREO satellite
- E) Transit of satellite Rhea across Saturn seen by Cassini spacecraft
- F) Rhea occulting Dione seen by Cassini spacecraft near Saturn
- G) Io transiting Jupiter seen by Galileo spacecraft
- H) Earth occulting moon seen by Space Shuttle astronauts
- I) Moon occulting the Pleiades star cluster
- J) Phobos transiting sun seen by Opportunity Rover on Mars.
- K) Venus transiting the sun seen by TRACE satellite.
- L) Moon occulting Saturn
- M) <u>Eclipse</u> of the sun by the moon.
- N) Transit of the space station across sun
- O) Hypothetical occultation of the andromeda galaxy by the moon.

#### Special Image credits:

Moon occulting Saturn - Pete Lawrence (http://www.digitalskyart.com/) pete.lawrence@pbl33.co.uk.

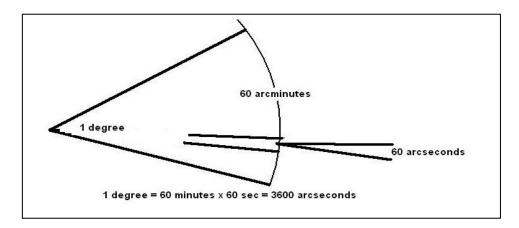
Space Station transiting sun - John Stetson

Moon and Andromeda - Adam Block (ngc1535@caelumobservatory.com - Tim Puckett (<u>tpuckett@mindspring.com</u>)

Moon and Star Cluster - Jerry Lodriguss (jerry5@astropix.com) http://www.astropix.com/

### Angular Measure in Astronomy

The easiest, and most basic, unit of measure in astronomy is the angular degree. Because the distances to objects in the sky are not directly measurable, a photograph of the sky will only indicate how large, or far apart, objects are in terms of degrees, or fractions of degrees. It is a basic fact in angle measurment in geometry, that 1 angular degree (or arc-degree) can be split into 60 arc-minutes of angle, and that 1 arc-minute equals 60 arc-seconds. A full degree is then equal to  $60 \times 60 = 3,600$  'arcseconds'. High-precision astronomy also uses the unit of milliarcsecond to represent angles as small as 0.001 arcseconds and microarcseconds to equal 0.000001 arcseconds.



**Problem 1** – The moon has a diameter of 0.5 degrees (a physical size of 3,474 km) A telescope sees a crater 1 arcsecond across. What is its diameter in meters?

**Problem 2** – A photograph has an image scale of 10 arcseconds/pixel. If the image has a size of  $512 \times 512$  pixels, what is the image field-of-view in degrees?

**Problem 3** – An astronomer wants to photograph the Orion Nebula (M-42) with an electronic camera with a CCD format of 4096x4096 pixels. If the nebula has a diameter of 85 arcminutes. What is the resolution of the camera in arcseconds/pixel when the nebula fills the entire field-of-view?

**Problem 4** – An electronic camera is used to photograph the Whirlpool Galaxy, M-51, which has a diameter of 11.2 arcminutes. The image will have 1024x1024 pixels. What is the resolution of the camera, in arcseconds/pixel, when the galaxy fills the entire field-of-view?

**Problem 5** – The angular diameter of Mars from Earth is about 25 arcseconds. This corresponds to a linear size of 6,800 km. The Mars Reconnaissance Orbiter's HiRISE camera, in orbit around Mars, can see details as small as 1 meter. What is the angular resolution of the camera in microarcseconds as viewed from Earth?

**Problem 6** – The Hubble Space Telescope can resolve details as small as 46 milliarcseconds. At the distance of the Moon, how large a crater could it resolve, in meters?

Space Math

Problem 1 – Answer: 0.5 degrees x 3600 arcsec/degree = 1800 arcseconds. Using proportions 1/1800 = x/3474 so X = 3474/1800 = 1.9 kilometers.

Problem 2 –Answer: 512 pixels x 10 arcsec/pixel x 1 degree/3600 arcseconds = 5120 arcseconds /3600 = 1.4 degrees, so the image is **1.4 x 1.4 degrees**.

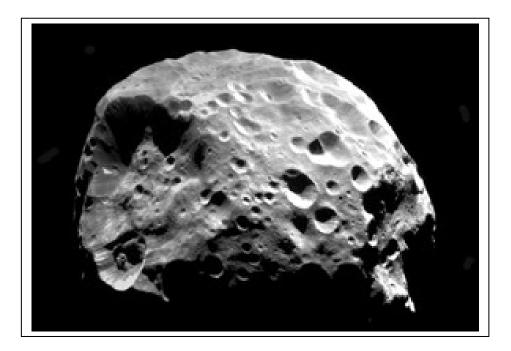
Problem 3 –Answer: 85 arcminutes x 60  $\operatorname{arcsec/arcmin} = 5,100 \operatorname{arcseconds}$ . This corresponds to 4096 pixels so the scale is 5,100  $\operatorname{arcsec/4096}$  pixels = **1.2**  $\operatorname{arcsec/pixel}$ .

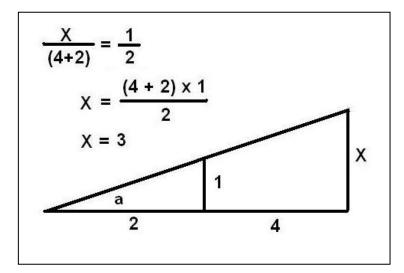
Problem 4 –Answer: 11.2 arcminutes x 60 arcsec/arcmin = 672 arcsec. This equals 1024 opixels so the scale is 672/1024 = 0.656 arcsec/pixel.

Problem 5 –Answer: 25 arcsec = 6800 km so 1 arcsec = 6800 km/25 = 272 km from Earth. For 1-meter resolution at Earth, the angular scale would have to be 1 sec x 1m/272000m = 0.0000037 arcseconds or **3.7 microarcseconds**.

Problem 6 – Answer: From Problem-1, 1 arcsecond = 1.9 kilometers. By proportions, 0.046 arcsec/1 arcsec = x/1.9 km so X = 0.046 x 1.9 km = **0.087** kilometers or 87 meters.

The picture below was taken by the Cassini spacecraft orbiting Saturn. It is of the satellite Phoebe, which from Earth subtends an angular size of about 32 milliarcsec. The smallest crater, about 1 km across, would subtend about 160 microarcseconds as seen from Earth.



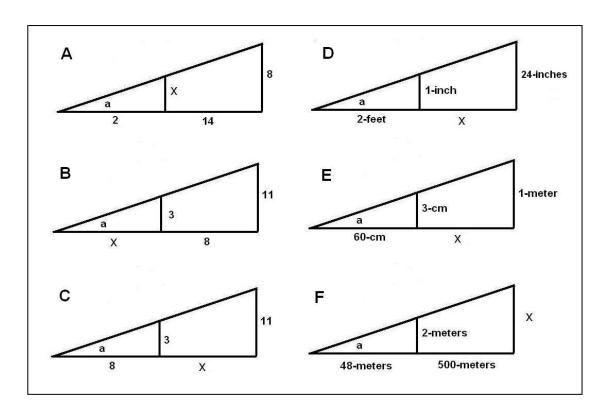


The corresponding sides of similar triangles are proportional to one another as the illustration to the left shows. Because the vertex angle of the triangles are identical in measure, two objects at different distances from the vertex will subtend the same angle, **a**. The corresponding side to 'X' is '1' and the corresponding side to '2' is the combined length of '2+4'.

**Problem 1:** Use the properties of similar triangles and the ratios of their sides to solve for 'X' in each of the diagrams below.

Problem 2: Which triangles must have the same measure for the indicated angle a ?

**Problem 3:** The sun is 400 times the diameter of the moon. Explain why they appear to have about the same angular size if the moon is at a distance of 384,000 kilometers, and the sun is 150 million kilometers from Earth?



**Problem 1:** Use the properties of similar triangles and the ratios of their sides to solve for 'X' in each of the diagrams below.

- A) X / 2 = 8 / 16 so X = 1
- B) 3/X = 11/(X+8) so 3(X+8) = 11X; 3X + 24 = 11X; 24 = 8X and so **X = 3**.
- C) 3/8 = 11/(x+8) so 3(x+8) = 88; 3X + 24 = 88; 3X = 64 and so **X = 21 1/3**
- D) 1-inch / 2-feet = 24 inches / (D + 2 feet); First convert all units to inches;
  1 / 24 = 24 / (D + 24); then solve (D + 24) = 24x24 so D = 576 24;
  D = 552 inches or 46 feet.
- E) 3 cm / 60 cm = 1 meter / (X + 60 cm). 3/60 = 1 meter / (X + 0.6 m) then 3(X + 0.60) = 60; 3X + 1.8 = 60; 3X = 58.2 meters so **X = 19.4 meters**.
- F) 2 meters / 48 meters = X / 548 meters ; 1/24 = X/548 ; X = 548 / 24 ; so X = 22.8.

Problem 2: Which triangles must have the same measure for the indicated angle a ?

Answer: Because the triangle (D) has the side proportion 1-inch /24-inches = 1/24 and triangle (F) has the side proportion 2 meters / 48 meters = 1/24 these two triangles, **D** and **F**, have the same angle measurement for angle a

**Problem 3:** The Sun is 400 times the diameter of the Moon. Explain why they appear to have the same angular size if the moon is at a distance of 384,000 kilometers, and the sun is 150 million kilometers from Earth?

Answer: From one of our similar triangles, the long vertical side would represent the diameter of the sun; the short vertical side would represent the diameter of the moon; the angle **a** is the same for both the sun and moon if the distance to the sun from Earth were 400x farther than the distance of the moon from Earth. Since the lunar distance is 384,000 kilometers, the sun must be at a distance of 154 million kilometers, which is close to the number given.

# Getting an Angle on the Sun and Moon



The Sun (Diameter = 696,000 km) and Moon (Diameter = 3,476 km) have very different physical diameters in kilometers, but in the sky they can appear to be nearly the same size. Astronomers use the angular measure of arcseconds (asec) to measure the apparent sizes of most astronomical objects. (1 degree equals 60 arcminutes, and 1 arcminute equals 60 arcseconds). The photos above show the Sun and Moon at a time when their angular diameters were both about 1,865 arcseconds.

**Problem 1** - Using a metric ruler, what is the angular scale of each image in arcseconds per millimeter?

**Problem 2** - In arcseconds, what is the size of the smallest feature you can see in the images of the Sun and Moon?

Problem 3 - About what is the area, in square arcseconds (asec<sup>2</sup>) of the circular Mare Serenitatis (A) region in the photo of the Moon?

**Problem 4** - At the distance of the Moon, 1 arcsecond of angular measure equals 1.9 kilometers. The Sun is exactly 400 times farther away than the Moon. On the photograph of the Sun, how many kilometers equals 1 arcsecond of angle?

Problem 5 - What is the area of Mare Serenitatis in square kilometers?

**Problem 6** - What would be the physical area, in square-kilometers, of an identical angular area to Mare Serenitatis if it were located on the surface of the sun?

**Problem 1** - Using a metric ruler, what is the angular scale of each image in arcseconds per millimeter? Answer: Moon diameter = 65 mm and sun diameter = 61 mm so the lunar image scale is 1,865 asec/65mm = 28.7 asec/mm and the solar scale is 1865 asec/61 mm = 30.6 asec/mm.

**Problem 2** - In arcseconds, what is the size of the smallest feature you can see in the images of the Sun and Moon? Answer: the smallest feature is about 0.5 mm or  $0.5 \times 28.7$  asec/mm = **14.4 asec for the Moon** and  $0.5 \times 30.6$  asec/mm = **15.3 asec for the Sun.** 

**Problem 3** - About what is the area, in square arcseconds (asec<sup>2</sup>) of the circular Mare Serenitatis (A) region in the photo of the Moon? Answer: The diameter of the mare is 1 centimeter, so the radius is 5 mm or 5 mm x 28.7 asec/mm = 143.5 asec. Assuming a circle, the area is  $A = \pi x (143.5 \text{ asec})^2 = 64,700 \text{ asec}^2$ .

**Problem 4** - At the distance of the Moon, 1 arcsecond of angular measure equals 1.9 kilometers. The Sun is exactly 400 times farther away than the Moon. On the photograph of the Sun, how many kilometers equals 1 arcsecond of angle? Answer: The angular scale at the sun would correspond to  $400 \times 1.9 \text{ km} = 760 \text{ kilometers per arcsecond}$ .

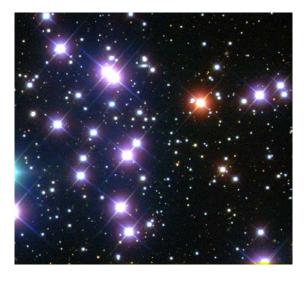
**Problem 5** - What is the area of Mare Serenitatis in square kilometers? Answer: We have to convert from square arcseconds to square kilometers using a two-step unit conversion 'ladder'.

 $64,700 \operatorname{asec}^2 x (1.9 \text{ km/asec}) x (1.9 \text{ km/asec}) = 234,000 \text{ km}^2$ .

**Problem 6** - What would be the physical area, in square-kilometers, of an identical angular area to Mare Serenitatis if it were located on the surface of the sun? Answer: The angular area is 400-times further away, so we have to use the scaling of 760 kilometers/asec deduced in Problem 4. The unit conversion for the solar area becomes:

64,700 asec<sup>2</sup> x (760 km/asec) x (760 km/asec) = 
$$37,400,000,000 \text{ km}^2$$
.





Although many astronomical objects may have the same angular size, most are at vastly different distance from Earth, so their actual sizes are very different. If your friends were standing 200 meters away from you, they would appear very small, even though they are as big as you are!

The pictures show the Moon (d = 384,000 km) and the star cluster Messier-34 (d = 1,400 light years). The star cluster photo was taken by the Sloan Digital Sky Survey, and although the cluster appears the same size as the Moon in the sky, its stars are vastly further apart than the diameter of the Moon!

#### In the problems below, round all answers to one significant figure.

**Problem 1** - The images are copied to the same scale. Use a metric ruler to measure the diameter of the Moon in millimeters. If the diameter of the moon is 1,900 arcseconds, what is the scale of the images in arcseconds per millimeter?

**Problem 2** - The relationship between angular size,  $\Theta$ , and actual size, L, and distance, D, is given by the formula:

Θ L = ----- D 206,265

Where  $\Theta$  is measured in arcseconds, and L and D are both given in the same units of length or distance (e.g. meters, kilometers, light years). A) In the image of the Moon, what does 1 arcsecond correspond to in kilometers? B) In the image of M-34, what does 1 arcsecond correspond to in light years?

**Problem 3** - What is the smallest detail you can see in the Moon image in A) arcseconds? B) kilometers?

**Problem 4** - What is the smallest star separation you can measure in Messier-34 in among the brightest stars in A) arcseconds? B) Light years?

Space Math

http://spacemath.gsfc.nasa.gov

**Problem 1** - The images are copied to the same scale. Use a metric ruler to measure the diameter of the Moon in millimeters. If the diameter of the moon is 1,900 arcseconds, what is the scale of the images in arcseconds per millimeter? Answer: The diameter of the Moon is about 64 millimeters, and since this corresponds to 1,900 arcseconds, the scale is 1,900 asec/64 mm = 29.68 or **30 asec/mm.** 

**Problem 2** - The relationship between angular size,  $\Theta$ , and actual size, L, and distance, D, is given by the formula:

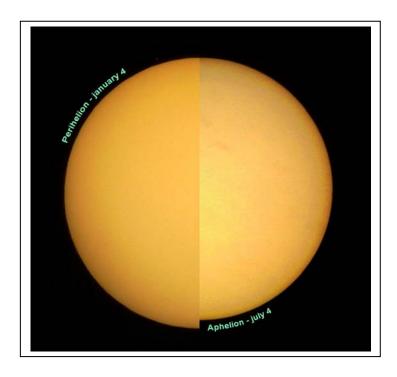


Where  $\Theta$  is measured in arcseconds, and L and D are both given in the same units of length or distance (e.g. meters, kilometers, light years). A) In the image of the Moon, what does 1 arcsecond correspond to in kilometers? B) In the image of M-34, what does 1 arcsecond correspond to in light years? Answer: A) For the Moon: L = 1 arcsec/206265 x (384,000 km) = 1.86 or **2.0 kilometers**. B) For the cluster, L = 1 arcsec/206265 x (1,400 light years) = **0.007 light years**.

**Problem 3** - What is the smallest detail you can see in the Moon image in A) arcseconds? B) kilometers? Answer: A) About 1 millimeter, which corresponds to **1.0 arcsec**. B) One arcsec corresponds to **2.0 kilometers**.

**Problem 4** - What is the smallest star separation you can measure in Messier-34 among the brightest stars in A) arcseconds? B) Light years? Answer: A) Students may find that some of the bright stars are about 3 millimeters apart, which corresponds to 3 mm x 30 asec/mm = **90 arcseconds**. B) At the distance of the cluster, 1 asec = 0.007 light years, so 90 asec corresponds to 90 x (0.007 light years/asec) = 0.63 or **0.6 light years** to 1 significant figure.

# Changing Perspectives on the Sun's Diameter



Earth's orbit is not a perfect circle centered on the Sun, but an ellipse! Because of this, in January, Earth is slightly closer to the Sun than in June. This means that the sun will actually appear to have a bigger disk in the sky in June than in January...but the difference is impossible to see with the eye, even if you could do so safely!

The figure above shows the sun's disk taken by the SOHO spacecraft. The left side shows the disk on January 4 and the right side shows the disk on June 4, 2009. As you can see, the diameter of the sun appears to change slightly between these two months.

**Problem 1** - What is the average diameter of the Sun, in millimeters, in this figure?

**Problem 2** - By what percentage did the diameter of the Sun change between January and June compared to its average diameter?

**Problem 3** - If the average distance to the Sun from Earth is 149,600,000 kilometers, how much closer is Earth to the Sun in June compared to January?

Problem 1 - What is the average diameter of the Sun, in millimeters, in this figure? Answer: Using a millimeter ruler, and measuring vertically along the join between the two images, the left-hand, January, image is 72 millimeters in diameter, while the right-hand image is 69 millimeters in diameter. The average of these two is (72 + 69)/2 = 70.5 millimeters.

Problem 2 - By what percentage did the diameter of the Sun change between January and June compared to its average diameter? Answer: In January the moon was larger then the average diameter by  $100\% \times (72 - 70.5)/70.5 = 2.1\%$ . In June it was smaller then the average diameter by  $100\% \times (70.5)$ 

- 69)/70.5 = 2.1%.

Problem 3 - If the average distance to the Sun from Earth is 149,600,000 kilometers, how much closer is Earth to the Sun in June compared to January?

Answer: The diameter of the sun appeared to change by 2.1% + 2.1% = 4.2% between January and June. Because the apparent size of an object is inversely related to its distance (i.e. the farther away it is the smaller it appears), this 4.2% change in apparent size occurred because of a 4.2% change in the distance between Earth and the Sun, so since  $0.042 \times 149,600,000 \text{ km} = 6,280,000 \text{ kilometers}$ , the change in the Sun's apparent diameter reflects the 6,280,000 kilometer change in earth's distance between January and June. The Earth is **6,280,000 kilometers closer to the Sun in June than in January.** 

# General Angles and Radian Measure



The relationship between the distance to an object, **R**, the object's size, **L**, and the angle that it subtends at that distance,  $\theta$ , is given by:

$$\theta = 57.29 \frac{L}{R} \text{ degrees}$$
$$\theta = 3,438 \frac{L}{R} \text{ arcminutes}$$
$$\theta = 206,265 \frac{L}{R} \text{ arcseconds}$$

To use these formulae, the units for length, L, and distance, R, must be identical.

**Problem 1** - You spot your friend (L = 2 meters) at a distance of 100 meters. What is her angular size in arcminutes?

**Problem 2** - The sun is located 150 million kilometers from Earth and has a radius of 696.000 kilometers, what is its angular diameter in arcminutes?

**Problem 3** - How far away, in meters, would a dime (1 centimeter) have to be so that its angular size is exactly one arcminute?

**Problem 4** - The spectacular photo above was taken by Jerry Lodriguss (Copyright 2007, http://www.astropix.com/HTML/SHOW\_DIG/055.HTM ) and shows the International Space Station streaking across the disk of the sun. If the ISS was located 379 kilometers from the camera, and the ISS measured 73 meters across, what was its angular size in arcseconds?

**Problem 5** - The orbital speed of the space station is 7.4 kilometers/second. If its distance traveled in 1 second is 7.4 kilometers, A) what was the angle, in arcminutes, that it moved through in one second as seen from the location of the camera? B) What was its angular speed in arcminutes/second?

**Problem 6** - Given the diameter of the sun in arcminutes (Problem 2), and the ISS angular speed (Problem 5) how long, in seconds, did it take the ISS to travel across the face of the sun?

**Problem 1** - Answer: Angle = 3,438 x (2 meters/100 meters) = **68.8 arcminutes**.

**Problem 2** - Answer:  $3,438 \times (696,000/150 \text{ million}) = 15.9 \text{ arcminutes in radius, so the diameter is <math>2 \times 15.9 = 31.8 \text{ arcminutes}$ .

**Problem 3** - Answer: From the second formula R = 3438 \* L/A = 3438 \* 1 cm/1 arcminute so R = 3,438 centimeters or a distance of **34.4 meters**.

**Problem 4** - Answer: From the third formula, Angle = 206,265 \* (73 meters/379,000 meters) = **39.7 arcseconds**.

**Problem 5** - Answer: The orbital speed of the space station is 7.4 kilometers/second. If its distance traveled in 1 second is 7.4 kilometers, A) The ISS traveled L = 7.4 kilometers so from the second formula Angle = 3,438 \* (7.4 km/379 km) = 67 arcminutes. B) The angular speed is just 67 arcminutes per second.

**Problem 6** - Answer: The time required is T = 31.8 arcminutes/ (67 arcminutes/sec) = **0.47** seconds.

The spectacular photo by Jerry Lodriguss had to be taken with careful planning beforehand. He had to know, to the split-second, when the sun and ISS would be in the right configuration in the sky as viewed from his exact geographic location. Here's an example of the photography considerations in his own words:

" I considered trying to monitor the transit visually with a remote release in my hand and just firing (the camera) when I saw the ISS in my guidescope. Then I worked out the numbers. I know that my reaction time is 0.19 seconds. This is actually quite good, but I make my living shooting sports where this is critical, so I better be good at it. I also know that the Canon 1D Mark IIn has a shutter lag of 55 milliseconds. Adding these together, plus a little bit of a fudge factor, the best I could hope for was about 1/4 of a second from when I saw it to when the shutter opened. Since the entire duration of the transit was only 1/2 of a second, in theory, I could capture the ISS at about the center of the disk if I fired as soon as I saw it start to cross. This was not much of a margin for error. I could easily blink and miss the whole thing... Out of 49 frames that the Mark IIn recorded, the ISS is visible in exactly one frame."

Planet	Distance	Diameter	Angular
	(million	(km)	Diameter
	km)		(arcseconds)
Mercury	100	4800	
Venus	40	12000	
Mars	75	7200	
Jupiter	630	144000	
Saturn	1275	120000	

The angular diameter of a planet depends on how far away it is from the Observer. This is a basic property of geometry that follows from the properties of similar triangles.

Once you know the physical distance and diameter of a planet in kilometers, it is easy to work out how big the planet will appear in degrees of arc. A simple formula that gives the angular diameter in degrees is just

 $\theta = 57.3 \frac{\text{Diameter}}{\text{Distance}}$ 

**Problem 1** - The table above gives the minimum distance to Earth of several planets, along with their diameters. Calculate the angular diameter of each planet to the nearest arcsecond if 1 degree = 3600 arcseconds.

**Problem 2** - As viewed from Earth, the planet Venus will transit the planet Jupiter on November 22, 2065. At a scale of 1 arcsecond per centimeter, draw two disks representing Mercury and Jupiter's apparent angular diameters just before the occultation begins, if the distance to Venus will be 225 million km and the distance to Jupiter at that time will be 955 million km.

**Problem 1** - The table above gives the minimum distance to Earth of several planets, along with their diameters. Calculate the angular diameter of each planet to the nearest arcsecond if 1 degree = 3600 arcseconds.

Planet	Distance	Diameter	Angular
	(million	(km)	Diameter
	km)		(arcseconds)
Mercury	100	4800	10
Venus	40	12000	62
Mars	75	7200	20
Jupiter	630	144000	47
Saturn	1275	120000	19

#### Table of planetary distances from the sun

**Problem 2** - As viewed from Earth, the planet Venus will transit the planet Jupiter on November 22, 2065. At a scale of 1 arcsecond per centimeter, draw two disks representing Mercury and Jupiter's apparent angular diameters just before the occultation begins, if the distance to Venus from Earth will be 225 million km and the distance to Jupiter at that time will be 955 million km.

Answer: The angular diameter of Venus will be 11 arcseconds, and Jupiter will be 31 arcseconds.

This is how the planets appeared on January 3, 1818 when Jupiter was at 930 million km from Earth and Venus was at 310 million km. Their angular sizes were 32 arcseconds and 8 arcseconds respectively. Unfortunately this event was lost in the glare of the sun.

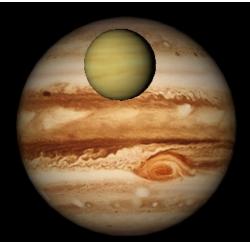
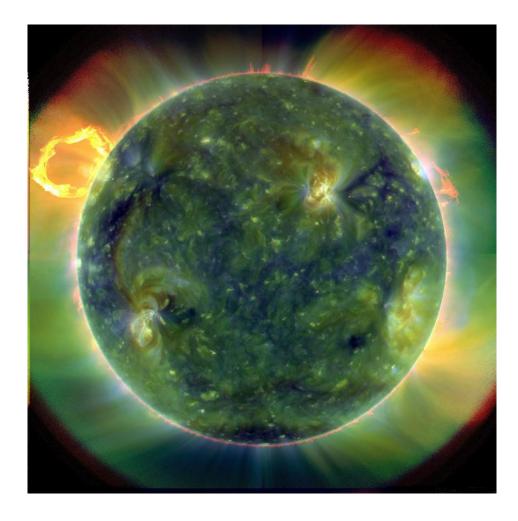


Image courtesy Wikimedia: Bart Benjamin and released to public domain.

# SDO Reveals Details on the Surface of the Sun



On April 21, 2010 NASA's Solar Dynamics Observatory (SDO) released its much-awaited 'First Light' images of the Sun. The image above shows a full-disk, multi-wavelength, extreme ultraviolet image of the sun taken by SDO on March 30, 2010. False colors trace different gas temperatures. Black indicates very low temperatures near 10,000 K close to the solar surface (photosphere). Reds are relatively cool plasma heated to 60,000 Kelvin (100,000<sup>°</sup> F); blues, greens and white are hotter plasma with temperatures greater than 1 million Kelvin (2,000,000<sup>°</sup> F).

**Problem 1** – The radius of the sun is 690,000 kilometers. Using a millimeter ruler, what is the scale of these images in kilometers/millimeter?

**Problem 2** – What are the smallest features you can find on this image, and how large are they in kilometers, and in comparison to Earth if the radius of Earth is 6378 kilometers?

**Problem 3** – Where is the coolest gas (coronal holes), and the hottest gas (micro flares), located in this image?

#### Space Math

Answer: The diameter of the Sun is 98 millimeters, so the scale is 1,380,000 km/98 mm = **14,000 km/mm**.

**Problem 2** – What are the smallest features you can find on this image, and how large are they in kilometers, and in comparison to Earth if the radius of Earth is 6378 kilometers?

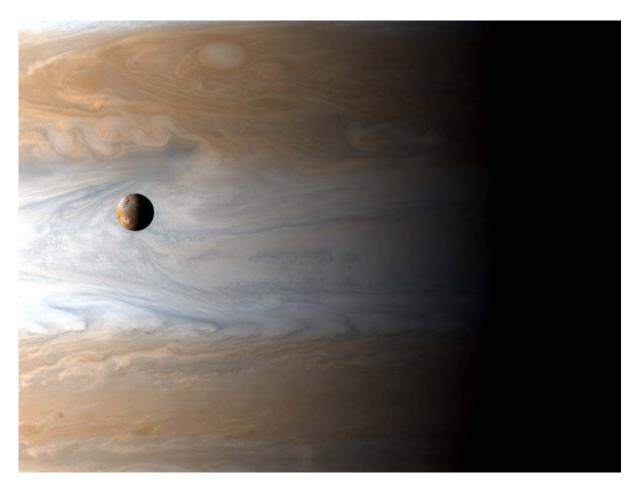
Answer: Students should see numerous bright points freckling the surface, the smallest of these are about 0.5 mm across or 7,000 km. This is slightly larger than  $\frac{1}{2}$  the diameter of Earth.

**Problem 3** – Where is the coolest gas (coronal holes), and the hottest gas (micro flares), located in this image?

Answer: There are large irregular blotches all across the disk of the sun that are dark blue-black. These are regions where thee is little of the hot coronal gas and only the 'cold' photosphere can be seen. The hottest gas seems to reside in the corona, and in the very small point-like 'microflare' regions that are generally no larger than the size of Earth.

Note: Microflares were first observed, clearly, by the Hinode satellite between 2007-2009. Some solar physicists believe that these microflares, which erupt violently, are ejecting hot plasma that eventually ends up in the corona to replenish it. Because the corona never disappears, these microflares happen all the time no matter what part of the sunspot cycle is occurring.

# Jupiter and Io



This NASA image of Jupiter with its satellite Io was taken by the Cassini spacecraft. (Credit: NASA/Cassini Imaging Team). The satellite is 3,600 kilometers in diameter.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the diameter of Io is 3,600 kilometers.

Step 1: Measure the diameter of Io with a metric ruler. How many millimeters in diameter?

Step 2: Use clues in the image description to determine a physical distance or length.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in kilometers per millimeter to two significant figures.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in kilometers to two significant figures.

- **Question 1:** What are the dimensions, in kilometers, of this image?
- **Question 2:** What is the width, in kilometers, of the largest feature in the atmosphere of Jupiter?
- Question 3: What is the width, in kilometers, of the smallest feature in the atmosphere of Jupiter?

Question 4: What is the size of the smallest feature on Io that you can see?

This NASA image of Jupiter with its satellite Io was taken by the Cassini spacecraft. (Credit: NASA/Cassini Imaging Team). The satellite is 3,600 kilometers in diameter.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the diameter of Io is 3,600 kilometers.

Step 1: Measure the diameter of Io with a metric ruler. How many millimeters in diameter? Answer: **10 mm** 

Step 2: Use clues in the image description to determine a physical distance or length. Answer: **3,600 km** 

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in kilometers per millimeter to two significant figures. Answer: 3600 km / 10 mm = **360 km/mm** 

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in kilometers to two significant figures.

Question 1: What are the dimensions, in kilometers, of this image? Answer: 160 mm x 119 mm = **58,000 km x 19,000 km** 

Question 2: What is the width, in kilometers, of the largest feature in the atmosphere of Jupiter? Answer: The width of the white equatorial band is 45 mm or **16,000 km** 

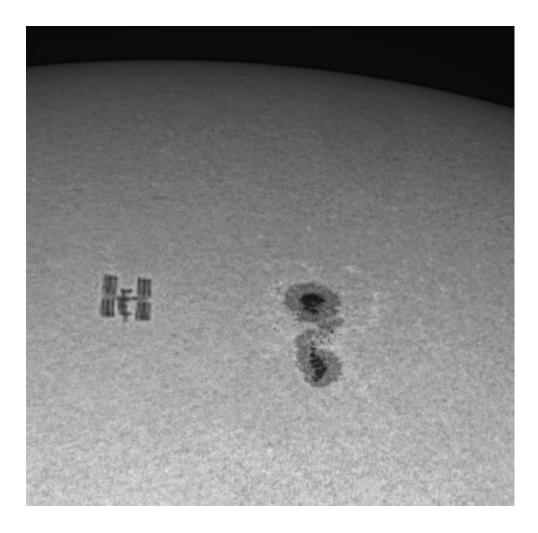
Question 3: What is the width, in kilometers, of the smallest feature in the atmosphere of Jupiter? Answer: The faint cloud streaks are 0.5 mm wide or **200 km** across to one significant figure.

Question 4: What is the size of the smallest feature on Io that you can see?

Answer: The white spots in the southern hemisphere are 0.5 mm across or 200 km to one significant figure. This is a good time to mention that some details in an image can be artifacts from the printing process or defects in the camera itself. Students may find photocopying artifacts at 0.5 mm or less.

**Note to teachers:** The correct scale for Io and Jupiter will be slightly different depending on how far away the camera was when taking the picture. If the camera was very close to Io, then the scale you will infer for Io will be very different than for the more distant Jupiter because Io will take up more of the field-of-view in the image. Geometrically, for a fixed angle of separation between features on Io, this angle will subtend a SMALLER number of kilometers than the same angle on the more-distant Jupiter. However, if the distance from the camera to Jupiter/Io is very large, then as seen from the camera, both objects are at essentially the same distance and so there will be little difference between the scales used for the two bodies. Students can check this result with an inquiry assignment.

# ISS and a Sunspot - Angular Scales



Photographer John Stetson took this photo on March 3, 2010 by carefully tracking his telescope at the right moment as the International Space Station passed across the disk of the sun.

The angular size,  $\theta$ , in arcseconds of an object with a length of L meters at a distance of D meters is given by

$$\theta = 206265 \frac{L}{D}$$

**Problem 1** - The ISS is 108 meters wide, and was at an altitude of 350 km when this photo was taken. If the sun is at a distance of 150 million kilometers, how large is the sunspot in A) kilometers? B) compared to the size of Earth if the diameter of Earth is 13,000 km?

**Problem 2** - The sun has an angular diameter of 0.5 degrees. If the speed of the ISS in its orbit is 10 km/sec, how long did it take for the ISS to cross the face of the sun as viewed from the ground on Earth?

**Problem 1** - The ISS is 108 meters wide, and was at an altitude of 350 km when this photo was taken. If the sun is at a distance of 150 million kilometers, how large is the sunspot in A) kilometers? B) compared to the size of Earth if the diameter of Earth is 13,000 km?

Answer: As viewed from the ground, the ISS subtends an angle of Angle =  $206265 \times (108 \text{ meters}/350,000 \text{ meters})$  so Angle = **63 arcseconds**.

At the distance of the sun, which is 150 million kilometers, the angular size of the ISS corresponds to a physical length of

- L = 150 million kilometers x (63/206265) so
- L = 46,000 kilometers.

The sunspot is comparable in width to that of the ISS and has a length about twice that of he ISS so its size is about **46,000 km x 92,000 km**.

As a comparison, Earth has a diameter of 13,000 km so the sunspot is about **3 times** the diameter of Earth in width, and 6 times the diameter of Earth in length.

**Problem 2** - The sun has an angular diameter of 0.5 degrees. If the speed of the ISS in its orbit is 10 km/sec, how long did it take for the ISS to cross the face of the sun as viewed from the ground on Earth?

**Answer:** From the ground, convert the speed of the ISS in km/sec to an angular speed in arcseconds/sec.

In one second, the ISS travels 10 km along its orbit. From the ground this corresponds to an angular distance of

Angle = 206265 x (10 km/350 km)

= 5900 arcseconds.

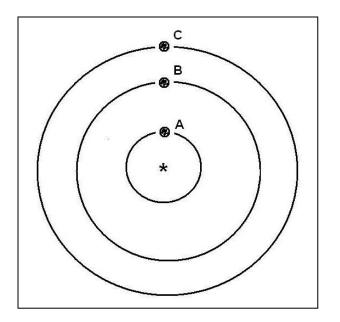
The speed is then 5900 arcseconds/sec. The diameter of the sun is 0.5 degrees which is 30 arcminutes or 1800 arcseconds. To cover this angular distance, the ISS will take

T = 1800 arcseconds / (5900 arcseconds/s) so

T = 0.3 seconds!

Space Math

http://spacemath.gsfc.nasa.gov



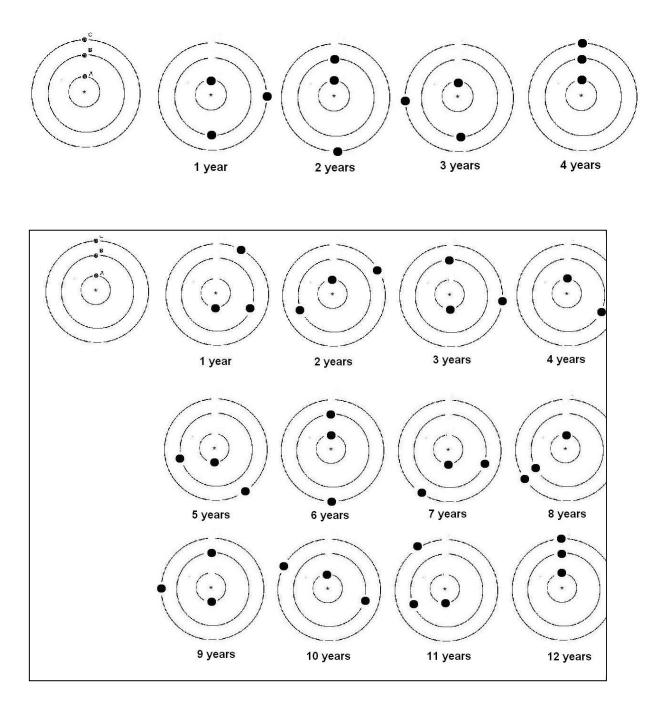
Since 1995, astronomers have detected over 1700 planets orbiting distant stars. Our solar system has 8 planets, and for thousands of years astronomers have studied their motions. The most interesting events happen when planets are seen close together in the sky. These are called conjunctions. or less-accurately. alignments. The figure shows a simple 3-planet solar system with the planets starting out 'lined up' with their star. Each planet revolves around the star at a different pace, so it is a challenge to predict when they will all line up again.

**Problem 1** - An astronomer detects three planets, A, B, C, that orbit their star once every 1, 2 and 4 earth-years in a clockwise direction. Using the diagram above, draw a series of new diagrams that show where will the planets be in their orbits after the following number of earth-years: A) 1 year? B) 2 years? C) 3 years? D) 4 years?

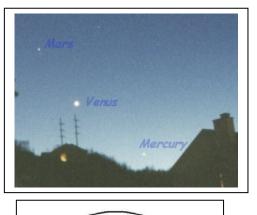
**Problem 2** - Suppose the three planets, A, B and C, orbited their star once every 2, 3 and 12 earth-years. A) How long would it take for all three planets to line up again? B) Where would the planets be after 6 earth-years?

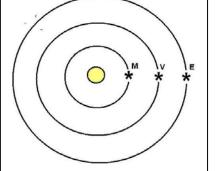
Problem 1 - Students will draw dots located as follows in the top diagram series:

Problem 2 - The bottom series of 12 possibilities indicates it will take 12 earthyears to return to the original line-up. The pattern after 6 earth-years is also shown.



# **Planetary Alignments**





One of the most interesting things to see in the night sky is two or more planets coming close together in the sky. Astronomers call this a conjunction. The picture to the left shows a conjunction involving Mercury, Venus and Mars on June 24, 2005.

As seen from their orbits, another kind of conjunction is called an 'alignment' which is shown in the figure to the lower left and involved Mercury, M, Venus, V, and Earth, E. As viewed from Earth's sky, Venus and Mercury would be very close to the Sun, and may even be seen as black disks 'transiting' the disk of the Sun at the same time, if this alignment were exact. How often do alignments happen?

Earth takes 365 days to travel one complete orbit, while Mercury takes 88 days and Venus takes 224 days, so the time between alignments will require each planet to make a whole number of orbits around the Sun and return to the pattern you see in the figure.

Suppose Mercury takes 1/4 earth-year and Venus takes 2/3 of an earth-year to make their complete orbits around the Sun. You can find the next line-up from two methods:

Method 1: Work out the three number series like this:

Earth = 0, 1, **2**, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12,...

Mercury = 0, 1/4, 2/4, 3/4, 4/4, 5/4, 6/4, 7/4, 8/4, 9/4, 10/4, 11/4, 12/4, 13/4, ...

Venus = 0, 2/3, 4/3, 6/3, 8/3, 10/3, 12/3, 14/3, 16/3, 18/3, 20/3, ...

Notice that the first time they all coincide with the same number is at 2 earth-years. So Mercury has to go around the Sun 8 times, Venus 3 times and Earth 2 times for them to line up again in their orbits.

**Method 2**: We need to find the Least Common Multiple (LCM) of 1/4, 2/3 and 1. First render the periods in multiples of a common time unit of 1/12, then the sequences are: Mercury = 0, 3, 6, 9, 12, 15, 18, 21, **24**,

Venus = 0, 8, 16, **24**, 32, 40, ... Earth, 0, 12, **24**, 36, 48, 60, ...

The LCM is 24 which can be found from prime factorization:

Mercury: 3 = 3Venus:  $8 = 2 \times 2 \times 2$ Earth:  $12 = 2 \times 2 \times 3$ 

The LCM the product of the highest powers of each prime number or  $3 \times 2 \times 2 \times 2 = 24$ . and so it will take 24/12 = 2 earth-years.

**Problem 1** - Suppose a more accurate estimate of their orbit periods is that Mercury takes 7/30 earth-years and Venus takes 26/42 earth-years. After how many earth-years will the alignment reoccur?

Space Math

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Mercury =  $7/30 \times 365 = 85$  days vs actual 88 days Venus =  $26/42 \times 365 = 226$  days vs actual 224 days Earth = 1

The common denominator is  $42 \times 30 = 1,260$  so the series periods are Mercury =  $7 \times 42 = 294$  so 7/30 = 294/1260Venus =  $26 \times 30 = 780$  so 26/42 = 780/1260Earth = 1260 so 1 = 1260/1260

The prime factorizations of these three numbers are

 $294 = 2 \times 2 \times 3 \times 7 \times 7$   $780 = 2 \times 2 \times 5 \times 3 \times 13$  $1260 = 2 \times 2 \times 3 \times 3 \times 5 \times 7$ 

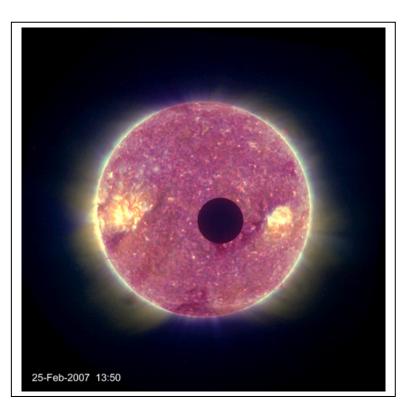
 $LCM = 2 \times 2 \times 3 \times 3 \times 5 \times 7 \times 7 \times 13 = 114,660$ 

So the time will be 114,660 / 1260 = 91 years! In this time, Mercury will have made exactly 114,660/294 = 390 orbits and Venus will have made 114,660/780 = 147 orbits

**Note to Teacher:** Why did the example problem give only 2 years while this problem gave 91 years for the 'same' alignment? Because we used a more accurate approximation for the orbit periods of the three planets. Mercury actual period = 88 days but 1/4 earth-year = 91.25 days compared to 7/30 earth year = 85 days. Venus actual period = 224 days but 2/3 earth-year = 243 days and 26/42 earth-year = 226 days.

This means that after 2 years and exactly 8 orbits (8 x 91.25 = 730 days), Mercury will be at  $8/4 \times 365 = 730$  days while the actual 88-day orbit will be at  $88 \times 8 = 704$  days or a timing error of 26 days. Mercury still has to travel another 26 days in its orbit to reach the alignment position. For Venus, its predicted orbit period is  $2/3 \times 365 = 243.3$  days so its 3 orbits in the two years would equal  $3 \times 243.3$  days = 730 days, however its actual period is 2/4 days so in 3 orbits it accumulates  $3 \times 224 = 672$  days and the difference is 730-672 = 58 days so it has to travel another 58 days to reach the alignment. In other words, the actual positions of Mercury and Venus in their orbits is far from the 'straight line' we were hoping to see after exactly 2 years, using the approximate periods of 1/4 and 2/3 earth-years!

With the more accurate period estimate of 7/30 earth-years (85 days) for Mercury and 26/42 earth-years (226 days) for Venus, after 91 years, Mercury will have orbited exactly 91 x 365 days/88 days = 377.44 times, and Venus will have orbited 91x365/224 = 148.28 times. This means that Mercury will be  $0.44 \times 88d = 38.7$  days ahead of its predicted alignment location, and Venus will be  $0.28 \times 224 = 62.7$  days behind its expected alignment location. Comparing the two predictions, Prediction 1: Mercury= - 26 days, Venus= - 58 days; Prediction 2: Mercury = +26 days and Venus = - 22 days. Our prediction for Venus has significantly improved while for Mercury our error has remained about the same in <u>absolute magnitude</u>. In the sky, the two planets will appear closer together for Prediction 2 in 1911 years than for Prediction 1 in 2 years. If we want an even 'tighter' alignment, we have to make the fractions for the orbit periods much closer to the actual periods of 88 and 224 days.



The twin STEREO satellites captured this picture of our Moon passing across the sun's disk on February 25, 2007. The two satellites are located approximately in the orbit of Earth, but are moving away from Earth in opposite directions. From this image, you can figure out how far away from the Moon the STEREO-B satellite was when it took this picture! To do this, all you need to know is the following:

1) The diameter of the Moon is 3,476 km

2) The distance to the Sun is 150 million km.

3) The diameter of the Sun is 0.54 degrees

Can you figure out how to do this using geometry?

<u>Problem 1:</u> Although the True Size of an object is measured in meters or kilometers, the Apparent Size of an object is measured in terms of the number of angular degrees it subtends. Although the True Size of an object remains the same no mater how far away it is from you, the Apparent Size gets smaller the further away it is. In the image above, the Apparent Size of the Sun was 0.54 degrees across on February 25. By using a millimeter ruler and a calculator, what is the angular size of the Moon?

<u>Problem 2:</u> As seen from the distance of Earth, the Moon has an Apparent Size of 0.53 degrees. If the Earth-Moon distance is 384,000 kilometers, how big would the Moon appear at twice this distance?

<u>Problem 3:</u> From your answer to Problem 1, and Problem 2, what is the distance to the Moon from where the above photo was taken by the STEREO-B satellite?

<u>Problem 4:</u> On February 25, 2007 there was a Half Moon as viewed from Earth, can you draw a scaled model of the Earth, Moon, Stereo-B and Sun distances and positions (but not diameters to the same scale!) with a compass, ruler and protractor?

<u>Problem 1:</u> Although the True Size of an object is measured in meters or kilometers, the Apparent Size of an object is measured in terms of the number of angular degrees it subtends. Although the True Size of an object remains the same no mater how far away it is from you, the Apparent Size gets smaller the further away it is. In the image above, the Apparent Size of the sun is 0.5 degrees across. By using a millimeter ruler and a calculator, what is the angular size of the Moon?

Answer: The diameter of the sun is 57 millimeters. This represents 0.54 degrees, so the image scale is 0.54 degrees / 57 millimeters = 0.0095 degrees/mm

The diameter of the Moon is 12 millimeters, so the angular size of the Moon is

12 mm x 0.0095 degrees/mm = 0.11 degrees.

<u>Problem 2:</u> As seen from the distance of Earth, the Moon has an Apparent Size of 0.53 degrees. If the Earth-Moon distance is 384,000 kilometers, how big would the Moon appear at twice this distance?

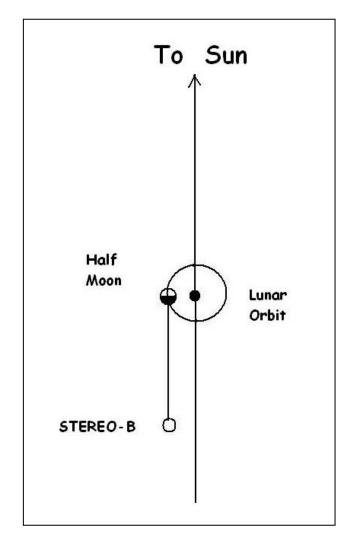
Answer: It would have an Apparent Size half as large, or 0.26 degrees.

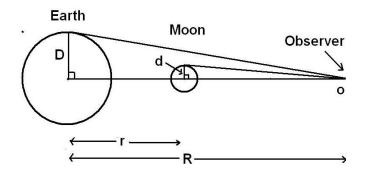
<u>Problem 3:</u> From your answer to Problem 1, and Problem 2, what is the distance to the moon from where the above photo was taken by the STEREO-B satellite?

Answer: The ratio of the solar diameter to the lunar diameter is 0.54 degrees/0.11 degrees = 4.9. This means that from the vantage point of STEREO, it is 4.9 times farther away than it would be at the Earth-Moon distance. This means it is 4.9 times farther away than 384,000 km, or 1.9 million kilometers.

<u>Problem 4:</u> On February 25, 2007 there was a Half Moon as viewed from Earth, can you draw a scaled model of the Earth, Moon, Stereo-B and Sun distances and positions (but not diameters!) using a compass, ruler and protractor?

Answer: The figure to the right shows the locations of the Earth, Moon and STEREO satellite. The line connecting the Moon and the Satellite is 4.9 times the Earth-Moon distance.

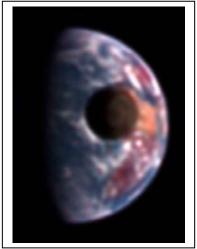




In space, your perspective can change in complicated ways that sometimes go against Common Sense unless you 'do the math'. This happens very commonly when we are looking at one object pass across the face of another. Even though the Moon is 1/4 the diameter of Earth, the simple ratio of their apparent diameters will depend on how far from them YOU are when you see them.

An important 'skinny triangle' relationship for triangles states that, if the angle is less than 1 degree (< 0.017 radians), the angle measure in radians equals very nearly the sine of the angle, which is just the ratio of the opposite side to the hypotenuse:  $\theta = D/R$  where D is the radius of the object in kilometers, R, is the distance to the object in kilometers, and  $\theta$  is the angular radius of the object in radians. For instance, the Moon is located r = 384,000 km from Earth and it has a radius of d = 1,738 km, so its angular radius is 1,738/384,000 = 0.0045 radians. Since 1 radian = 57.3 degrees, the angular radius of the Moon is 0.0045 x 57.3 = 0.26 degrees, so its diameter is 0.52 degrees as viewed from Earth.





**Problem 1** - In the figure above, assume that the diameter of the Moon is less than 1 degree when spotted by the spacecraft located at 'O'. What is the angular diameter of: A) the Moon,  $\theta_m$ , in terms of d and R? B) The Earth,  $\theta_e$ , in terms of D and R? and C) What is the ratio of the angular diameter of the Moon to the Earth in terms of d, D, r and R?

**Problem 2** - A spacecraft is headed directly away from the Moon along the line connecting the center of Earth and the Moon. At what distance will the angular diameter of the Moon equal the angular diameter of Earth?

**Problem 3** - The figure to the top left shows the martian satellite Phobos passing across the disk of the sun as viewed from the surface of Mars by the Rover Opportunity. If the ratio of the diameters is 1/2, and if r = 228 million km, d = 10 km, and D=696,000 km, about how far from Phobos was Opportunity at the time the photo was taken?

**Problem 4** - The Deep Impact spacecraft observed the Moon pass across the disk of Earth as shown in the photo to the bottom left. The ratio of the disk diameters is 1/3.9, and if r = 384,000 km, d=1,786 km and D = 6,378 km, about how far from Earth, R, was the spacecraft?

**Problem 5** - As the distance, R, becomes very large, in the limit, what does the angular ratio of the disk approach in the equation defined in Problem 1?

**Problem 1** - What is the angular diameter of: A) the Moon,  $\theta_m$ , in terms of d and R? Answer:

$$\theta m = \frac{2\pi d}{R - r}$$

B) The Earth,  $\theta_e$ , in terms of D and R? Answer:

$$\theta e = \frac{2\pi D}{R}$$

C) What is the ratio of the angular diameter of the Moon to the Earth in terms of d, D, r and R? Answer:

$$\frac{\theta m}{\theta e} = \frac{R}{R-r}\frac{d}{D}$$

**Problem 2** - At what distance will the angular diameter of the Moon equal the angular diameter of Earth? Answer:

$$1 = \frac{R}{R - 384,000} \frac{(1,786)}{(6,378)}$$
 so 3/4 R = 384,000 km, and so **R = 512,000 km**.

**Problem 3** - If the ratio of the diameters is 1/2, and if r = 228 million km, d = 10 km, and D=696,000 km, about how far from Phobos was Opportunity at the time the photo was taken? Answer:

$$\frac{1}{2} = \frac{R}{R - 228 million} \frac{(10 km)}{(696,000 km)}$$
 so 34799 R = 228 million km, and so R = 6,552 km.

**Problem 4** - The ratio of the disk diameters is 1/3.9, and if r = 384,000 km, d=1,786 km and D = 6,378 km, about how far from Earth, R, was the spacecraft? Answer:

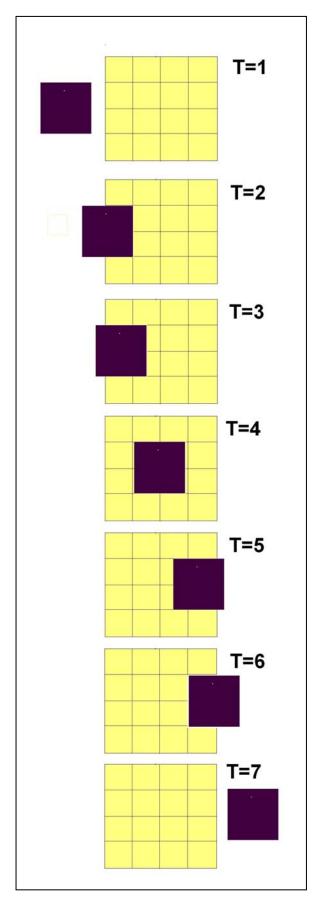
$$\frac{1}{20} = \frac{R}{R} \frac{(1,786km)}{(6,278km)}$$

 $3.9 \quad R-384,000 \text{km} \ (6,378 \text{km})$  so 0.02 R = 384,000 km, so R = 19.2 million km. Note: the actual distance was about 30 million km for the photo shown in this problem.

**Problem 5** - As the distance, R, becomes very large, in the limit, what does the angular ratio of the disk approach in the equation defined in Problem 1? Answer: As R becomes much, much larger than r (e.g the limit of r approaches infinity), then the equation approaches

$$\frac{\theta m}{\theta e} = \frac{d}{R} \frac{R}{D}$$

and since the 'R' terms cancel, we get the angular ratio approaching the physical ratio d/D of the diameters of the two bodies. In other words, although the apparent angular sizes change rapidly when you are very close to the bodies and the value of R is comparable to 'r', at very great distances, the angular ratio approaches a constant value d/D. This has many practical consequences in the search for planets around other stars as they' transit' their stars.



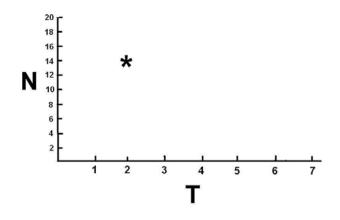
Space Math

NASA's Kepler spacecraft recently announced the discovery of five new planets orbiting distant stars. The satellite measures the dimming of the light from these stars as planets pass across the face of the star as viewed from Earth. To see how this works, lets look at a simple model.

In the Bizarro Universe, stars and planets are cubical, hot spherical. Bizarro astronomers search for distant planets around other stars by watching planets pass across the face of the stars and cause the light to dim.

**Problem 1** - The sequence of figures shows the transit of one such planet, Osiris (black). Complete the 'light curve' for this star by counting the number of exposed 'star squares' not shaded by the planet. At each time, T, create a graph of the number of star brightness squares. The panel for T=2 has been completed and plotted on the graph below.

**Problem 2** - If you knew that the width of the star was 1 million kilometers, how could you use the data in the figure to estimate the width of the planet?



http://spacemath.gsfc.nasa.gov

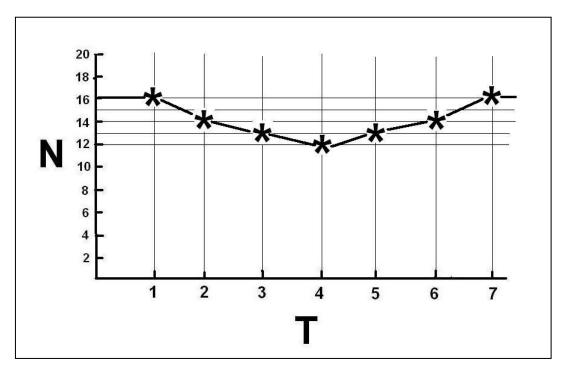
**Problem 1** - The sequence of figures shows the transit of one such planet, Osiris (black). Complete the 'light curve' for this star by counting the number of exposed 'star squares' not shaded by the planet. At each time, T, create a graph of the number of star brightness squares. The panel for T=2 has been completed and plotted on the graph below.

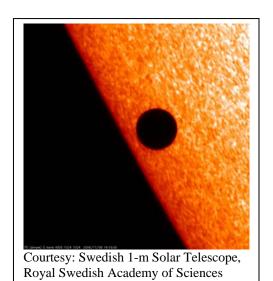
Answer: Count the number of yellow squares in the star and plot these for each value of T in the graph as shown below. Note, for T= 3 and 5, the black square of the planet occupies 2 full squares and 2 half squares for a total of 2 + 1/2 + 1/2 = 3 squares covered, so there are 16 - 3 = 13 squares remaining that are yellow.

**Problem 2** - If you knew that the width of the star was 1 million kilometers, how could you use the data in the figure to estimate the width of the planet?

Answer: The light curve shows that the planet caused the light from the star to decrease from 16 units to 12 units because the planet blocked 16-12 = 4 units of the stars surface area. That means that the planet squares occupy 4/16 of the stars area as seen by the astronomers. The area of the star is just the area of a square, so the area of the square planet is 4/16 of the stars area or Ap = 4/16 x Astar. Since the star as a width of Wstar = 1 million kilometers, the planet will have a width of  $Wp = Wstar\sqrt{\frac{4}{16}}$  or **500,000 kilometers**.

The amount of star light dimming is proportional to the ratio of the area of the planet and the star facing the observer. The Kepler satellite can detect changes by as little as 0.0001 in the light from a star, so the smallest planets it can detect have diameters about 1/100 the size of the stars that they orbit. For a star with a diameter of the sun, 1.4 million kilometers, the smallest planet detectable by the Transit Method has a diameter about equal to 14,000 kilometers or about the size of Earth.





When a dark object like a planet passes across the disk of a brighter object such as a star, the light from the star will become dimmer as it is blocked by the planet from our vantage point in space.

The figure to the left shows the planet Mercury as it transited the face of the sun on May 7, 2003. Its black disk reduced the brightness of the sun by an amount equal to the area covered by the planet's disk.

Because the shapes of both the sun and planet are very-nearly circular, the percentage of brightness change in the sun's light is just

$$B = 100\% \frac{\pi r^2}{\pi R^2}$$

Where R is the angular radius of the sun in degrees and r is the radius of the planet in degrees.

**Problem 1** – During the transit of Venus on June 8, 2004, from the vantage point of Earth, the angular diameter of the sun was 0.53 degrees and the angular diameter of Venus was 0.016 degrees. A) By what percentage was the light from the sun reduced during the transit of Venus? B) If the brightness of the sun is 1300 watts/meter<sup>2</sup> at the surface of Earth, by what amount was the power reaching Earth reduced?

**Problem 2** – Someday we may have spacecraft orbiting Saturn that can capture a view of Jupiter transiting the sun. If the angular diameter of the sun at the distance of Saturn is 0.056 degrees and the angular diameter of Jupiter from Saturn is 0.013 degrees, by what percentage will the light from the sun be reduced during a transit of Jupiter?

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Answer:

A) We will use the formula

$$B = 100\% \frac{\pi r^2}{\pi R^2}$$

where the radius of the sun is R = 0.53/2 = 0.265 degrees and the radius of Venus is r = 0.016/2 = 0.008 degrees. Then  $B = 100\% (0.008/0.265)^2$  so B = 0.091 %

B) If 100% = 1300 watts/meter<sup>2</sup>, then the brightness change is  $W = 1,300 \times 0.00091$ 

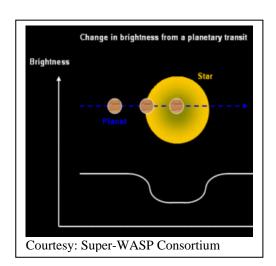
= 1.2 watts/meter<sup>2</sup>.

So instead of 1,300 watts falling on each square meter of Earth's surface, only 1,300 - 1.2 = 1,299 watts were delivered.

**Problem 2** – Someday we may have spacecraft orbiting Saturn that can capture a view of Jupiter transiting the sun. If the angular diameter of the sun at the distance of Saturn is 0.056 degrees and the angular diameter of Jupiter from Saturn is 0.013 degrees, by what percentage will the light from the sun be reduced during a transit of Jupiter?

Answer: R = 0.056/2 = 0.028 degrees and r = 0.013/2 = 0.0065 degrees then B = 100% (0.0065/0.028)<sup>2</sup> = **5.4** %

When Jupiter eclipses the sun as viewed from Saturn, the sun's brightness dims by 5.4%. This is a much larger change than what is observed at Earth during the transit of Venus, which is only 0.09%.



An amazing thing about studying transits and eclipses is that <u>you do not have</u> to actually see the star's disk to know that they happen!

**Eclipses:** If you were measuring the brightness of a star and it suddenly faded <u>completely</u> out of sight, this means that some object with its same angular size as the star passed between you and the star from your vantage point in space.

**Transits:** If you were measuring the brightness of a star and it suddenly lost <u>some, but not all</u> of its original brightness, this means that some object with a smaller angular size than the star passed between you and the star from your vantage point in space.

Suppose you discovered that the brightness of a star decreased by 1% and then brightened again to its normal level after a few days. You know that the <u>maximum</u> amount of fading is just the ratio of the circular area of the planet compared to the circular area of its star.

Problem 1 – What is the ratio of the radius of the planet to the radius of its star?

**Problem 2** – If the radius of the star is 500,000 kilometers, what is the radius of the planet?

**Problem 3** – How would you classify this transiting planet compared to the planet Neptune in our solar system if the radius of Neptune is 25,000 kilometers?

Problem 1 – What is the ratio of the radius of the planet to the radius of its star?

B = 1% so  $0.01 = \frac{\pi r^2}{\pi R^2}$  and so  $\frac{r}{R} = 0.1$ 

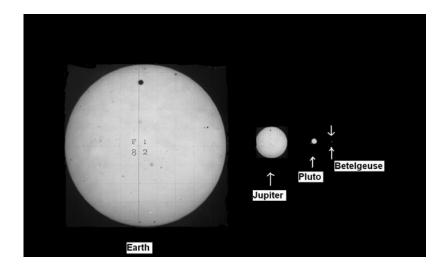
**Problem 2** – If the radius of the star is 500,000 kilometers, what is the radius of the planet?

Answer: R = 500,000 km so since r/R = 0.1 we have  $r = 0.1 \times 500,000$  **r = 50,000 km** 

**Problem 3** – How would you classify this transiting planet compared to the planet Neptune in our solar system if the radius of Neptune is 25,000 kilometers?

Answer: The planet is about **one-half the size of Neptune.** 

Note: The diameter of Earth is 13,000 km, so this planet is about 3.8 times the diameter of Earth, and would be classified as a super-Earth planet.



The approximately-scaled figures above show the transit of Venus in 1882 (Photograph courtesy the United States Naval Observatory) as seen at various distances from Earth. The amount by which the sun's light faded during the transit is given by the formula

$$B = 100\% \frac{\pi r^2}{\pi R^2}$$

where R is the sun's radius in degrees and r is the radius of Venus in degrees.

**Problem 1** – From Earth, r = 0.008 degrees and R = 0.27 degrees. By what percentage did the sun's light fade as viewed from Earth during the transit of Venus?

**Problem 2** – From Jupiter, r = 0.0015 degrees and R = 0.052 degrees. By what percentage would the sun's light have faded had the transit been viewed from Jupiter?

**Problem 3** - From Pluto, r = 0.0002 degrees and R = 0.0068 degrees. By how much would the sun's light have faded had the transit been viewed from Pluto?

**Problem 4** - From the star Betelgeuse located 650 light years from Earth, r =  $2.0 \times 10^{-10}$  degrees and R =  $6.8 \times 10^{-9}$  degrees. By how much would the sun's light have faded had the transit been viewed from the vicinity of the star Betelgeuse?

**Problem 5** - Explain why you do not need to actually see the surface of a star in order to detect the transit of a planet.

Answer:  $B = 100\% (0.008/0.27)^2 = 0.088\%$ 

**Problem 2** – From Jupiter, r = 0.0015 degrees and R = 0.052 degrees. By what percentage would the sun's light have faded had the transit been viewed from Jupiter?

Answer:  $B = 100\% (0.0015/0.052)^2 = 0.083\%$ 

**Problem 3** - From Pluto, r = 0.0002 degrees and R = 0.0068 degrees. By how much would the sun's light have faded had the transit been viewed from Pluto?

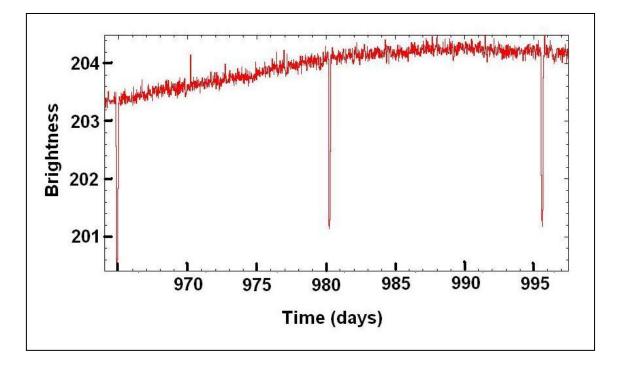
Answer:  $B = 100\% (0.0002/0.0068)^2 = 0.087\%$ 

**Problem 4** - From the star Betelgeuse located 650 light years from Earth,  $r = 2.0 \times 10^{-10}$  degrees and R = 6.8 x  $10^{-9}$  degrees. By how much would the sun's light have faded had the transit been viewed from the vicinity of the star Betelgeuse?

Answer:  $B = 100\% (2.0x10^{-10}/6.8x10^{-9})^2 = 0.087 \%$ 

**Problem 5** - Explain why you do not need to actually see the surface of a star in order to detect the transit of a planet.

Answer: The percentage of dimming that you calculated for each of the four examples was essentially identical: 0.08%. The percentage change in the brightness of a star during the transit only depends on the <u>ratio</u> of the planet's angular diameter to the star's angular diameter. It does not depend on the actual diameter of the star at the time of the transit. Because, by the property of similar triangles, your distance from the transit affects both the angular size of the star and the angular size of the planet at the same time, your distance from the transit 'cancels out' in the ratio used in the calculation, and so you will see exactly the same amount of dimming in the star's brightness whether you can fully see the disk of the star, or the star is an unresolved point of light in the sky.



Planets travel in elliptical orbits around their parent stars. The time it takes for one complete revolution is called the period of the planet. For Earth, the period of its orbit is exactly 1 'Earth Year'. For more distant planets such as Neptune, the period is as long as 164 years, while for Mercury, its period is only 88 days!

When a planet passes in front of its star, called a transit, the light dims slightly as shown in the graph above taken by NASA's Kepler spacecraft. The time between successive transits is equal to the period of revolution of the planet around its star.

**Problem 1** - The stellar brightness data for the star Kepler 005972334 is shown in the graph above. What is the orbital period of the planet?

**Problem 2** - The brightness of the star near the middle transit at T = 980 days is about 204 units. From the 'depth' of the brightness change and the known radius of this sun-like star of 635,000 kilometers, what is the diameter of the transiting planet?

**Problem 3** - If Jupiter has a radius of 73,000 km, how does the size of this transiting planet compared to that of Jupiter?

Space Math

Answer: The dips in the star's brightness occur at times T = 965 days, T = 980 days and T = 995 days so the average period is **P = 15 days**.

**Problem 2** - The brightness of the star near the middle transit at T = 980 days is about 204 units. From the 'depth' of the brightness change and the known radius of the star of 700,000 kilometers, what is the diameter of the transiting planet?

Answer: The amount of light lost by the star during the transit is B = 204 - 201 = 3 units. As a fraction of the light lost this is 3/204 = 0.0147. Since this is equal to the ratio of the apparent angular areas of the planet and the star we have

$$0.0147 = \frac{\pi r^2}{\pi R^2}$$
  
so that 
$$0.121 = \frac{r}{R}$$

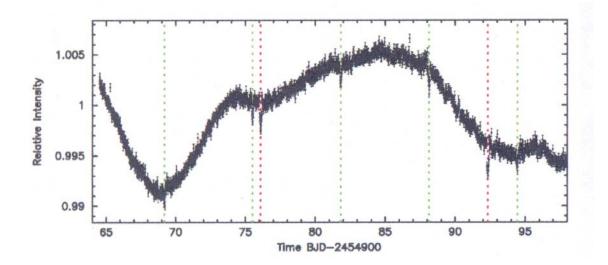
Since R = 635,000 km, the radius of the planet is about  $r = 0.121 \times 635,000$ **r = 77,000 km**.

**Problem 3** - If Jupiter has a radius of 73,000 km, how does the size of this transiting planet compared to that of Jupiter?

Answer: The radius of Jupiter is 73,000 km so this transiting planet is about the same size as Jupiter!

Note: From the Kepler data archives at http://archive.stsci.edu/kepler/data\_search/search.php

This star is located at RA(2000) = 19h 48m 08.93s and Dec(2000) = +41d 13' 19.1"Its apparent magnitude is 13.7m in the infrared 'J-band', and it has a surface temperature of 5,495 K, which is similar to our sun's temperature of 5,770 K, making this a sun-like 'yellow' star. The distance to this star is about 300 light years.



During its first few months of observation, NASA's Kepler Space Observatory discovered that five of its nearly 150,000 target stars had multiple planets. A study of the transits in the star's brightness time record - called a light curve by astronomers showed the tell-tail 'blips' as the planets passed in front of their star. The light curve above is for the star KOI-896 in the Kepler catalog of 'Objects of Interest'. It shows two planet transits, one with a period of about 6 days (green dashed lines) and the other with a period of about 16 days (red dashed lines).

**Problem 1** - Create a simulated light curve for two planets with periods of 50 days and 200 days.

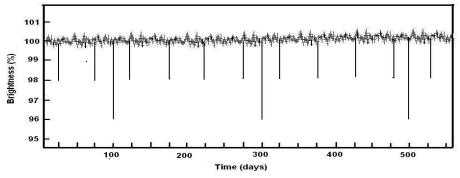
**Problem 2** - Create a simulated light curve for 3 planets with periods of 15 days, 75 days and 100 days.

**Problem 3** - If you could only observe your two simulated stars for 80 consecutive days, which planets might you miss having discovered?

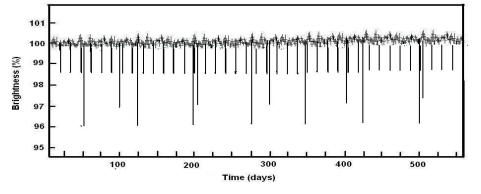
**Problem 4** - How many days of consecutive data-taking would you need to gather in order to have a good chance of detecting a planet like Earth with a period of 365 days?

The light curve for KOI-896 was obtained from the paper by Jason Steffen et al *"Five Kepler target stars that show multiple transiting exoplanet candidates"* arXiv:1006.2763v1[astro-ph.EP] which is a pre-print available at http://arxiv.org/abs/1006.2763

**Problem 1** - Create a simulated light curve for two planets with periods of 50 days and 200 days.



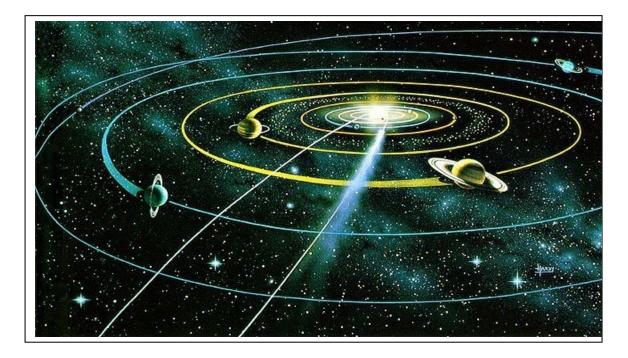
**Problem 2** - Create a simulated light curve for 3 planets with periods of 15 days, 75 days and 100 days.



**Problem 3** - If you could only observe your two simulated stars for 80 consecutive days, which planets might you miss having discovered? Answer: **The planets with periods of 100 and 200 days.** 

**Problem 4** - How many days of consecutive data-taking would you need to gather in order to have a good chance of detecting a planet like Earth with a period of 365 days?

Answer: You would need **at least 365 days** in order to see two consecutive transits. IF there are more than one planet in this system, you would need considerably more days to avoid confusion.



For stars with masses similar to our sun, there is a relationship between the period of the planet in its orbit, and the average radius of the planet's orbit. Called Kepler's Third Law, it says that the square of the orbit period is proportional to the cube of the average orbital radius. When the orbit distance, **a**, is stated as a multiple of the Earth-Sun distance (150 million km), called the Astronomical Unit (AU), and the orbit period, **T**, is in multiples of Earth Years, Kepler's Third Law becomes

$$T^2 = a^3$$

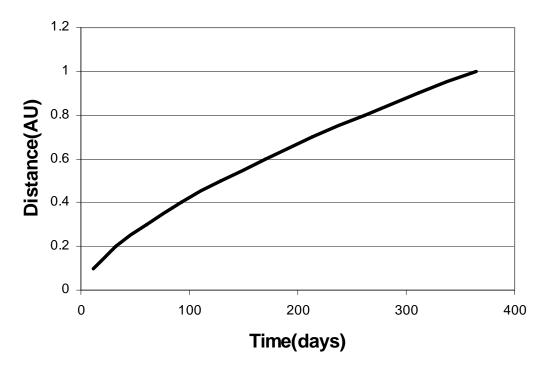
**Problem 1** - Plot this relationship for orbital distances between 0.1 AU and 5 AU with the orbital period, in days, along the horizontal axis.

**Problem 2** - The table below gives the data for 10 known exoplanets. Calculate the missing tabular entries using Kepler's Third Law.

Planet	Orbit Period (days)	Orbit Distance (AU)	
Epsilon Eridani-b	2,502		
Kepler-8b		0.048	
WASP-5b	1.6		
55 Cancri-b		5.7	
Fomalhaut-b		115	
HD209458-b	3.5		
Gliese-876-b		0.2	
MOA-2005-BLG-390lb	4,800		
HD-80606-b	111		
Upsilon Andromedae-d		2.6	

Table of exc	planet orbita	I properties
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**Problem 1** - Plot this relationship for orbital distances between 0.1 AU and 5 AU with the orbital period, in days, along the horizontal axis.

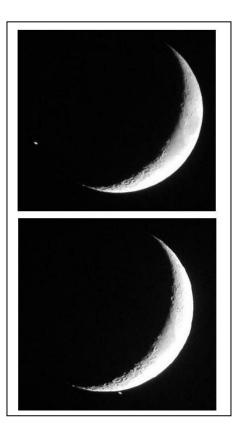


**Problem 2** - The table below gives the data for 10 known exoplanets. Calculate the missing tabular entries using Kepler's Third Law.

Table et exept	anot orbital pro	
Planet	Orbit Period (days)	Orbit Distance (AU)
Epsilon Eridani-b	2,502	3.6
Kepler-8b	3.8	0.048
WASP-5b	1.6	0.026
55 Cancri-b	4967	5.7
Fomalhaut-b	450,000	115
HD209458-b	3.5	0.044
Gliese-876-b	32.7	0.2
MOA-2005-BLG-390lb	4,800	5.6
HD-80606-b	111	0.5
Upsilon Andromedae-d	1530	2.6

	Table of e	exoplanet	orbital	properties
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# Occultation Timing and the Diameter of the Moon



Eclipses are spectacular to see, and transits can be dramatic especially when they involve solar system bodies, but the vastly more common occultations can provide hard-to-get information about distant celestial bodies.

Extracting this information begins with determining how long an occultation lasted, which is the most basic information you can gather.

These two images were taken by Amateur Astronomer Ray Emery in Rothwell, UK on April 2, 2002. It shows the occultation of Saturn by the Moon. The top image was taken at 20:59:50 UTC when the western-most edge of the ring of Saturn just touched the eastern edge of the disk of the moon. The bottom image was taken at 21:26:15 UTC when the easternmost edge of Saturn's ring emerged from behind the western edge of the moon at the end of the occultation.

Problem 1 - How many seconds did the occultation take from start to end?

**Problem 2** - Draw a chord through the disk of the moon along the track of Saturn. What fraction of the full diameter of the moon did the length of the occultation chord occupy?

**Problem 3** - If the occultation of Saturn followed exactly along the full diameter of the moon, how long, in seconds, would the occultation have taken?

**Problem 4** - Based on the orbit and distance of the moon, the speed of the moon perpendicular to the line connecting the center of Earth with the center of the moon is 1.3 km/sec. From the occultation timing information, what is the diameter of the moon in kilometers?

Problem 1 - How many seconds did the occultation take from start to end?

#### or 26 minutes and 25 seconds

**Problem 2** - Draw a chord through the disk of the moon along the track of Saturn. What fraction of the full diameter of the moon did the length of the occultation chord occupy?

Answer: The exact numbers will depend on the magnification used in reproducing the images. The ratio of the chord to the full diameter is about **0.6 or 3/5** 

**Problem 3** - If the occultation of Saturn followed exactly along the full diameter of the moon, how long, in seconds, would the occultation have taken?

Answer: 26:25 is decimal form = 26.42 which is 3/5 of the full diameter, so the full time to travel the diameter would be  $26.42 \times 5/3 = 44.0$  minutes which is **2,640** seconds.

**Problem 4** - Based on the orbit and distance of the moon, the speed of the moon perpendicular to the line connecting the center of Earth with the center of the moon is 1.3 km/sec. From the occultation timing information, what is the diameter of the moon in kilometers?

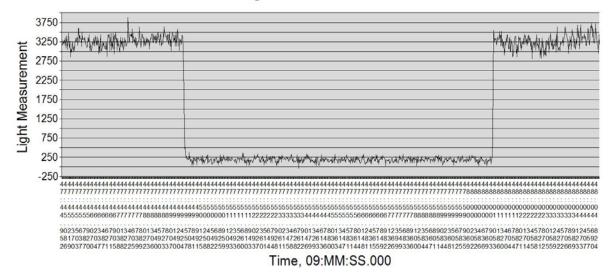
Answer: The diameter of the moon is just 1.3 km/sec x (2640 seconds) = 3,400 kilometers. The actual value is about 3,432 kilometers.

Image Credit: See http://www.popastro.com/sections/occ/satapr16.htm

Space Math

# Occultation Timing Light Curves

Palma Light Measurement Plot



Many occultations involve a planet or asteroid passing in front of a star. Because the start and end times must be known to fractions of a second for small occulting bodies, humans cannot be involved in taking the data. Instead, sensitive light meters called photometers measure the brightness of the target star, and then automatically record the brightness hundreds of time a second.

One such occultation is recorded in the photometric data in the above graph. The data were taken by amateur astronomer Ed Morana (edmor@pacbell.net) who lives in Livermore California and brings his equipment to many locations in the western United States to record occultations. His other timing data can be found at http://pictures.ed-morana.com/AsteroidOccultations/.

**Problem 1** - The horizontal axis gives the star brightness measurements from 09h 47m 45.952s to 09h 48m 04.824s. What is the time interval between measurements in units of seconds?

Problem 2 - At about what time did the occultation of asteroid Palma begin?

Problem 3 - At about what time did the occultation of asteroid Palma end?

Problem 4 - What was the total duration of the occultation?

**Problem 5** - At the time of the observation, Asteroid Palma was traveling at 1 kilometers/sec. About what is the width of this asteroid along the unknown chord measured by this occultation measurement.

**Problem 1** - The horizontal axis gives the star brightness measurements from 09h 47m 45.952s to 09h 48m 04.824s. What is the time interval between measurements in units of seconds?

Answer: From the two consecutive measurements at 45.086s and 45.219s the time interval is **0.133 seconds**.

Problem 2 - At about what time did the occultation of asteroid Palma begin?

Answer: Reading from left to right, the brightness of the star dipped at about 47:49.424

Problem 3 - At about what time did the occultation of asteroid Palma end?

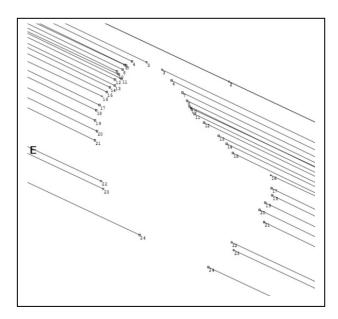
Answer: The star regained its former brightness at about **48:00.903**.

**Problem 4** - What was the total duration of the occultation?

Answer: 48m 00.903s - 47m 49.424 = 60.903 - 49.424 = **11.479 seconds**.

**Problem 5** - At the time of the observation, Asteroid Palma was traveling at 1 kilometers/sec. About what is the width of this asteroid along the unknown chord measured by this occultation measurement.

Answer: The width is about 1.0 km/sec x 11.5 seconds = **11.5 kilometers**.



If you know that the object doing the occulting has a circular shape, you can use one occultation timing measurement for a single chord of the circle to determine the physical diameter of the object once you know its speed. But suppose you cannot even see the shape of the object because it is too small to clearly distinguish in your telescope?

When a number of observers make occultation timings of an asteroid from a number of different locations, their timing 'tracks' outline the many different chords that make up the asteroid.

Amateur astronomers are very active in measuring asteroid occultations. The above map was created by combining the data from ten different amateur astronomers located in California, Oklahoma, and New Mexico. On December 11, 2008 the asteroid 135-Hertha occulted the faint star HIP-13021 located in the constellation Aries. The asteroid was at a distance of about 135 million kilometers from Earth.

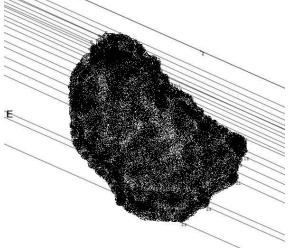
Like the path of a total solar eclipse, the shadow of the asteroid passes across the surface of Earth. The lines in the above figure show the location of this path at the various locations of the Observers who took part in the occultation study. The gaps in the lines show the locations (times) when the asteroid occulted the star from each station.

**Problem 1** - At the time of the occultation, draw a continuous line that shows the shape of the cross-section of asteroid 135-Hertha. Fill-in the shape to look like a dark asteroid!

**Problem 2** - The 'shadow' of the asteroid as it passes across a star can be thought of as a parallel cylinder of rays that strike Earth's surface and create a moving path across its surface. The width of this path, in kilometers, is equal to the projected width of the asteroid in kilometers. If the perpendicular distances between the northern-most (top) and southern-most (bottom) tracks in the above diagram was 80 kilometers, in kilometers, what is:

- A) The projected width of the asteroid?
- B) The projected length of the asteroid?
- C) The average diameter of asteroid 135-Hertha?

**Problem 1** - At the time of the occultation, draw a continuous line that shows the shape of the cross-section of asteroid 135-Hertha. Fill-in the shape to look like a dark asteroid!



**Problem 2** - The 'shadow' of the asteroid as it passes across a star can be thought of as a parallel cylinder of rays that strike Earth's surface and create a moving path across its surface. The width of this path, in kilometers, is equal to the width of the asteroid. If the perpendicular distances between the northern-most (top) and southern-most (bottom) tracks in the above diagram was 80 kilometers, in kilometers, what is: A) The projected width of the asteroid? B) The projected length of the asteroid? C) The average diameter of asteroid 135-Hertha?

Answer: A) Use a millimeter ruler to determine the scale of the image. For example, The perpendicular distance between the two tracks is about 50 millimeters so the scale is 80 km/50 mm = 1.6 km/mm. The width is about **60 kilometers**.

- B) The length is about 103 kilometers.
- C) The average diameter is about (103+60)/2 = 82 kilometers.

For more examples of amateur astronomers conducting occultation measurements see:

### http://scottysmightymini.com/PR/20090719Pretoria\_pr.htm

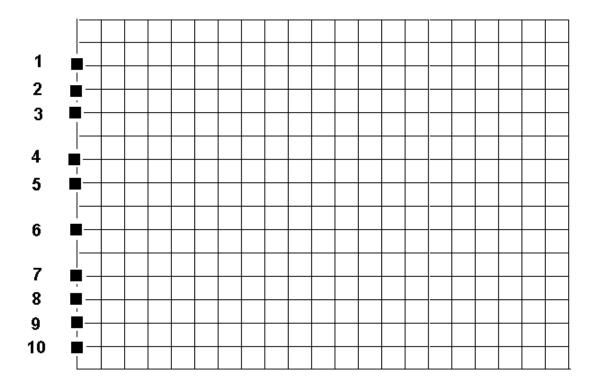
Also visit IOTA: the International Occultation Timing Organization http://www.lunar-occultations.com/iota/iotandx.htm

Observer	Start	End
1	11:15:08	11:15:13
2	11:15:07	11:15:21
3	11:15:04	11:15:14
4	11:15:02	11:15:14
5	11:15:04	11:15:15
6	11:15:06	11:15:17
7	11:15:08	11:15:16
8	11:15:09	11:15:16
9	11:15:11	11:15:18
10	11:15:13	11:15:16

When occultation timings from several stations are combined, the shape of the occulting body can be determined (i.e. its cross-section).

The table to the left gives the locations and timings for when the star was behind the asteroid as viewed by 10 Observers who watched the occultation of the star XYZ1325 by Asteroid Phoenix.

The geographic locations (X,Y) of the Observers are given in kilometers on a N-S and E-W grid. The times are given in minutes and seconds from the start of 11:00 p.m Local Time.



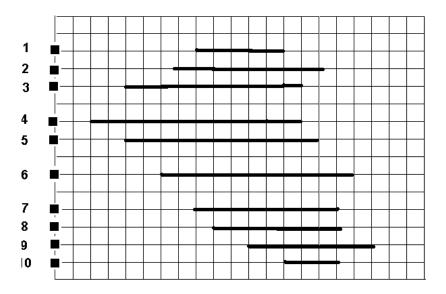
**Problem 1** - The position-time grid above is marked in 1-km squares. In the horizontal direction, each grid represents the distance traveled by the shadow in 1 second. Assuming that the path of the asteroid shadow is from left to right exactly parallel to the horizontal axis at each station, and that the left-hand edge is the location of the shadow at exactly 11:15 p.m, draw the occultation chords.

**Problem 2** - What were the approximate dimensions of the asteroid in kilometers?

#### Space Math

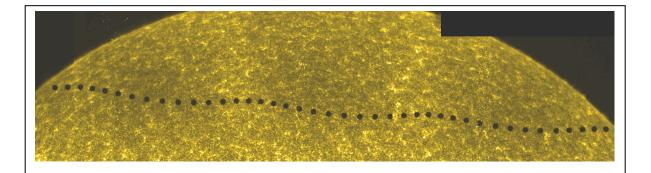
**Problem 1** - The position-time grid above is marked in 1-km squares. In the horizontal direction, each grid represents the distance traveled by the shadow in 1 second. Assuming that the path of the asteroid shadow is from left to right exactly parallel to the horizontal axis at each station, and that the left-hand edge is the location of the shadow at exactly 11:15 p.m, draw the occultation chords.

Answer: Example: For Station 1, the occultation chord begins at 11:15:08 and ends at 11:15:13. Since each horizontal cell corresponds to 1 second, the chord will be drawn starting at 8-cells (8 seconds) to the right of the left vertical edge of the graph (at 11:15:00), and extend 5 cells to the right.



Problem 2 - What were the approximate dimensions of the asteroid in kilometers?

Answer: Each square corresponds to a vertical distance of 1 km, and a time-interval that also corresponds to 1 kilometer. The vertical 'length' of the asteroid is about 12 kilometers, and the horizontal width varies from about 8 kilometers to 11 kilometers.

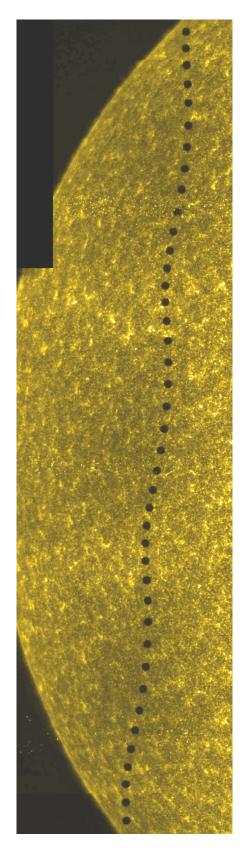


NASA's TRACE satellite is in a polar orbit around Earth. On May 7, 2003 it took a series of pictures of the sun during the time that the planet Mercury was in transit. The track of the planet disks across the face of the sun has a pronounced wiggle caused by the satellite's north-south movement along its orbit as it took the images. The TRACE satellite orbit had a radius of R=6,698 kilometers. The parallax shift seen in the above wiggle of Mercury disks was caused by the satellite as its vantage point changed by 2R.

**Problem 1** - At the time of the transit, the angular diameter of the sun was 0.53 degrees. The angular scale of the image above is 0.0028 degrees/millimeter. About what is the diameter of Mercury in A) degrees? B) seconds of arc?

**Problem 2** - Draw two parallel lines from left to right so that: A) The first line connects the centers of the Mercury disks at the top of the sequence of disks. B) The second line connects the centers of the Mercury disks at the bottom of the sequence of disks. What is the angular separation,  $\alpha$ , of the two parallel lines in degrees?

**Problem 3** - The above photo montage was created by removing the solar parallax angle from each disk image and then superimposing the disks. The total parallax angle,  $\theta$ , is then  $\theta = \alpha/2 + 0.0019$  degrees. The parallax formula states that Tan( $\theta$ ) = R/D. If R is the radius of the TRACE satellite orbit, what is the distance, D, from Earth to Mercury at the time of the transit?



**Problem 1** - Answer: Measure several mercury disks in the image and take their average. The images are about 1.2mm in diameter, so their angular diameter is A)  $1.2 \times 0.0028 \text{ deg/mm} = 0.0033 \text{ degrees}$  B)  $0.0033 \text{ degrees} \times (60 \text{ minutes/1 degree}) \times (60 \text{ seconds/1 minute}) = 12 \text{ arcseconds}$  in diameter.

**Problem 2** - The separation of the lines should be about 2.0 millimeters. Since the scale is 0.0028 degrees/mm, the separation is 2.0 mm x 0.0028 deg/mm = **0.0056 degrees**.

**Problem 3** - The above photo montage was created by removing the solar parallax angle  $\alpha/2$  from each disk image and then superimposing the disks. The total parallax angle,  $\theta$ , is then  $\theta = \alpha/2 + 0.0019$  degrees. The parallax formula states that Tan( $\theta$ ) = R/D. What is the distance, D, from Earth to Mercury at the time of the transit given the orbital radius, R, of the satellite?

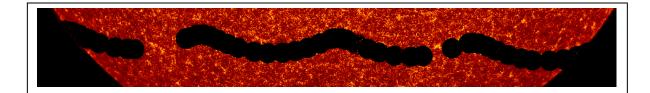
Answer:  $\theta = (0.0056/2) + 0.0019$ = 0.0047 degrees.

D = 6698/tan(0.0047) = 82,000,000 km.

Note that a 0.2mm uncertainty in measuring the parallel line separations yields distances between

 $\theta$  = 0.0050 to 0.0044 so D = 77 million to 87 million km.

the actual distance between Earth and Mercury was 82 million km.

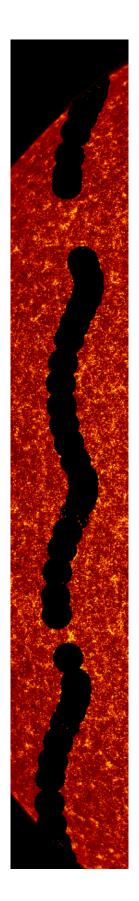


NASA's TRACE satellite is in a polar orbit around Earth. On June 8, 2004 it took a series of pictures of the sun during the time that the Venus was in transit. The wavy string of superposed Venus disks is caused by the satellite, which orbits Earth in a polar orbit, changes its vantage point from one side to the other of Earth's center by a projected distance of 6,698 km, and so the disk of Venus appears, first lower, then higher than its average dead-on position. This is a familiar parallax effect that can be used to determine the distance to Venus.

**Problem 1** - The width of the disks in the wavy black line is equal to the diameter of Venus, which had an angular diameter of 0.0016 degrees on this date. Draw two parallel lines that connect the A) centers of the disks at the top of the wave and B) the centers of the disks at the bottom of the wave. If the scale of this image is 0.0032 degrees/millimeter, what is the angular separation of the lines in degrees which defines the angle  $\alpha$ ?

**Problem 2** - The image was created by subtracting a solar parallax angle of 0.0024 degrees from each image, and then superimposing the images to show the movement of Venus across the sun. The total parallax angle,  $\theta$ , is then  $\theta = \alpha/2 + 0.0024$  degrees. The parallax formula states that  $Tan(\theta) = R/D$ . What is the distance, D, from Earth to Venus at the time of the transit given the radius, R, of the satellite's orbit around Earth?

**Problem 3** - During the previous Transits of Venus in the years 1874 and 1882, one of the most important aspects of our model of the solar system was to convert the predicted distances given in terms of the Earth-Sun distance unit ( called the Astronomical Unit) into actual physical distances such as miles or kilometers. Careful studies of the track taken by the transit of Venus across the sun's disk could be geometrically used to determine the parallax distance to Venus at the time of the transit. For the 2004 transit of Venus, the modeled distance from Earth to Venus was 0.29 Astronomical Units. From your answer to Problem 2, what would you estimate as the length of the Astronomical Unit in kilometers?



**Problem 1** - The width of the disks in the wavy black line is equal to the diameter of Venus, which had an angular diameter of 0.0016 degrees on this date. Draw two parallel lines that connect the A) centers of the disks at the top of the wave and B) the centers of the disks at the bottom of the wave. If the scale of this image is 0.0032 degrees/millimeter, what is the angular separation of the lines in degrees which defines the angle  $\alpha$ ?

Answer: The separation is about 4 millimeters, so the angular separation is 4 mm x 0.0032 degrees/mm = 0.013 degrees.

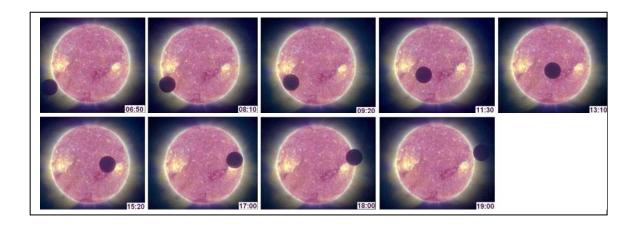
**Problem 2** - The image was created by subtracting a solar parallax angle of 0.0024 degrees from each image, and then superimposing the images to show the movement of Venus across the sun. The total parallax angle,  $\theta$ , is then  $\theta = \alpha/2 + 0.0024$  degrees. The parallax formula states that Tan( $\theta$ ) = R/D. What is the distance, D, from Earth to Mercury at the time of the transit given the radius, R, of the satellite's orbit around Earth?

Answer:  $\theta = 0.013/2 + 0.0024 = 0.0089$  degrees. so D = 6,698 km / tan(0.0089) = **43,000,000 km**.

**Problem 3** - For the 2004 transit of Venus, the modeled distance from Earth to Venus was 0.29 Astronomical Units. From your answer to Problem 2, what would you estimate as the length of the Astronomical Unit in kilometers?

Answer: 0.29 AU = 43 million km, so 1.0 AU = 43 million/0.29 AU = **145 million km**.

Note: The actual, very accurate, adopted length has been established as 1 AU = 149,597,870.7 kilometers. This has been determined by bouncing radio signals off of a variety of solar system bodies (e.g. asteroids) whose distances in AU have been precisely predicted, and using the time it takes light to travel out and back at a speed of 299,792 km/sec.

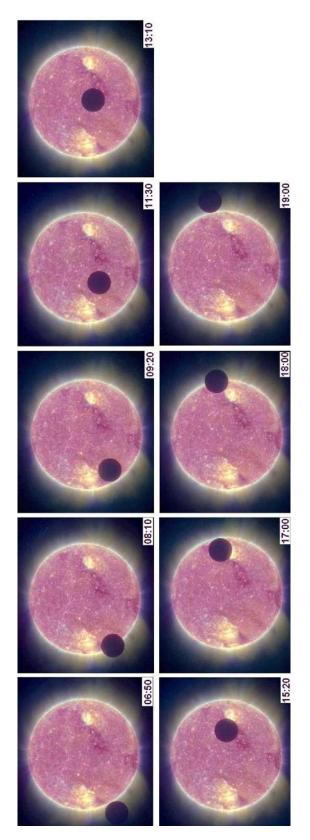


NASA's STEREO-B satellite is in an orbit around the sun at the same distance as Earth. On February 25, 2007 it took a series of pictures of the sun during the time when the moon was in transit across the sun, and when the satellite was 1.7 million km from the moon. The normal Earth-Moon distance is 380,000 km. By comparison, the distance to the sun from Earth is 149 million km.

**Problem 1** - If the angular diameter of the sun was 2100 arcseconds at the time of the transit as viewed by STEREO-B, what was the diameter of the moon as viewed by STEREO-B in A) arcseconds? B) degrees?

**Problem 2** - By what percentage was the sun's light dimmed during the times when the full, circular, lunar disk covered the solar surface in these images?

**Problem 3** - Based on the sequence of images in the above series, with the Universal Time (hour:minutes) indicated in the lower right corner of each image, draw the light curve of this lunar transit from start to finish in terms of the percentage of sunlight visible by the STEREO-B satellite, from 93% to 100%, and the Universal Time in decimal hours since 06:00.



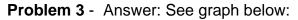
**Problem 1** - Method 1: The diameter of the solar disk in each image is about 21 millimeters, so the scale of the images is 2100 arcseconds/21 mm = 100 arcseconds/mm. The lunar disk measures about 4mm, so the angular diameter is 4mm (100 asec/mm) = **400 arcseconds**.

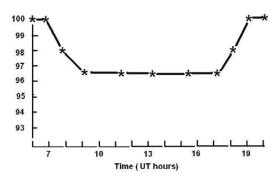
Method 2: Using the trigonometric formula  $Tan(\theta) = D/R$   $Tan(\theta) = 3400/1,700,000 = 0.002$ So  $\theta = 0.11$  degrees = **410 arcseconds**.

So the diameter of the moon is about **400** arcseconds

**Problem 2** - By what percentage was the sun's light dimmed during the times when the full, circular, lunar disk covered the solar surface in these images?

Answer: By the ratio of their circular areas.  $100\% \times (400/2100)^2 = 3.6\%$ 

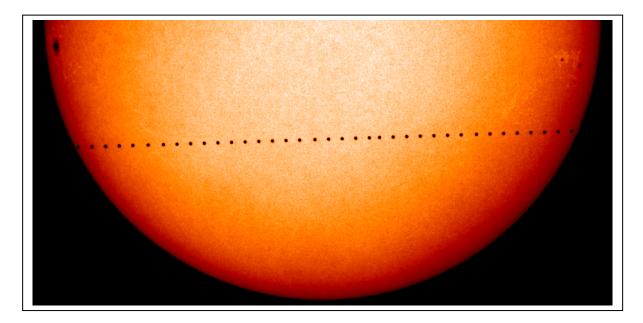




Movies of the transit can be found at the STEREO website

http://stereo.gsfc.nasa.gov/gallery/item.p hp?id=selects&iid=8

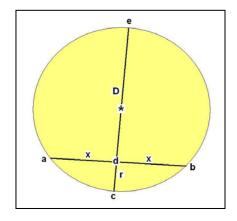
http://spacemath.gsfc.nasa.gov



On November 8, 2006 NASA's SOHO satellite took a series of pictures of the sun during the time that the Mercury was in transit. This photo montage shows the transit between 19:12:04 Universal Time on November 8 and 00:10:08 Universal Time on November 9. The path of the transit follows a <u>chord</u> across the circular face of the sun. The image only shows a portion of the face of the sun, not the full disk. This provides an opportunity to use a basic property of chords top determine the full diameter of the solar disk, and the perpendicular distance between the chord and the center of the sun.

**Problem 1** - For any chord, the perpendicular bisector of the chord coincides with a diameter of the circle, and passes through the center of the circle. From the figure below, and the properties of similar triangles, D/X = X/r. Use this proportionality, and a millimeter ruler, to determine the diameter of the sun at the scale of the image above.

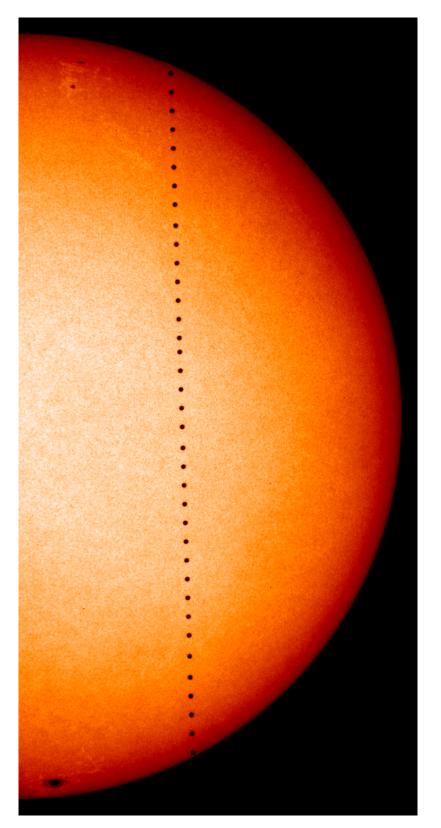
**Problem 2** - The angular diameter of the solar disk at the time of the transit was 1850 seconds of arc. From your answer to Problem 1, A) what is the scale of the image in arcseconds/millimeter? B) What was the minimum distance between the center of the sun and the path of the transit, to the nearest second of arc?



In the figure to the left, the segment lengths are related to the proportion variables, x, r, and D as follows:

> Segment cd = rSegment ad = bd = xSegment de = D

http://spacemath.gsfc.nasa.gov



**Problem 1** - Answer: On the front image, the transit path has a length of 134 millimeters, so x = 134/2 = 66 millimeters. Constructing the perpendicular bisector of this transit chord, the segment corresponding to DC in the figure measures 42 millimeters, then from the proportion:

 $\frac{D}{x} = \frac{x}{r}$  we have  $D = \frac{66^2}{42}$ So D = 104 mm.

The full diameter d = D + r so d = 146 mm.

Problem 2 - Answer: A) Scale = 1850 asec/146 mm = 12.7 arcseconds/mm.

B) The shortest distance is just

S = (D+r)/2 - r = (146 mm)/2 - 42 mm = 31 mm

And the corresponding angular distance is just 31 mm x (12.7 asec/mm) = **394 arcseconds**.



On December 29, 2009 amateur astronomer Alex McConahay captured this sequence of images of the International Space Station (ISS) transiting the moon at 8:30:43 p.m from a location in Moreno Valley, California.

The center to center distance of the moon from Earth was 364,794 km with a lunar radius of 1,738 km. The image speed was 15 frames per second. The ISS has the dimensions 108 x 73 meters.

The angular size in degrees, q, of a distant object depends on its actual physical size, R, and its distance from the Observer, D, according to the basic formula:

$$Tan(\theta) = \frac{R}{D}$$
 which for angles less than a degree becomes  $\theta = 57.3 \frac{R}{D}$ 

**Problem 1** - From the given measurements, what is the angular diameter in arcseconds of the Moon?

**Problem 2** - The maximum angular width of the ISS at the time of the transit was 44 seconds of arc. How far from the ISS was the photographer when this image was taken?

Problem 3 - About how long, in seconds, did the transit of the ISS take?

Problem 4 - At the distance of the ISS, how fast was the ISS moving in km/s?

**Problem 5** - How many meters from the photographer's location would the track of the ISS have just missed the lower-right limb of the moon in this picture?



Problem 1 - Answer: 57.3 x (2x1738)/364794 = 0.546 degrees or 1,966 arcseconds

Problem 2 - Answer;  $D = 57.3 \times (3600 \text{ asec/deg}) \times 108 \text{ meters}/44 = 506 \text{ kilometers}.$ 

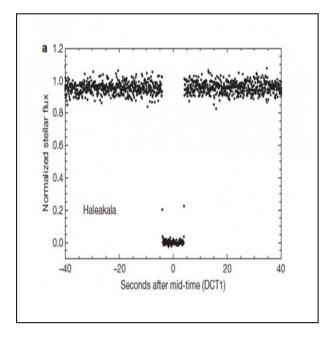
**Problem 3** - Answer: About seven time intervals x 1/15 sec/interval = 7/15 sec = **0.47** seconds

**Problem 4** - Answer: The scale of the image is 1966 asec/68mm = 29 asec/mm. The length of the transit chord is 62mm so the angular length is 1798 asec. At 506 km, this corresponds to R =  $1798/(57.3x3600) \times 506$  km = 4.4 kilometers. This took 0.47 seconds so the speed along its orbit was V=4.4 km /0.47sec ; V = 9.4 km/sec.

**Problem 5** - At the distance of the ISS, the chord must shift an angular distance equal to the midpoint of the chord to the limb of the moon. On the photo this is a distance of 23 mm or  $23 \times (29 \text{ asec/mm}) = 667 \text{ asec}$ . From the formula, R = 667 / (57.3 x 3600) x 506 km R = 1.6 kilometers.

Image courtesy Alex McConahay alexmcconahay@roadrunner.com Amateur astronomers use CalSky http://www.calsky.com/ to predict when the ISS will transit the sun and moon from a given geographic location.

Space Math



The Kuiper Belt Object KBO-55636 orbits the sun beyond the orbit of Neptune. It is believed to be one of over 70,000 similar bodies traveling in the dark outskirts of our solar system at a geocentric speed of about 25 km/s.

Knowing the detailed orbit of this previously-discovered body, astronomers predicted that on October 9, 2009 it would occult an unnamed star. They succeeded in detecting its occultation light curve shown to the left, and 55636 became the first KBO to be observed during an occultation.

**Problem 1** - About how long did the transit take as viewed from Haleakala, Hawaii according to the light curve above?

**Problem 2** - As viewed from Earth, a 1-meter shift at the distance of the KBO corresponds to 1-meter displacement of the KBO shadow along the occultation track at Earth's surface. From the timing information and the geocentric speed of the occultation, what was the approximate diameter of the KBO assuming that it was perfectly round?

**Problem 3** - If the angular size in seconds of arc,  $\theta$ , is related to the diameter, L, and distance, D, to an object in kilometers according to  $\theta$  = 206265 (L/D), what is the average angular diameter of KBO-55636 if the distance to this object is D = 6,422 million km ?

**Problem 1** - About how long did the transit take as viewed from Haleakala, Hawaii according to the light curve above?

Answer: The light curve shows that before the transit the brightness was about 0.95 stellar flux units, and dimmed to zero for a full occultation (eclipse actually!) between -5 sec and +5 seconds around the center of the occultation, for a duration of **10 seconds**.

**Problem 2** - As viewed from Earth, a 1-meter shift at the distance of the KBO corresponds to 1-meter displacement of the KBO shadow along the occultation track at Earth's surface. From the timing information and the geocentric speed of the occultation, what was the approximate diameter of the KBO assuming that it was perfectly round?

Answer:  $D = 10 \sec x (25 \text{ km/sec}) = 250 \text{ kilometers.}$ 

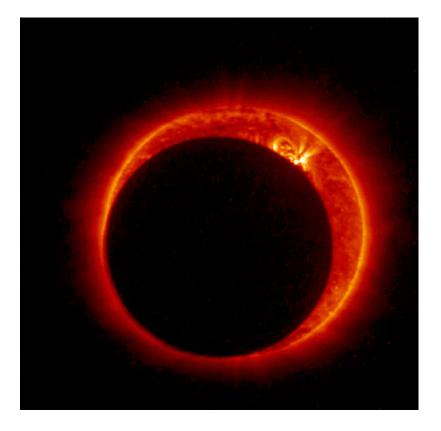
**Problem 3** - If the angular size in seconds of arc,  $\theta$ , is related to the diameter, L, and distance, D, to an object in kilometers according to  $\theta = 206265$  (L/D), what is the average angular diameter of KBO-55636 if the distance to this object is D = 6,422 million km ?

Answer: From the angle formula:  $\theta = 206265 \times (250 \text{ km}/6422 \text{ million km})$  so  $\theta = 0.008 \text{ arcseconds}.$ 

Note: After measuring the exact amount of time that the star was blocked from view, as well as the velocity with which the shadow of 55636 moved across Earth, the researchers calculated that the KBO has a diameter of about 286 kilometers.

J. L. Elliot, M. J. Person, C. A. Zuluaga, A. S. Bosh, E. R. Adams, T. C. Brothers, A. A. S. Gulbis, S. E. Levine, M. Lockhart, A. M. Zangari, B. A. Babcock, K. DuPré, J. M. Pasachoff, S. P. Souza, W. Rosing, N. Secrest, L. Bright, E. W. Dunham, S. S. Sheppard, M. Kakkala, T. Tilleman, B. Berger, J. W. Briggs, G. Jacobson, P. Valleli, B. Volz, S. Rapoport, R. Hart, M. Brucker, R. Michel, A. Mattingly, L. Zambrano-Marin, A. W. Meyer, J. Wolf, E. V. Ryan, W. H. Ryan, K. Morzinski, B. Grigsby, J. Brimacombe, D. Ragozzine, H. G. Montano, A. Gilmore. Size and albedo of Kuiper Belt object 55636 from a stellar occultation. Nature, 2010; vol. 465, pp. 897

# An Eclipse of the Sun from Space



This spectacular image of the moon passing across the face of the sun was taken by ESA's Proba 2 satellite on January 15, 2010 while millions of people in South Africa watched the 'annular eclipse' event from the ground. The satellite orbits Earth at an altitude of 750 km.

The face of the sun is imaged by Proba-2 at ultraviolet wavelengths similar to its sister satellite, SOHO, located at the L1 point over 1.5 million km from Earth. A spectacular 'active region' can be seen in the image in the upper right.

No matter where you are in the solar system, or beyond, total solar eclipses are actually very rare. They require that the diameter of the star, the eclipsing body, and the observer be just-right so that as seen from the observer's location, the eclipsing body exactly covers the disk of the star as viewed by the observer.

**Problem 1** - The sun has a diameter of 1,392,000 km. At the time of this photograph, Earth was located 147 million km from the sun, and the solar angular diameter was 1,953 arcseconds. Meanwhile, the moon had a diameter of 3,476 kilometers. How far from the moon was the Proba-2 spacecraft at the time of this photo?

**Problem 2** - In the distant future, a photographer from National Geographic wants to capture the total eclipse of the sun by Europa as viewed from a spacecraft near Jupiter's satellite Europa. Jupiter is located 770 million km from the sun, and Europa has a diameter of 2,900 km. How far from Europa must the spacecraft be in order that the photographer can view a total solar eclipse?

**Problem 1** - The sun has a diameter of 1,392,000 km. At the time of this photograph, Earth was located 147 million km from the sun, and the solar angular diameter was 1,953 arcseconds. Meanwhile, the moon had a diameter of 3,476 kilometers. How far from the moon was the Proba-2 spacecraft at the time of this photo?

Answer: At what distance from the moon, D, will the moon have the same angular diameter as the sun of 1,953 seconds of arc? Since the angles are the same, the distances have to be in proportion:

 $\frac{1,392,000}{147,000,000} = \frac{3,476}{X}$  so X = **367,000 kilometers.** 

**Problem 2** - In the distant future, a photographer from National Geographic wants to capture the total eclipse of the sun by Europa as viewed from a spacecraft near Jupiter's satellite Europa. Jupiter is located 770 million km from the sun, and Europa has a diameter of 2,900 km. How far from Europa must the spacecraft be in order that the photographer can view a total solar eclipse?

Answer: At this distance, the angular diameter, x, of the sun can be deduced from the proportion:

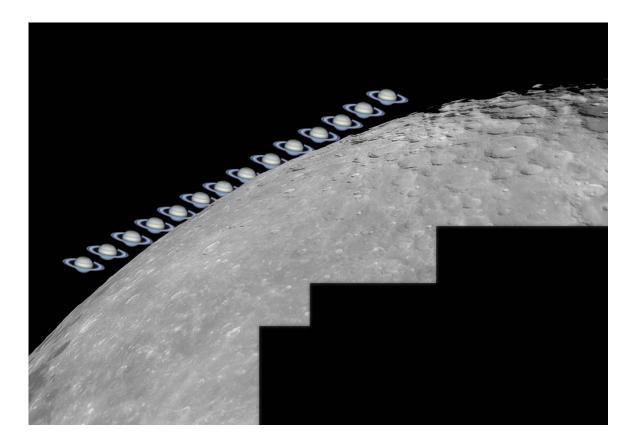
 $\frac{x}{1953} = \frac{147 million}{770 million}$  so x = 373 arcseconds.

The angular size of Europa from the spacecraft must equal 373 arcseconds, so since the angles are the same, the distances have to be in proportion:

 $\frac{d}{770 million} = \frac{2900 km}{1,392,000 km}$ 

so d = **1.6 million kilometers** from Europa.

# A Grazing Occultation of Saturn by the Moon



Although transits and eclipses are dramatic, many interesting events called grazing occultations also occur, especially with our moon. This image was taken on March 3, 2007 by UK amateur astronomer Pete Lawrence (Image courtesy *Digital-Astronomy*) and shows the moon passing close-by Saturn, and partially occulting its rings in a series of images spaced every 90 seconds. To catch transits, occultations and grazing occultations, the astronomer has to be very close to the track of the event as it races across the surface of Earth. Because small angles are involved in getting the occultation track exactly right, we use the formula  $\theta = 206265(L/D)$  where  $\theta$  is the angular size in arcseconds; L is the diameter of the object in km, and D is the distance in km of the occulting object from the Observer's location.

**Problem 1** - At the time of the grazing occultation, Saturn had a maximum angular diameter (disk + rings) of about 25 arcseconds, the moon's distance from the surface of Earth was about 384,000 km, and the moon's diameter is (as always!) 3,476 km. How many kilometers, L, would you have to move to shift the moon's sky location by exactly the angular diameter of Saturn and its rings?

**Problem 2** – From the beginning to the end of this photo sequence, how much time elapsed?

**Problem 3** – Occultation timing is an important tool to verify our understanding of the exact orbit of a body such as the moon. If the predicted center of the moon had been shifted 60 kilometers to the upper right, what would the photo have shown?

**Problem 1** - At the time of the grazing occultation, Saturn had a maximum angular diameter (disk + rings) of about 25 arcseconds, the moon's distance was about 384,000 km and the moon's diameter is (as always!) 3,476 km. AS viewed from Earth, how many kilometers, L, would you have to move to shift the moon's sky location by exactly the angular diameter of Saturn?

Answer: Solve for L:  $25 = 206265 \frac{L}{384000 km}$  to get L = **47 kilometers.** 

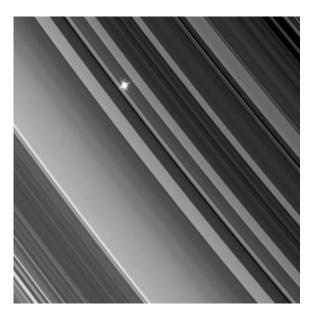
**Problem 2** – From the beginning to the end of this photo sequence, how much time elapsed?

Answer: There were 14 images spaced 90 seconds apart, so the total elapsed time was  $T = 14 \times 90 \text{ s} = 1,260$  seconds or **21 minutes**.

**Problem 3** – Occultation timing is an important tool to verify our understanding of the exact orbit of a body such as the moon. If the predicted center of the moon had been shifted 1 kilometer to the upper right, what would the photo have shown?

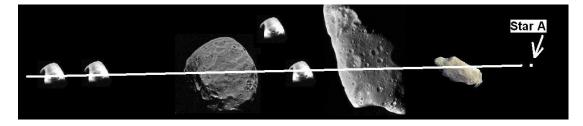
Answer: At the distance of the moon, a 60 km shift corresponds top an angular shift of  $Q = 206265 \times (60 \text{ km}/384000 \text{ km}) = 32 \text{ arcseconds}.$ 

This is much larger than the 25 arcsecond diameter of Saturn and its rings, so instead of a grazing occultation, the photograph would have shown an actual full occultation with the disk of Saturn completely hidden by the limb of the moon in the 7<sup>th</sup> and 8<sup>th</sup> images.



On September 26, 2006 NASA's Cassini spacecraft took this picture of a portion of Saturn's rings occulting a distant star from a distance of 543,000 km.

Occultation observations point the Cassini camera toward a star whose brightness is well known. Then, as Cassini watches the rings pass in front of the star, the star's brightness changes as it is blocked by numerous particles too small to directly image.



The figure above shows the path taken by Star A from right to left as viewed from a spacecraft equipped with a sensitive light meter. Suppose that for this occultation, there are seven pieces of rock/ice in this imaginary portion of the ring of Saturn. At the location of the spacecraft, the ring particles are too small to be visible. The relative speeds of the spacecraft and the ring particles, however, causes the star to occult these particles along the path shown, and the occultation takes exactly 1 second from the time the star disappears behind the right-hand edge of the rock on the far-right, to time that the star emerges from behind the left-hand edge of the rock on the far-left.

**Problem 1** - Using a millimeter ruler, what is the scale of the image above in seconds/millimeter?

**Problem 2** - From the locations and widths of the rocks along the occultation path, draw a curve that represents the brightness changes of the star during the occultation of the ring particles.

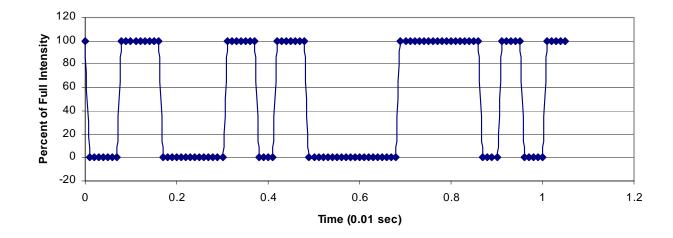
**Problem 3** - If the relative speed of the spacecraft and the rocks is 1 km/sec, how wide are the rocks, in meters, based on the occultation information?

**Problem 1** - Answer: the distance between the asteroid edges is about 112 millimeters, so this takes 1 second during the occultation and so the scale is S = 1 sec/112 mm

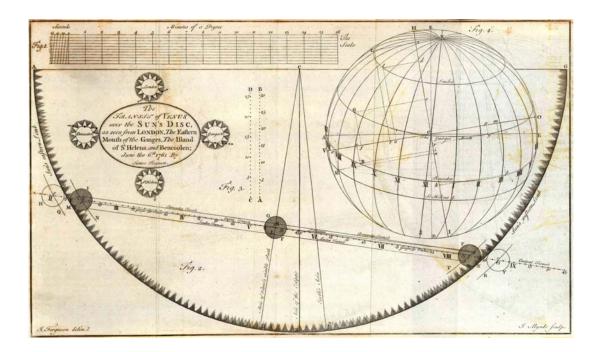
#### = 0.009 seconds/mm.

	0	
Occulting	Distance	Time
Rock Number	from start	from start
(from right to	(mm)	(seconds)
left)		
1	0	0
1	8	0.072
2	19	0.171
2	34	0.306
3	41	0.369
3	46	0.414
4	54	0.486
4	76	0.684
5	96	0.864
5	101	0.909
6	107	0.963
6	112	1.000

**Problem 2** - Table gives time intervals, which are graphed below:



Problem 3 - Answer: Rock1: Duration of occultation = 0.072 sec, length = 0.072 sec x 1000 meters/sec, so Rock 1 = 72 meters. Rock 2 = 135 meters; Rock 3 = 45 meters; Rock 4= 198 meters; Rock 5= 45 meters and Rock 6= 45 meters.



As Venus transits the solar disk, its track defines a chord, whose length and location depend on the position of the observer on Earth's surface. Two observers on Earth located far apart (North-South) will observe slightly different chord tracks parallel to each other. From this parallax shift, the distance to Venus can be determined.

The complicated geometric figure above was drawn by James Ferguson in his 1790 book 'Astronomy Explained Upon Sir Isaac Newton's Principles', for the June 6, 1761 Transit of Venus. It shows the chord taken by this transit as observed from London, England. The scale of this image reproduction is 13 arcseconds per millimeter.

**Problem 1** - The distance to Venus from Earth during the 1761 transit was about 40 million km. Suppose that astronomers located at the same longitude as London, but 8,000 km due south viewed the same transit. The parallax formula states that

$$\theta = 206265 \frac{L}{D}$$

where L is the viewing shift distance in km, D is the distance to Venus, and q is the resulting angular shift in seconds of arc. Where will the new transit chord be located on the solar disk from this southern position for the observers?

**Problem 1** - The distance to Venus from Earth during the 1761 transit was about 40 million km. Suppose that astronomers located at the same longitude as London, but 8,000 km due south viewed the same transit. The parallax formula states that

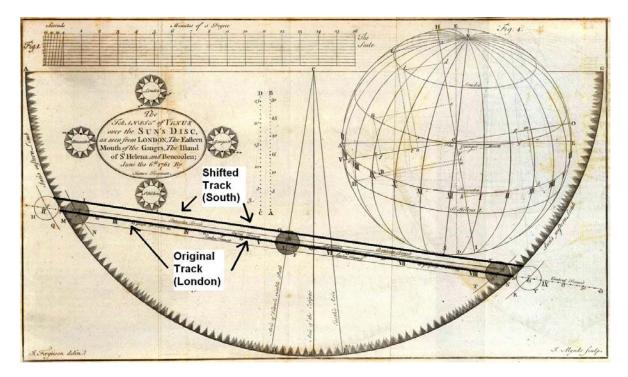
$$\theta = 206265 \frac{L}{D}$$

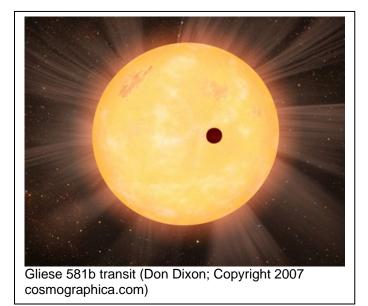
where L is the viewing shift distance in km, D is the distance to Venus, and q is the resulting angular shift in seconds of arc. Where will the new transit chord be located on the solar disk from this southern position for the observers?

Answer: The parallax shift angle would be

 $\theta = 206265 \frac{8,000 \text{ km}}{40 \text{ million km}}$  so  $\theta = 41 \text{ arcseconds}$ .

The scale of this figure is 13 arcseconds/mm, so on the figure, the shift will be about 41 asec x (1 mm/13 asec) = 3 millimeters. Because the new observers are located south of the London observers, the new track will shift upwards by 3 millimeters, and parallel to the London track as shown below.





As viewed from Earth. Mercury and Venus can transit our sun and provide a rare glimpse of planetary motion in our solar system. In the last 10 years, discovered astronomers have nearlv 1000 planets orbitina hundreds of other stars near our sun. One of these, the red dwarf star Gleise 581 has 6 planets orbiting within 150 million kilometers of their star. This provides some interesting transit opportunities in this very compact planetary system!

The angular diameter,  $\theta$ , in seconds of arc of an object whose physical diameter is L kilometers, as viewed from a distance of D kilometers, is given by

$$\theta = 206265 \frac{L}{D}$$

For example, our moon has a diameter of L=3,400 kilometers and distance, D = 384,000 kilometers, so from Earth, its angular diameter is about  $\theta$  = 1,800 arc seconds.

**Problem 1** - Suppose that observers were located on the outermost planet Gliese 581f and viewing the other planets as they transited the face of Gliese 581. The diameter of this star is about 400,000 km. A) What is the angular diameter of the star from the surface of Gliese 581f? B) From the table below, what are the angular diameters of the interior planets b-g viewed from Gliese 581f? C) Which planet would appear to be the largest dark 'spot' on the face of Gliese 581?

			1	
Planet	Relative	Distance	Diameter	Angular
	Speed	(million	(km)	Diameter
	(asec/hr)	km)		(asec)
Gliese 581 e	94	5	15,000	
Gliese 581 b	68	6	50,000	
Gliese 581 c	45	11	20,000	
Gliese 581 g	33	22	20,000	
Gliese 581 d	25	33	25,000	
Gliese 581 f		114	25,000	

#### Properties of exoplanets in the Gliese system

**Problem 2** - Assuming that the planets transit across the full diameter of the star, from the orbital speeds noted in the table, how long would each transit take as viewed from Gliese 581f? (Note: In our solar system, the 2012 transit of Venus will take about 7 hours.

Space Math

**Problem 1** - Suppose that observers were located on the outermost planet Gliese 581f and viewing the other planets as they transited the face of Gliese 581. The diameter of this star is about 400,000 km. A) What is the angular diameter of the star from the surface of Gliese 581f? B) From the table below, what are the angular diameters of the interior planets b-g viewed from Gliese 581f? C) Which planet would appear to be the largest dark 'spot' on the face of Gliese 581?

Answer: A) Star diameter = 206265 (400,000 km/114,000,000 km) = **724 arcseconds**.

Note Our sun has a diameter of 1800 arcseconds as viewed from Earth so Gliese 581 is about 40% as big as our sun as viewed from Gliese 581f.

B) See entries in last column. Example For Gliese 581e is located 114 million -5 million = 109 million km from Gliese 581f, so its angular diameter will be, q = 206265 (15000/109,000,000) = 28 arcseconds.

C) The star has a diameter of 724 arcseconds, so the planets will subtend the following percentages of the solar disk;

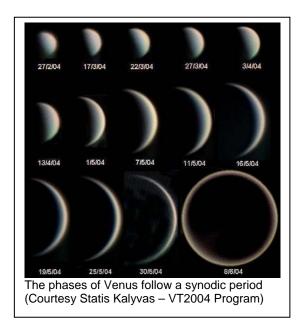
Gliese  $581e = 100\% \times 28/724 = 4\%$ . Similarly: Gliese 581b = 13%, Gliese 581c = 6%; Gliese 581g = 6% and Gliese 581d = 9%.

Note: By comparison, Venus subtends only 3 % of the solar diameter as viewed from Earth.

Planet	Speed (asec/hr)	Distance (million	Diameter (km)	Angular Diameter
		km)		(asec)
Gliese 581 e	94	5	15,000	28
Gliese 581 b	68	6	50,000	95
Gliese 581 c	45	11	20,000	40
Gliese 581 g	33	22	20,000	45
Gliese 581 d	25	33	25,000	64
Gliese 581 f		114	25,000	

**Problem 2** - Assuming that the planets transit across the full diameter of the star, from the orbital speeds noted in the table, how long would each transit take as viewed from Gliese 581f? (Note: In our solar system, the 2012 transit of Venus will take about 7 hours.)

Answer: The star's diameter from Gliese 581f is 724 arcseconds, so for Gliese 581e the transit time is about T = 724/94 = 7.7 hours. The other times are as follows: Gliese 581b = 10.6 hours; Gliese 581c = 16.1 hours; Gliese 581g = 21.9 hours and Gliese 581d = 29.0 hours.



One evening at 6:00 PM, you happen to look towards the western sky at sunset and notice the very bright planet Venus high up in the western sky. A few weeks later you look for it again, but now at 6:00 PM it is lower in the sky, and a week later it is even lower. Then, it sets at the same time as the sun, and now you have to get up before sunrise to start seeing Venus climb higher up in the sky.

You would really like to know how long you need to wait in order to see Venus at its highest point in the sky just at sunset. To do this calculation, you need to know the synodic period of Venus as seen from Earth.

The synodic period, S, of a body is defined as the time it takes the body to re-appear in the same spot of the observer's sky. Or example, the phases of Venus depend on the location of Venus, Earth and the sun, so from Earth, you have to wait for the proper positions to re-occur so that you can see the illuminated portion of Venus in the same phase as before. The distance of Venus from the Sun as seen from Earth also follows a synodic cycle. If you were to plot the distance of Venus from the sun in degrees, over a period of time, you would get a 'sine' curve.

For a solar system body inside the orbit of Earth viewed from Earth, where E = 365 days, and the orbital period of the body is P, the synodic period, S is given by the simple formula:

$$\frac{1}{S} = \frac{1}{P} - \frac{1}{E}$$

**Problem 1** – An astronomer notes that the separation between Venus and the sun on the near-side of the sun is exactly zero degrees on January 1, 2010. If E = 365.24 days and P = 224.7 days, A) how many days will the astronomer have to wait to see Venus once again reach a separation of zero degrees on the near-side of the sun? B) On what date will this occur?

**Problem 2** – As viewed from Mars for which E = 687 days, how long must you wait to see Earth once again reach its greatest distance from the sun in the martian sky? Explain how this is also the time between situations where the planets are at their closest positions to each other in their orbits.

**Problem 1** – An astronomer notes that the separation between Venus and the sun on the near-side of the sun is exactly zero degrees on January 1, 2010. If E = 365.24 days and P = 224.7 days, A) how many days will the astronomer have to wait to see Venus once again reach a separation of zero degrees on the near-side of the sun? B) On what date will this occur?

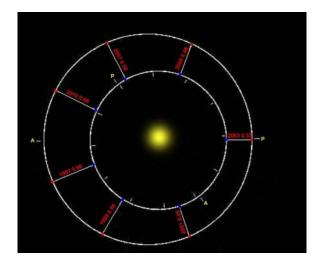
Answer: 1/S = 1/224.7 - 1/(365.24) 1/S = 0.00445 - 0.00274 1/S = 0.00171 S = 1/0.00171and so **S = 584.84 days**.

**Problem 2** – As viewed from Mars for which E = 687 days, how long must you wait to see Earth once again reach its greatest distance from the sun in the martian sky? Explain how this is also the time between situations where the planets are at their closest positions to each other in their orbits.

Answer:

1/S = 1/365.24 - 1/(687) 1/S = 0.00274 - 0.00146 1/S = 0.00128 S = 1/0.00128and so **S = 781 days**.

This is also the time between the closest distances between Earth and Mars, which is a time when NASA can send spacecraft and future manned missions to mars while traveling the shortest distance. See figure below:



Orbit of the E

Because the orbit plane of Venus is tilted with respect to Earth's, there are exactly two times during the year when Venus is located at the intersection point of these tilted planes – a location called the Node. The two nodes can be connected with a 'line of nodes' that passes through the center of the sun.

Because the tilt of the orbit planes is only 3.4 degrees, but the sun has a diameter of 0.5 degrees, there will be many times when Venus is located between Earth and sun, but for Venus to be close enough to the sun for it to be seen as a dark spot 'transiting' the sun is very rare. First let's have a look at the synodic cycle of Venus and Earth. This will tell us how many days will elapse between Earth, Venus and the sun being roughly lined up.

The <u>synodic period</u> is the time it takes a planet viewed from Earth to be observed at exactly the same illumination phase as it had previously. This depends on the planet, Earth and sun being in exactly the same geometric relationship as before. For example, the time between lunar Full Moons is the moon's synodic period. The time between the centers of Earth, Venus and the sun falling on exactly the same straight line on two successive times is called the synodic period or Venus. This is also the time between seeing Venus at its farthest distance (in angular measure) in the evening sky from the sun on two successive occasions.

**Problem 1** – If the orbit period of Earth is 365.24 days and Venus is 224.7 days, the synodic period is just 584.84 days. Show that the period of time that elapses between 5 synodic periods of Venus is nearly equal to 8 Earth years.

**Problem 2** – Draw two concentric circles with the sun at the center, with the inner circle being the orbit of Venus and the outer circle being the orbit of Earth. Place a dot on each circle to represent Earth and Venus at their closest positions, called oppositions, and draw a line between them that intercepts the sun. From this starting position, mark 4 new dots on the orbit circle for Earth that represent the passage of 5 synodic time intervals. Mark the location of Venus at these times and draw lines that connect Earth and Venus to the sun at each new opposition time. What inscribed figure can you draw by connecting the five Earth points?

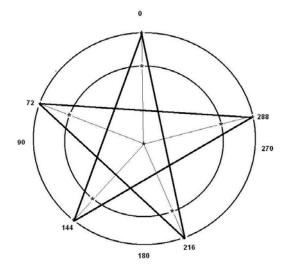
**Problem 3** – If one of these 5 oppositions represents a time when Venus is exactly at one of the orbital nodes so that you can see a transit of Venus across the sun, how long will you have to wait to see the next transit of Venus?

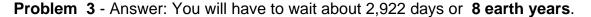
**Problem 1** –Answer:  $5 \times (584.84) = 2,924$  days.  $8 \times (365.24) = 2,922$  days.

**Problem 2** – Answer: The synodic period of 584.84 days equals 1.60 Earth years, so on the Earth orbit, we place dots for Earth that are spaced 1.6 Earth years apart. Since one full circle represents 1.0 Earth years, the 'remainder' is a distance of 0.6 of a year or 216 degrees apart, so the dots are placed as follows:

Dot 1 = 0.0 (Start) Dot 2 =  $(1.6 - 1)^*360 = 216$  degrees Dot 3 =  $(3.2 - 3)^*360 = 72$  degrees Dot 4 =  $(4.8 - 4)^*360 = 288$  degrees Dot 5 =  $(6.4 - 6)^*360 = 144$  degrees Dot 6 =  $(8.0 - 8)^*360 = 0$  degrees (Start)

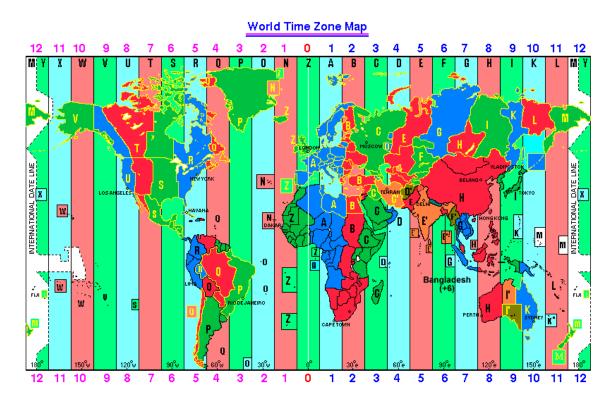
Plot these points on the Earth circle and draw a line from the earth dot to the sun, placing a dot for Venus on the Venus circle where the intersection. Notice the pentagram figure formed by connecting the dots in the order above, also, the 5 points (0, 72, 144, 216, 288) are exactly 72 degrees apart, forming a regular pentagon. Students may construct a more accurate drawing using a protractor to measure the angles along the earth circle.





Note: Because the orbits of Earth and Venus are not exact circles, there are actually two cycles for transits of Venus; an approximately 121 / 105-year cycle, and a pair of transits separated by 8 years. The two transits in 1874 and 1882 were separated by 8 years, followed by the transits of 2004 and 2012 after a gap of 122 years. The next transit after 2012 will occur after 105 years in the year 2117 AD. In addition, although the transits occurring 8 years apart appear during the same month, they switch from June to December and then back to June during the longer cycle. The 2004 and 2012 transits occurred in June, but the 2117 and 2125 transits will occur in December.

## What time is that transit?



Have you ever traveled a long distance across the United States only to discover that the current time is an hour or more different than what is one your watch? With millions of air travelers moving across the globe every day, the concept of the 'time zone' has started to become well known in a practical way. When astronomers calculate when a transit or other celestial event will occur, they use a Universal Time (UTC) standard rather than Local Time.

Universal Time is the Local Time precisely in Greenwich, England. For all other observers at other longitudes, you have to convert from UTC to Local Time to figure out what your clock time should be for an event. The map above shows the internationally-adopted time zones. The top line indicates the number of hours in the westward direction (left) that have to be <u>subtracted</u> from 0:00 UTC to convert to Local 'standard' time. Moving to the right (eastward) gives the number of hours <u>added</u> to 0:00 UTC to get to Local Standard Time.

**Problem 1** - A total solar eclipse is supposed to start at 19:00 UTC in the afternoon in London. At what Local Time will it start in Los Angeles?

**Problem 2** - On June 8, 2012 the Transit of Venus is predicted start at 22:09 UTC. What local time will that be in Hawaii (Time Zone W)?

**Problem 3** - Sunset occurs at 5:47 p.m Local Time in Rio de Janeiro; A) What UTC is this? B) If the Transit of Venus occurs between 22:09 UTC and 04:49 UTC, will it be visible?

**Problem 4** - For what UTC will it be 9:00 a.m Local Time in New York and 10:00 p.m Local Time in Perth, Australia?

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**Problem 1** - A total solar eclipse is supposed to start at 19:00 UTC in the afternoon in London. At what Local Time will it start in Los Angeles?

Answer: From the time zone map, Los Angeles is 8 hours to the west of Greenwich, so it is 8 hours behind UTC, and so its Local Time is 19:00 UTC - 8:00 = 11:00 a.m Local Time.

**Problem 2** - On June 8, 2012 the Transit of Venus is predicted start at 22:09 UTC. What local time will that be in Hawaii (Time Zone W)?

Answer: Hawaii is located 10 behind Greenwich, so the Local Time will be 22:09 UTC - 10:00 = **12:09 p.m Local Time in Hawaii.** 

**Problem 3** - Sunset occurs at 5:47 p.m Local Time in Rio de Janeiro; A) What UTC is this? B) If the Transit of Venus occurs between 22:09 and 04:49 UTC, will it be visible?

Answer: A) Rio de Janeiro is 2 hours behind UTC. The local time 5:47 p.m is the same as 17:47 on the '24-hour' clock, so sunset happens at 17:47 + 2:00 = 19:47 UTC.

The Transit of Venus occurs between 22:09 and 04:49 UTC, so sunset occurs about 2 hours (2:22) before the transit starts. **The transit will not be visible from Rio de Janeiro.** 

**Problem 4** - For what UTC will it be 9:00 a.m Local Time in New York and 10:00 p.m Local Time in Perth, Australia?

Answer: New York is 5 hours behind UTC so UTC = 09:00 + 5:00 = 14:00 UTC. Perth is 8 hours ahead of UTC so 10:00 PM - 8:00 = 2:00 PM or 14:00 UTC.



At any given moment, some part of earth is in daylight, some part is in nighttime. Along a specific geographic band, observers are watching sunrise, while along another band, the sun is just setting. The above 'world clock' displays all of these conditions so that travelers can see where these different 'diurnal' events are taking place at a specific moment in time. The dot is the location where the sun is directly over head at Noon.

**Problem 1** - From your knowledge of the direction of Earth's rotation, along which arcs are sunrise and sunset occurring?

Problem 2 - Has western Australia witnessed sunrise or sunset?

**Problem 3** - The equator runs along the horizontal mid-line of the figure, and the figure is drawn for a day near the summer solstice. Where would an observer see 'High Noon' with the sun directly overhead?

Problem 4 - What will the figure look like in 6 hours?

Problem 5 - What would an observer witness along the top edge of the figure?

Space Math

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**Problem 1** - From your knowledge of the direction of Earth's rotation, along which arcs are sunrise and sunset occurring?

Answer; The sun rotates from west to east. Moving to the left from the vertical centerline of the daylight region, you are moving into time zones that are earlier than Noon, so the sun will be lower and lower in the East until you reach the far-left edge of the daylight boundary, which represents sunrise. Similarly, moving to the right is in the direction of sunset on the far-right boundary of the sunlit zone.

Problem 2 - Has western Australia witnessed sunrise or sunset?

Answer: It has witnessed sunset and is now entering nighttime.

**Problem 3** - The equator runs along the horizontal mid-line of the figure, and the figure is drawn for a day near the summer solstice. Where would an observer see 'High Noon' with the sun directly overhead?

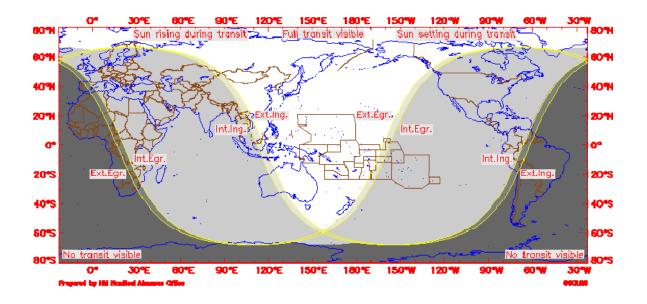
Answer: Near the white dot in the middle of the African continent.

**Problem 4** - What will the figure look like in 6 hours?

Answer: The continents will all be shifted to the right by 6/24 = 1/4 of the Earth's surface, and so the sunset line will pass through eastern Africa, South America will be towards the center of the daylight zone, and High Noon will be somewhere near Venezuela.

**Problem 5** - What would an observer witness along the top edge of the figure?

Answer: This region shows a band of light extending across all time zones, so this must be Arctic Summer during the Summer Solstice in June, when the sun is above the horizon 24 hours a day.



The figure above shows the visibility of the Transit of Venus for June 6, 2012 using a global map. Although this is a compact way to present transit information, it can be very confusing to read.

The transit progresses from Exterior Ingress with the solar disk (around 22:09 UTC) to Exterior Egress with the solar disk (around 04:49 UTC) some six hours later. Because you can only see the transit when in the daytime, you also have to keep track of local sunrise and sunset. On June 6, sunrise will occur at 03:46 UTC, and sunset at 20:32 UTC. Locations to the left of the center line near a longitude of  $165^{\circ}$  East represent locations for which the sun is rising during the transit, and to the right of center, locations where the sun is setting during the transit. Note: Since 180 degrees in Longitude equals 12 time zones, there are 15 degrees per time zone.

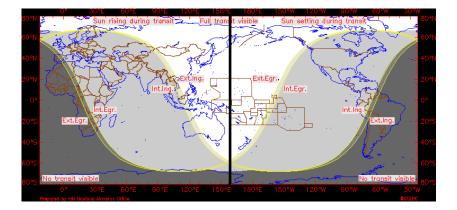
**Problem 1** - If the middle of the transit occurs at 01:30 UTC, where will this occur at High Noon (12:00 p.m) Local Time? Draw a vertical line at this location.

**Problem 2** - In New York City, (Local Time = UTC - 4 hours) sunrise and sunset occur at 5:25 a.m and 8:23 p.m. How much of the transit will be visible at this location?

**Problem 3** - In San Francisco, (Local Time = UTC - 7 hours) sunrise and sunset occur at 5:48 a.m and 8:28 p.m. How much of the transit will be visible at this location?

**Problem 1** - If the middle of the transit occurs at 01:30 UTC, where will this occur at High Noon (12:00 p.m) Local Time? Draw a vertical line at this location.

Answer: This will occur at a longitude that is 12:00 - 01:30 = 10.5 hours east of Greenwich England (0 Longitude). Since 180 degrees in Longitude equals 12 time zones, there are 15 degrees per time zone, so 10.5 hours east equals a Longitude of  $15 \times 10.5 = 158$  East of Greenwich, England (0 longitude).

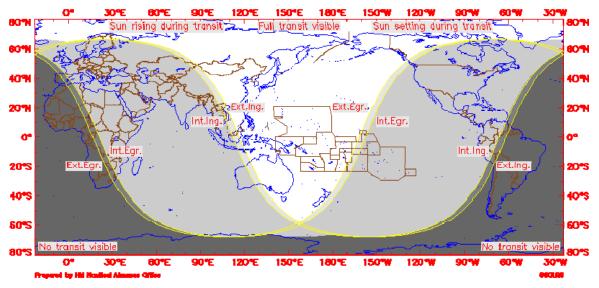


**Problem 2** - In New York City, (Local Time = UTC - 4 hours) sunrise and sunset occur at 5:25 a.m and 8:23 p.m. How much of the transit will be visible at this location?

Answer: The transit information is given in UTC, so we first convert the New York times to UTC. New York Local Time is 4 hours <u>behind</u> UTC so sunrise = 05:25 + 4:00 = 09:25 UTC and sunset = 8:23 PM + 4:00 = 00:23 UTC. The transit lasts from 22:09 UTC to 04:49 UTC, so from New York the observer gets to see the first **22:09 UTC - 00:23 UTC = 2h 14m of the transit <u>before</u> the sun sets** 

**Problem 3** - In San Francisco, (Local Time = UTC - 7 hours) sunrise and sunset occur at 5:48 a.m and 8:28 p.m. How much of the transit will be visible at this location?

Answer: The transit information is given in UTC, so we first convert the San Francisco times to UTC. San Francisco Local Time is 7 hours <u>behind</u> UTC so sunrise = 05:48 + 7:00 = 12:28 UTC and sunset = 8:28 PM + 7:00 = 03:28 UTC. The transit lasts from 22:09 UTC to 04:49 UTC, so from San Francisco the observer gets to see the first 22:09 UTC - 03:28 UTC = 5h 19m of the transit <u>before</u> the sun sets



The figure above shows the visibility of the Transit of Venus for June 6, 2012 using a global map. Although this is a compact way to present transit information, it can be very confusing to read.

The transit progresses from Exterior Ingress with the solar disk (around 22:09 UTC) to Exterior Egress with the solar disk (around 04:49 UTC) some six hours later. Because you can only see the transit when in the daytime, you also have to keep track of local sunrise and sunset. On June 6, sunrise will occur at 03:46 UTC, and sunset at 20:32 UTC. Locations to the left of the center line near a longitude of  $165^{\circ}$  East represent locations for which the sun is rising during the transit, and to the right of center, locations where the sun is setting during the transit. Note: Since 180 degrees in Longitude equals 12 time zones, there are 15 degrees per time zone.

**Problem 1** - On the day of the transit, the sun will be at its highest point in the sky, High Noon, at exactly 12:00 Local Time. If Hawaii is located 10 hours to the west of Greenwich, England, what will be the Universal Time (UTC) of Hawaii High Noon?

**Problem 2** - If the start of the transit occurs at 22:09 UTC, and sunrise from Honolulu occurs at 5:49 a.m, how soon after sunrise with the transit start?

**Problem 3** - If sunset occurs at 7:11 p.m Local Time, how long before sunset did the transit end?

Problem 4 - At what Local Times in Hawaii does the transit begin and end?

Answer: UT = 12:00 + 10:00 = **22:00 UTC**.

**Problem 2** - If the start of the transit occurs at 22:09 UTC, and sunrise from Honolulu occurs at 5:49 AM, how soon after sunrise with the transit start?

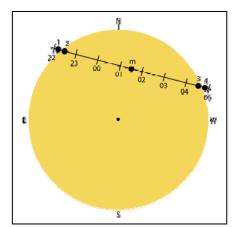
Answer: UTC for sunrise is 05:49 a.m + 10:00 = 15:48 UTC. The transit starts at 22:09 UTC, so it begins 22:09 - 15:48 = 6h 21m or **6 hours and 21 minutes <u>after</u> sunrise in Honolulu.** 

**Problem 3** - If sunset occurs at 7:11 p.m Local Time, how long before sunset did the transit end?

Answer: Sunset is at 19:11 + 10:00 = 05:11 UTC. The transit ended at 04:49 UTC, so it ended 05:11 - 04:49 = 22 minutes <u>before</u> sunset.

Problem 4 - At what Local Times in Hawaii does the transit begin and end?

Answer: In terms of Local Time, the transit starts at 22:09 UTC - 10h = 12:09 p.mLocal Time, and ends at 04:49 - 10h = 28:49 - 10:00 = 18:49 or **6:49 p.m** Local Time.



Hawaii and Alaska are US locations where the entire 6-hour transit can be observed:

Anchorage: Latitude = +61.218 Honolulu: +21.307 Longitude= -149.900 -157.858

Here are the Local Hawaii Times for the four stages (contacts) of the transit shown in the diagram to the left:

Local times for transit Contacts				
Contact=	1	2	3	4
Anchorage	14:06:28	14:24:02	20:30:44	20:48:31
Honolulu	12:10:06	12:27:45	18:26:37	18:44:36

When astronomers and other observers want to experience and record the exact transit events of 'first contact' and 'last contact' it is not sufficient to know the timing of the transit events to the nearest minute. Because of the speed of movement of the disk of Venus as it crosses the sun, you can easily miss these events by being too late or too early. Forecasts can now be calculated years or decades in advance. They use accurate mathematical and physical models of the orbits of Earth, Venus and the location of the sun, to make predictions that are 'good' to fractions of a second. To do this, the latitude and longitude of the observer have to be known to fractions of a degree, and the observer's altitude to meter-accuracy.

**Problem 1** - Between First Contact (1) and Second Contact (2) in the table, the disk of Venus travels exactly its own angular diameter of 58.26 arcseconds in the indicated time. The diameter of the sun at this time is 1,891.4 arcseconds. How many seconds elapse between these events as observed at A) Anchorage and B) Honolulu?

**Problem 2** - What is the full duration, in seconds, of the transit between Second Contact and Third Contact as viewed from A) Anchorage and B) Honolulu?

**Problem 3** - From your answer to Problem 1, and using the angular diameter of Venus, what is the average speed of the transit in arcseconds/second between First and Second Contacts in A) Anchorage? B) Honolulu?

**Problem 4** - What is the length, to the nearest arcsecond, of the transit chord between Second and Third Contact as seen from A) Anchorage? B) Honolulu?

Contact=	1	2	3	4
Anchorage	14:06:28	14:24:02	20:30:44	20:48:31
Honolulu	12:10:06	12:27:45	18:26:37	18:44:36

**Problem 1** - Between First Contact (1) and Second Contact (2) in the table, the disk of Venus travels exactly its own angular diameter of 58.3 arcseconds in the indicated time. The diameter of the sun at this time is 1,891.4 arcseconds. How many seconds elapse between these events as observed at A) Anchorage and B) Honolulu?

Answer:	Anchorage:	T = 14:24:02-14:06:28	Honolulu: T = 12:27:45 - 12:10:06
	_	= 24:02 - 06:28	= 27:45 - 10:06
		= 17:34	= 17:39
		= 17 minutes 34 seconds	= 17 minutes 39 sec
		= 17x60 + 34	= 17x60+ 39
		= 1,054 seconds	= 1,059 seconds

**Problem 2** - What is the full duration, in seconds, of the transit between Second Contact and Third Contact as viewed from A) Anchorage and B) Honolulu?

Answer:	Anchorage: T = 20:30:44 - 14:24:02	Honolulu: 18:26:37 - 12:27:45
	= 6:06:42	= 5:58:52
	= 6x3600 + 6x60 + 42	= 5x3600 + 58x60 + 52
	= 22,002 seconds	= 21,532 seconds

**Problem 3** - From your answer to Problem 1, and using the angular diameter of Venus, what is the average speed of the transit in arcseconds/second between First and Second Contacts in A) Anchorage? B) Honolulu?

Answer: A) Anchorage: The diameter of Venus is 58.26 arcseconds, and it takes 1,054 seconds to travel this distance, so the average speed is 58.26 arcseconds /1,054 seconds = **0.05527** arcseconds/sec.

B) Honolulu: It takes 1,059 seconds to travel this distance, so the average speed is 58.26 arcseconds /1,059 seconds = **0.05501 arcseconds/sec.** 

**Problem 4 -** What is the length, to the nearest arcsecond, of the transit chord between Second and Third Contact as seen from A) Anchorage? and B) Honolulu?

Answer: A) Anchorage: The average speed of the transit is 0.05527 arcseconds/sec, so the length of the chord is just 22,002x0.05527 = 1,216 arcseconds. B) Honolulu: The average speed of the transit is 0.05501 arcseconds/sec, so the length of the chord is just 21,532x0.05501 = 1184 arcseconds.

This website provides a calculator for your specific latitude and longitude http://www.transitofvenus.nl/details.html

Space Math

#### Appendix A: The Transits of Venus: Ancient History to 1882 AD

History credits the English Astronomer Jeromiah Horrocks as the first human to ever witness a transit by Venus, which occurred in 1631, but could other more ancient people have also seen transits of Venus as well? There have been 52 transits of Venus across the Sun between 2000 B.C and 1882 A.D. As seen on the sun, Venus is as big as a large sunspot. You could see it with the naked eye if you knew exactly when to look. But, because you cannot look directly at the sun except when it is close to the horizon, you would have only a very short time to be lucky to see it, and would need a reason for wanting to look at the sun on the horizon in this way at all! Four transits occurred during the Babylonian Era on May 20, 1641 BC, November 20, 1540 BC, November 18, 1512 BC and May 23, 1406 BC Could any of these have been seen?

In the British journal 'Monthly Notices of the Royal Astronomical Society' for November 1882 (vol. XLIII page 41) you can find a curious article written by Rev. S.J. Johnson that asks whether the ancient Assyrians had observed the Venus Transit. He said that an article in the journal Nature published a few years earlier, and written by the



BAKED CLAY TABLET IN AKKADIAN CUNEIFORM, EGYPT, CA. 1400 BC

well-known Oriental scholar Sayce, Rev. mentioned a broken Assyrian cuneiform tablet. The tablet was about Venus, and a translated sentence on the tablet had breaks in it which seemed to indicate that such a transit had been seen. "the planet Venus --- it passed across ---- the Sun --across the face of the Sun". The data of the tablet was apparently before the 16th Century B.C.. So, what was this mysterious tablet mentioned by Sayce, and had it really been translated correctly? The implication is that sometime before ca 1500 'something' involving Venus and the Sun at close quarters did occur from Babylonia. If it was perhaps one of the four transits in the list above, this would be one of the earliest astronomical phenomena ever recorded by humans that survived to the present time! Since it is impossible to tell from the articles exactly which cuneiform tablet the inscription appeared upon, we cannot subject this tablet to a modern translation to see if its message stands up.

The Venus Tables of Ammizaduga were discovered in 1850 in Nineveh by Sir Henry Layard in excavations of the library of Asurbanipal. The translations were published a few years later by Sir Henry Rawlinson and George Smith as "Tables of the movements of the planet Venus and their influences". One of the large tablets called K.160 contains 14 observations of Venus and For example, in section 1 we read "If on the 21st of Ab, Venus disappears in the east, remains absent in the sky for two months and 11 days, and in the month of Arahsamna on the 2nd day, there will be rains in the land ;desolation will be wrought". None of these tablets have any inscription suggesting a transit. The tablets indicate that the Babylonians knew that every 8 solar years (8 x 365.24 = 2921.92 days) Venus reappears in the exact same place in the sky ( 5 x 583.9d = 2919.5 days). Because this also equals 99 lunar months (99 x 29.5 = 2920.5d)

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Venus returns to the same place in the sky at the same lunar month (and phase) too, but the return happens 2 1/2 days later each time (2921.92 - 2919.5 = 2.42d). After 150.8 years the return is exact ( $2.42 \times 150.8 = 365.24$ ).

How about Chinese observers? Chinese astrologers kept close track of the sun, especially large sunspots that could be seen at sunrise and sunset before the sun became too bright to see with the unaided eye. The earliest records of sunspot sightings began around 800 BC., but their observations apparently began in earnest around 167 BC. Astronomers Zhuang and Wang (1988) compiled a list of over 270 sunspot sightings from ancient Chinese, Korean and Japanese records. A comparison by Wittman and Zu (1987) and Yao and Stephenson (1988) of sunspots and the expected Venus transits shows no examples of near-misses.

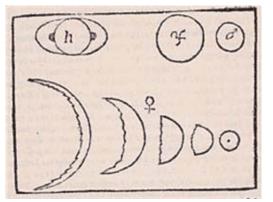
Mideavel Arab astronomers often explained dark spots on the sun as transits of Mercury or Venus, examples are 840, 1030, 1068 and 1130 AD, but no Venus transits occurred during these years so they were probably very large sunspots.



Did Montezuma see the Venus transit in 1518? Montezuma, the leader of the Aztec people in pre-Columbus Mexico, was a careful observer of the sun which he used in his divination practices. Venus was a very important celestial body in Aztec mythology as well as Mayan. The Transit of May 25, 1518 would have been visible to him at sunset. It is said that a jade figure at the British Museum of the god Quetzalcoatl, an aspect of Venus, wearing the Sun as his neck ornament, is a memorial of this rare event. Since Montezuma and the Aztec civilization were conquered by Cortez in 1520, this would certainly have been an ill-omen of impending doom!

#### The Modern Era of Transit Observations

We now arrive at the modern era of Venus transit observing as scientists first sorted out the shape and scale of the solar system and planetary orbits, and then began to make very accurate forecasts of where planets would be in the sky. Galileo Galilee in ca 1610 was the first human to actually see Venus as more than just a bright point of light in the sky. With his telescope, he made the discovery that it has a disk shape that changed its illumination phase just the way the Moon does as it circles Earth. This only made sense if Venus orbited the Sun, and so Venus played a very important role in confirming the heliocentric model of Copernicus. In September 1610, he sent an anagram to a friend of his announcing his discovery which translates as: "*The mother of love* [Venus] emulates the shapes of Cynthia [the Moon]".



Galileo's drawings of Saturn and the phases of Venus

Johannes Kepler, meanwhile, was shaking up the world by his meticulous use of astronomical data assembled by Tycho Brahe. The result was his discovery of three important laws of planetary motion, and later on, the publication of the Rudolphine Tables in September 1627. These tables were superior to commonly used tables based on Ptolmey's epicycle models, and included planetary position predictions to 1636. What he discovered during these laborious hand calculations was that Venus would pass in front of the Sun in 1631, so he wrote a 'Notice to the Curious in Things Celestial' to alert them to the

Venus transit of 1631 as well as a second transit to take place in the 1700's. The December 6th 1631 transit was looked for by Gassendi from Paris but not seen. It was actually not visible from Europe at all. Kepler himself died in 1630, but he had actually missed a second transit of Venus which would occur 9 years later.

## **Transit of Venus December 4, 1639.**

Jeremiah Horrocks (b. 1619) was an 'amateur' astronomer who had made numerous planetary observations, and noticed that they didn't agree with the established Tables based on Kepler's work.



His observations, and updated calculations, led to his predicting that on December 4, 1639, the planet Venus would pass across the face of the sun. This would happen 8 years after the previous 'Transit of Venus' that had been predicted by Kepler. Let's see what he had to say in his own words, taken from the article that he wrote on the event:

"Anxiously intent therefore on the undertaking through the greater part of the 23rd, and the whole of the 24th, I omitted no available opportunity of observing her ingress. I watched carefully on the 24th from sunrise to nine o'clock, and from a little before ten until noon, and at one in the afternoon, being called away in the intervals by business of the highest importance, which, for these ornamental pursuits I could not with propriety

#### Space Math

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neglect. But during all this time I saw nothing in the sun except a small and common spot, consisting as it were of three points at a distance from the center towards the left, which I noticed on the preceding and following days. This evidently had nothing to do with Venus. About fifteen minutes past three in the afternoon, when I was again at liberty to continue my labors, the clouds, as if by divine interposition, were entirely dispersed, and I was once more invited to the grateful task of repeating my observations. I then beheld a most agreeable spectacle, the object of my sanguine wishes, a spot of unusual magnitude and of a perfectly circular shape, which had already fully entered upon the sun's disc on the left, so that the limbs of the Sun and Venus precisely coincided, forming an angle of contact. Not doubting that this was really the shadow of the planet, I immediately applied myself sedulously to observe it"

"...I wrote therefore immediately to my most esteemed friend William Crabtree, a person who has few superiors in mathematical learning, inviting him to be present at this Uranian banquet, if the weather permitted; and my letter, which arrived in good time, found him ready to oblige me; he therefore carefully prepared for the observation, in a manner similar to that which has been mentioned. But the sky was very unfavorable, being obscured during the greater part of the day with thick clouds; and as he was unable to obtain a view of the Sun, he despaired of making an observation, and resolved to take no further trouble in the matter. But a little before sunset, namely about thirty-five minutes past three, certainly between thirty and forty minutes after three, the Sun bursting forth from behind the clouds, he at once began to observe, and was gratified by beholding the pleasing spectacle of Venus upon the Sun's disc. ... but Crabtree's opportunity was so limited that he was not able to observe very minutely either the distance itself; or the inclination of the planet. As well as he could guess by his eye, and to the best of his recollection, he drew upon paper the situation of Venus, which I found to differ little or nothing from my own observation;...

I wrote also of the expected transit to my younger brother, who then resided at Liverpool, hoping that he would exert himself on the occasion. This indeed he did, but it was in vain, for on the 24th, the sky was overcast, and he was unable to see anything, although he watched very carefully....I hope to be excused for not informing other of my friends of the expected phenomenon, but most of them care little for trifles of this kind, preferring rather their hawks and hounds, to say no worse; and although England is not without votaries of astronomy, with some of whom I am acquainted, I was unable to convey to them the agreeable tidings, having myself had so little notice... At Goesa, in Zealand, where Lansberg lately flourished, it [the Transit] commenced at fourteen minutes past three, and the Sun set at fifty-five minutes past three, consequently it might have been seen there. But no one excepting Lansberg and his friend Hortensius, both of whom I hear are dead, would trouble themselves about the matter; nor is it probable that, if living, they would be willing to acknowledge a phenomenon which would convict their much-vaunted tables of gross inaccuracy...In short, Venus was visible in the Sun throughout nearly the whole of Italy, France, and Spain; but in none of those countries during the entire continuance of the transit. But America! O fortunatos nimium bona Si sua norit! Venus! Which riches dost thou squander on unworthy regions, which attempt to repay such favors with gold, the paltry product of their mines. Let these barbarians keep their precious metals to themselves, the incentives to evil, which we are content to do without. These rude people would indeed ask from us too much should they deprive us of those celestial riches, the use of which they are not able to comprehend. But let us cease this complaint O Venus! and attend to thee ere thou dost depart.'

Space Math

In 1663, James Gregory, a Scottish mathematician and astronomer, suggested that a more accurate measurement of the Solar Parallax could be gained from observations of the transit of Venus made from various widely separate geographical locations.



Edmund Halley (1656-1742) Sir realized importance of transits in determining sun's distance during 1677. During a stay on the island of Saint-Helena, Halley observed a Mercury transit in that year and made careful note of the times of entry and exit of Mercury over the solar disk. He realized that if a transit would be observed from different latitudes on Earth, the different observers would see Mercury cross the Sun along at a different angle. This effect is known as parallax (this is even more noticable for Venus transits, since Venus is closer to us than Mercury, which increases the difference in angles) and could be used to determine an accurate Earth-Sun distance.Halley published past and future transit predictions in 1691, then in 1716 he published a greatly refined version of a paper originally read before the Royal

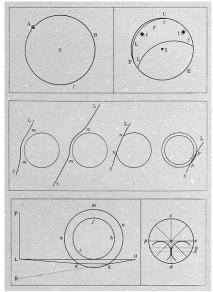
Society in 1691, entitled 'A new Method of determining the Parallax of the Sun, or his Distance from the Earth'. In the paper he championed the idea of scientists from various nations observing the 1761 and 1769 transits of Venus in as many parts of the world as possible. This, he argued, would result in a 'certain and adequate solution of the noblest, and otherwise most difficult problem' of accurately establishing the distance between the Earth and the Sun.

In 1716, Halley formally proposes Venus transit observations and shows how to use them to find exact value of the astronomical unit - the distance from the sun to earth. In his article published in the Philosophical Transactions and titled "A new Method of determining the Parallax of the Sun, or his Distance from the Earth" he notes:

"We therefore recommend again and again, to the curious investigators of the stars to whom, when our lives are over, these observations are entrusted, that they, mindful of our advice, apply themselves to the undertaking of these observations vigorously. And for them we desire and pray for all good luck, especially that they be not deprived of this coveted spectacle by the unfortunate obscuration of cloudy heavens, and that the immensities of the celestial spheres, compelled to more precise boundaries, may at last yield to their glory and eternal fame."

## Transit of Venus, June 5, 1761.

On June 5, 1761 the transit of Venus was observed by 176 scientists from 117 stations all over the world. The curious 'Black Drop Effect' was first spotted, and the Russian astronomer Mikhail Lomonosov (see center panel of figure) was the first to deduce that Venus had an atmosphere because of the beautiful halo of light that



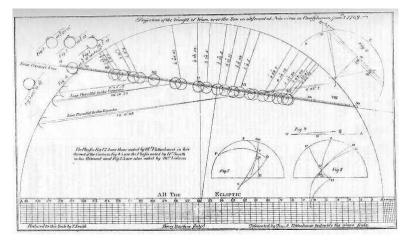
surrounded its dark disk just as it crossed the edge of the sun. This transit was not one of the best ones to observe to determine the distance to Venus and the sun. It actually took nearly 50 years for the astronomer Encke to finally collect all of the observations, analyze them mathematically, and report an improved estimate for the distance of 95 million miles.

Some of the scientists were involved in spectacular international events in carrying out their observations in remote corners of the world. Most were French or English, and the transit occurred during the peak of the Seven Year's War between these two international empires. Special letters of passage were carried by these scientists so that they could safely pass into 'enemy' territory.

Here's what the newspapers had to say about this transit in a very short announcement on June 8, 1761. By the way, written english used the letter 'f' instead of 's' in many words!:

"The tranfit of Venus over the Sun on Saturday laft was carefully obferred by many curiouf Gentlemen and differ'd confiderably from every Computation made of it. Its Emerfion was at about 35 Minutes after Eight, but the Morning being cloudy, was not vifible in London til it had paff'd three Fourths of the Sun's Diameter."

## Transit of Venus, June 3, 1769

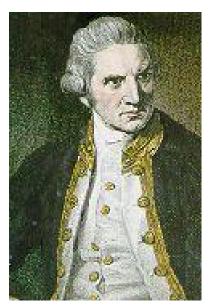


"Projection of the Transit of Venus over the Sun as observed at Norrington in Pennsylvania June 3, 1769."

## The Distance to the Sun

Although you cannot measure this distance directly, it is possible to use the Parallax Effect together with careful measurements of the transit of Venus to determine the distance to Venus from Earth, and relate this directly to the predicted distance given in terms of the distance from the sun to Earth, called the Astronomical Unit.

The sun is located at a distance of 150 million km, so if you were to view the sun from the poles of Earth, a distance of 13,000 kilometers, the angular 'parallax' shift would be about 18 seconds of arc. Mounting expeditions to Earth's North and South Poles is an impractical experiment, especially for the technology available during the



17- 19<sup>th</sup> centuries! However, if you deployed ships to islands separated by Earth's radius, 6,378 km, this angular difference would be 9 arcseconds. As a comparison, the angular diameter of the sun is about 1,800 arcseconds.

Although there were many international expeditions involved in these observations, the is perhaps the most famous expedition at the time, under the lead of Captain James Cook, who set up an observation post in Tahiti with his ship, the Endeavour. The expedition astronomers setup an observatory on what is now called "Point Venus".

Other expeditions were no so successful. The French scientist Guillaume Le Gentil was a passenger on a Spanish ship headed for Manila in May, 1766. Accused of being a French spy, he managed to escape and find his

way to Pondicherry where the best transit observing was expected on French territory, but after all this effort, the transit happened on a cloudy day. After nearly 12 years abroad, and traveling nearly 70,000 miles, he wrote in his journal:



"I was more than two weeks in a singular dejection and almost did not have the courage to take up my pen to continue my journal; and several times it fell from my hands, when the moment came to report to France the fate of my operations."

Le Gentil's journeys, carefully documented in his diaries, are a must-read, and recount his various run-ins and near-imprisonment in the hands of France's English enemies. He returned to France on October 8, 1771, having been gone for nearly 12 years.

## Transit of Venus, December 8, 1874

Hundreds of photographs taken of this transit. This was the first use of the new technology of photography, but few photographic plates were scientifically useful. Congress allocated \$75,000 for international scientific expeditions. Over \$1 million expended internationally. We now begin to see far more curiosity about this



phenomenon in the newspaper accounts.

December 9. The Chicago Daily Telegraph

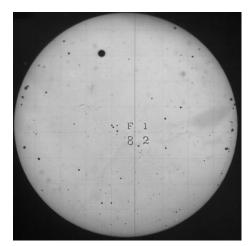
"Hence the probability is that observations of the transit of Venus in 1874, one which more that one million dollars have been expended, and involving the equivalent of not less than 200 years of labor on the part of one man, will only reduce the uncertainty by about one-third of its present magnitude".

December 10 Chicago Tribune. "All scientific men, and all others who are sufficiently informed to respect scientific pursuits, will be rejoiced at the news that the transit of Venus has been successfully observed at several stations. Fortunately the preparations for this great event were so complete that failure was scarcely possible"

Photographic studies of the transit were extensive, but the bottom line seemed to be that the measuring of the plates led to disappointing results in establishing the sun-earth distance. The problem was in getting the plate scale accurately enough, and the orientation of the plate to the sun.

December 31, 1874 Spectroscopic study of Venus first attempted, and reported in the international journal Nature. They were unable to detect anything of interest.

Eight American expeditions were fitted out in 1874, organized by the Transit of Venus Commission, with Simon Newcomb as Secretary. The U. S. Congress appropriated funds totaling an astounding \$177,000 for the expeditions. Although Newcomb considered the result of the 1874 observations disappointing due to inherent difficulties in the method, at the urging of Naval Observatory astronomer William



Harkness, in 1882 Congress once again appropriated some \$10,000 for improving the instruments, and \$75,000 for sending eight more expeditions.

# Transit of Venus, December 6 1882.

There was enormous public interest in this event. Smoked glass and amateur telescopes abound. Eventually, astronomers were able to get a distance to the sun from earth of 92.4 million miles with an

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uncertainty of about 1 million miles. Harvard Observatory astronomers tried to use a spectroscope to identify the atmosphere of Venus but could find no spectral evidence for such an atmosphere. A moon to Venus was also searched for but not found. Astronomer Henry Draper died so suddenly that no one knew how to operate his transit equipment. Among the public reactions to this event, the newspaper accounts give lots of details:



#### 1882 December 6 Philadelphia Enquirer.

"Scores of Columbia College students wearing morter boards climbed to the top of the new law-school building to catch a glimpse."

1882 December 7. Boston Daily Globe "Visit of Venus. She crosses the disk of the God of Day. The spectacle is viewed through telescopes and smoked glass'

1882 December 7. San Francisco Chronicle "Transit of Venus: The Planet's Course Across the Face of the Sun. A Grand Sight From the Observatory'

- "Many of the residents of San Francisco were noticed yesterday with a piece of smoked glass to their eye, looking curiously at the sun, between the hours of about sunrise and noon,

during which time Venus was visible; and even under these disadvantages without the aid of a suitable telescope, it was still a grand and beautiful spectacle. All who missed a view of the transit of Venus are to be commiserated, for should they live to be 100 years old the chance will not come again occur."

1882. December 16 Scientific American "It is possibly the last time that so much scientific stress will be laid upon the transit of Venus. For before the next one in 2004, we have faith to believe that other and more accurate methods will be found for computing the sun's distance"

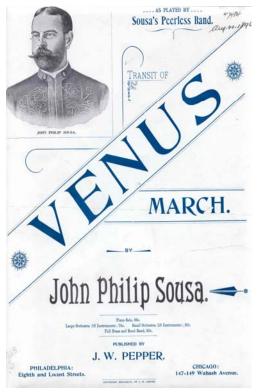
In May, 1883 after considerable detective work, American astronomer Simon Newcomb announced that Maximillian Hell was innocent of falsifying his data for the 1769 transit of Venus. Georgetown's Jesuit astronomer John G. Hagan, S.J. wrote to Newcomb. "By this act you have obliged the Jesuits of all times and all places. It was fitting that this act of justice should be reserved to an American astronomer, who stands aloof from the petty quarrels of the old world."

In 1891, Newcomb's refined calculation of the distance between earth and sun was finally produced, based on all of the assembled data from several transits. He concludes that the distance is 8.79 = -0.051".



William Harkness, an astronomer at the U.S. Naval Observatory, also spent a considerable part of his career analyzing the data, producing a value for the solar parallax, and putting it in context of other astronomical constants. This he did in his lengthy monograph "The Solar Parallax and its Related Constants," published in 1891 in Washington Observations. Simon Newcomb in fact used this value in his famous volume The Elements of the Four Inner Planets and the Fundamental Constants of Astronomy (1895), but gave it a much lower weight than most other methods. Harkness ultimately had the last, inspired reflection about what was to come. The next transit of Venus will occur, as Harkness put it will be "when the June flowers are blooming in 2004,"

The transit of Venus also inspired many other creative avenues of expression at the uniqueness of this event. John Philip Sousa (1854 to 1932) was very interested in the 1882 transit of Venus. In 1882. he wrote his 'Venus Transit March'. He didn't write it specifically to commemorate the transit itself, but wrote it to honor the great American physicist Prof. Joseph Henry who had died on May 13, 1878. The Smithsonian Institution in Washington D.C. asked Sousa to write this march for the planned unveiling of the statue of Henry in front of the Smithsonian Institution in 1883. The music was to be played while dignitaries walked from the museum to a special receiving stand in front of the Smithsonian.



Sousa was a Freemason, as were many 'intellectuals' of that time. His march was performed at a specific time and date: April 19, 1883 at 4:00 PM. While 10,000 people and representatives from many branches of government filed in a stately procession at this propitious hour, the planet Venus, invisible to the participants, had completed its arc in the sky and was setting in the west. At the same time, Virgo was rising in the east, and Jupiter was directly over head. Why did this time and date get selected for the unveiling of the magnificent bronze statue? In the mystical circles in which Sousa and the event organizers orbited, Venus was associated with the element copper, Joseph Henry had used large quantities of copper to create his powerful electromagnets, which at that time operated some of America's newest technology. The connection of the 'passing of Henry' commemorated by the statue, and the 'passing of Venus' in the west may have seemed like a fitting bond between two separate worlds: human and cosmic.

Sousa's Transit of Venus March remains a delightful, and rarely-played addition to Sousa's opus of compositions. The Transit of Venus March never caught on during Sousa's lifetime. It went unplayed for more than 100 years after Sousa's copies of the music were destroyed in a flood. As reported in The Space Math http://spacemath.gsfc.nasa.gov Washington Post, Library of Congress employee Loras Schissel recently found copies of the old sheet music for Venus "languishing in the library's files". In 2004, for the first time in over a century, Souza's Transit of Venus march was re-performed in October, 2003 by the Virginia Grand Military Band conducted by Mr. Loras Schissel. A recording of this march can be heard on YouTube http://www.youtube.com/watch?v=t08ZXaA\_0z0 . A detailed description of its re-discovery at the Library of Congress can be found at their website http://lcweb2.loc.gov/diglib/ihas/html/venus/venus-home.html

Sousa's March doesn't exhaust all of the musical possibilities for this event that can be found. Other musical compositions written in 1874-1882 probably have something to do with the transit of Venus in one way or another, especially since they 'appeared' at about the same time as the 1874 and 1882 events! No other musical compositions with similar titles are cataloged at the Library of Congress during the period.

Going back even earlier, at a time near the June 6, 1769 transit of Venus the British Public Library has a copy of an old song '*Come ye lads and lasses with speed. The Transit of Venus*' published in London ca 1774 [G.307.(125.)] Although the author and circumstances are unknown.

Considering how spectacular and mysterious the Transit seemed to most people, it's not too surprising that some creative souls decided to capture their inspired thoughts in poetry. Here is an excerpt of a poem reflecting on the 1769 Transit viewed during the legendary Captain Cook Expedition.

'Welcomed the Maori, shuddered at their meat, And in the galley watched them grill and eat. Yet for all this, some arrow of disaster Sped with the wisdom of that sailing master. A paler shadow overhung the rocks, Where Venus, still in transit, brought the pox. Plague to the peoples, slaughter to the whales, Bitter the fannings of Endeavour's sails! Still, in the wake of unsought devastations, Came the first stages of the wealth of nations. Shepards and farmers, in their sober glory, Began to write their unhistoric story."

Later-still, we have the charming poem "The Flaneur: Boston Common, During the Transit of Venus' written by American poet Oliver Wendell Holmes (1809-1894). Here is a short excerpt from this poem:

Grudge not to-day the scanty fee To him who farms the firmament, To whom the Milky Way is free; Who holds the wondrous crystal key, The silent Open Sesame That Science to her sons has lent; Who takes his toll, and lifts the bar That shuts the road to sun and star. If Venus only comes to time, (And prophets say she must and shall,) To-day will hear the tinkling chime

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Of many a ringing silver dime, For him whose optic glass supplies The crowd with astronomic eyes, --The Galileo of the Mall.

Dimly the transit morning broke; The sun seemed doubting what to do, As one who questions how to dress, And takes his doublets from the press, And halts between the old and new, Please Heaven he wear his suit of blue, Or don, at least, his ragged cloak, With rents that show the azure through!

In later years, during the 20th century astronomers continued to study Venus and found it to be a much different world than Earth. It was hardly our 'Twin' as had been so often imagined based on its size and mass alone. In 1922, astronomers St. John and Nicholson investigated spectrum of Venus near 5900, 6300 and 6867 Angstroms where oxygen should be detectable. No trace was found using a sophisticated spectroscope. Twelve years later, a second team of astronomers were able to detect carbon dioxide in the atmosphere for the first time. The reason that Walter Adams and Theodore Dunham succeeded was that, because of the Doppler shift between Venus and Earth, the wavelength shift between the carbon dioxide in Venus' atmosphere and in Earth's atmosphere was so slight that only by this time were spectroscopes sensitive enough to discern the difference in wavelength.

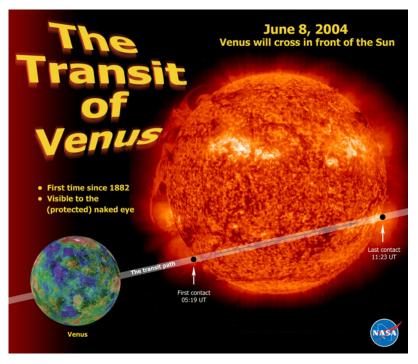
Radio observations of Venus during the 1960's indicated an enormous temperature of 700<sup>o</sup>C, which was later interpreted by astronomers such as Carl Sagan as evidence of a Greenhouse Effect. The atmosphere was dense (100 x Earth at sea-level) and composed almost entirely of carbon dioxide; an effective greenhouse gas. NASA's Mariner and Magellan spacecraft in the 1960's and 1970's eventually explored the detail composition of the atmosphere, and using penetrating radar, created a high-resolution map of the cloud-covered surface showing extensive volcanism and cratering.

## Appendix B The Transit of Venus – June 8, 2004.

It was a long and historically eventful 122 years since it was last glimpsed by humankind. It was a gap that was filled by technologies and scientific advancements that could scarsely be dreamed of by the denizens of 1882 as they looked forward in time. William Harkness, imagining June flowers blooming in time for the 2004 transit, had long-since departed this world, as had all of the other humans who had seen this astronomical event the last time. It is sobering to grasp astronomical times scales of this magnitude and the flux of human events that utterly pass into history in a mere 122 years. But the new generations of observers were ready for its reoccurrence. The sheer magnitude of this modern public event is hardly done iustice in а short recapitulation of its scope and impact world-wide. It would not be thousands of lucky viewers who would watch it through smoked glass this time. It would be nearly 1 billion people spanning all the major continents, and tied together by communications and imaging technology that knit the transit of venus together into a single, global experience.

Although astronomers had known the precise circumstances of this event for over 200 years, modern astronomical needs had long since bypassed a compelling research focus to this modern event. There had come to pass much better technologies and observing techniques for determining the distance from Earth to sun, to study the Venusian atmosphere, and to determine its precise orbit around the sun. Despite the intensive astronomical scrutiny of the past centuries, the modern scientific attitude towards the Transit of Venus was one of jaded interest in a rare but utterly 'mundane' astronomical event. It would have, largely, passed unnoticed to humanity were it not for the efforts of amateur astronomers, and a variety of education professionals across the world who realized that there was still intense public interest in anything involving rare planetary 'alignments'. They also hoped that it might serve as a teachable moment for millions of students around the world, who needed more awareness of science and technology in a demanding world that now places a premium on 'STEM' careers of every kind.

In June 2002, at a meeting of NASA educators at the Goddard Spaceflight Center, Dr. Sten Odenwald, a NASA astronomer presented the 2004 Transit of Venus as an opportunity for NASA to use this event to connect students and the general public to themes in solar science and exploration. The planning for NASA's education activities began in ernest in October 2002 with the selection of the Transit of Venus as the primier education theme for NASA's 'Sun Earth Day' event. The program would produce an educator resource folder containing education materials from NASA's many science missions. There would be a series of workshops for teachers during the 2003-2004 academic year. To capture the excitement of this rare event, the Transit would be webcast in real time from the Observatory in Athens, Greece so that millions of people around the world could enjoy this event from their homes. Announcement of the Transit of Venus and NASA's education intentions at the January, 2003 national convention of the American Astronomical Society soon followed.



The Sun-Earth Dav program developed a number of web-based resources to support this event. These resources are still available at NASA Sun-Earth th Dav archive: http://sunearthday.nasa.gov/2011/past days.php

The NASA education webbased resources were so heavily used by millions of visitors that the traffic shut down the NASA web servers briefly!



NASA partnered with the Exploratorium in San Francisco to conduct the webcast, which can be found at

#### http://www.exploratorium.edu/venus/

The one-hour live Webcast from Athens was viewed by over 100,000 visitors. The program was hosted by Dr. Sten Odenwald (NASA) and by Dr. Isabel Hawkins (University of California) who discussed the history of the transit. its scientific gave a blow-by-blow importance. and narration of the evant as it occurred.

Dr. Odenwald's colorful and illustrated online diary of this event can be found at the Astronomy Café website:

http://www.astronomycafe.net/Venus/VenusDiari es.html

Plans are now in progress for NASA's Sun-Earth Day to feature the 2012 Transit of Venus through a webcast from Hawaii on June 5-6, 2012, along with other educational resources during the 2011-2012 academic year.

## Appendix C Transits in the News

#### The 1769 Transit of Venus.

Archived copies at the Library of Congress of <u>The Boston Weekly Newsletter</u>, <u>The Essex Gazette</u> and the <u>New York Chronicle</u> revealed no articles about the Transit of Venus for the period from June 2-5. Between June 8-15, The New York Chronicle did publish a single drawing by Robert Harpur, which was reproduced in a letter to the Editor in the June 15-22 issue of The New York Chronicle on page 63 with the notation: *"June 10 .Please insert in your next paper the following scheme of the transit of Venus over the sun as observed at Kings College on the 3rd xxx which will oblige your most humble servant Robert Harpur."* Following the drawing is a paragraph summarizing the contact times but nothing more interesting.

Although the newspaper reportage of this event seemed to be non-existent or at best brief and sporadic, this transit did not entirely come and go without leaving in its wake a number of subtle but important changes in the way that American 'pre-Revolutionary' science was conducted. Linda Kerber, in 1972, wrote an article "Science in the Early Republic: The Society for the Study of Natural Philsophy" (William and Mary Quarterly 3rd Ser. Vol 29, No 2 p. 264.) and tries to describe how....

"Conceptual changes in science were accompanied by a radical revision of possible roles for scientists; the growth of full-time professionalism made it increasingly difficult to be an amateur. Perhaps the last major occasion on which the advance of knowledge was seen to depend on the contributions of the amateur was the observation of the Transit of Venus in 1769. That observation was, as Donald Fleming has remarked, "the symbolic act of allegiance to science and learning". Those who observed the transits and made measurements may have been amateurs, but they were not casual ones. In Providence, Joseph Brown spent nearly one hundred pounds sterling on a telescope and other equipment; he and Benjamin West accomplished the delicate, demanding task of calibrating the instruments. West was primarily a bookseller and almanac maker, but he had a serious interest in mathematics, and his reports of the transit are sophisticated. By the end of the century, however, it was less likely that part-time participants could make contributions to knowledge. the 'characteristic home of the American scientist' shifted from the private laboratory or privately organized observations .. to the college."

#### The 1874 Transit of Venus

The impact of these transits on the average American was, however, not really made clear until the next pair of transits in 1874 and 1882, when many domestic newspapers went to great trouble to run detailed articles on the progress of various scientific expeditions to distant lands. The 1874 Transit went off with barely a mention in many newspapers:

#### **December 9** The Chicago Daily Telegraph

"Hence the probability is that observations of the transit of Venus in 1874, one which more that one million dollars have been expended, and involving the equivalent of not less than 200 years of labor on the part of one man, will only reduce the uncertainty by about one-third of its present magnitude".

**December 10** Chicago Tribune. "All scientific men, and all others who are sufficiently informed to respect scientific pursuits, will be rejoiced at the news that the transit of Venus

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has been successfully observed at several stations. Fortunately the preparations for this great event were so complete that failure was scarcely possible...All scientific men, and all others who are sufficiently informed to respect scientific pursuits, will be rejoiced at the news that the transit of Venus has been successfully observed at several stations. Fortunately the preparations for this great event were so complete that failure was scarcely possible"

But by the 1882 transit, a far more rousing burst of interest seems to have been unleashed. This sense of 'cosmic adventure' was infectious and caused many columnists and reporters to wax poetic on these exotic events. Although articles tended to report the details of the measurements as though the average citizen understood what these numbers meant, occasionally articles would appear that captured some of the sense of excitement during these events. The newspaper accounts of the 1882 transit were especially lucid and fun to read. In the first one, from the San Francisco Chronicle (December 6, 1882) one can almost hear the voice of Mark Twain!

#### The 1882 Transit of Venus

**December 6** San Francisco Chronicle. "Why the astronomical community should be so exercised over a planet's movements; why their emotions should culminate when a shadow impinges upon or fades from the solar substance; why the transit of the planet should be called apparent; why Professor This should perch himself upon a pile of volcanic rocks in the Southern Pacific and Professor That shiver in the snows of Siberia to pry into an orbital incident happening millions of miles from either - these or something like them, are questions which the untutored many might wish to put to the erudite few."

**December 6** Philadelphia Enquirer. "Scores of Columbia College students wearing morter boards climbed to the top of the new law-school building to catch a glimpse."

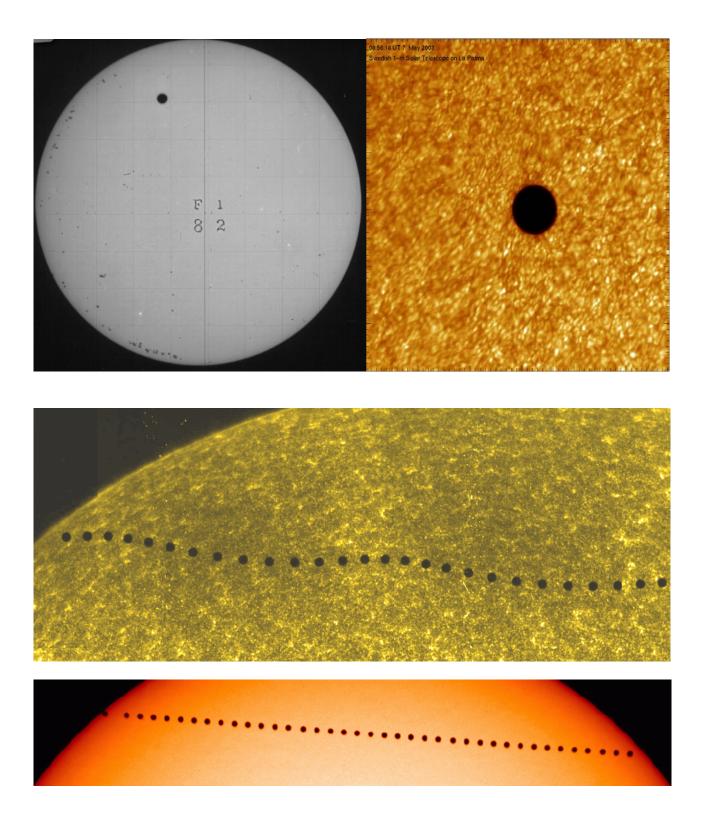
**December 7**. Boston Daily Globe "Visit of Venus. She crosses the disk of the God of Day. The spectacle is viewed through telescopes and smoked glass'

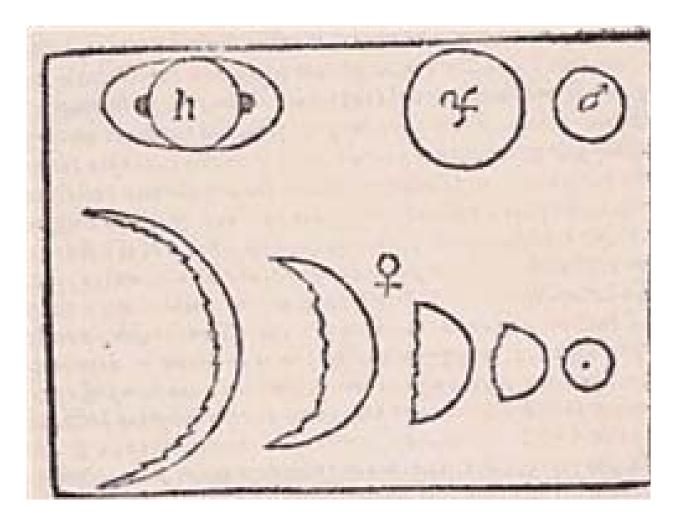
**December 7**. San Francisco Chronicle "Transit of Venus: The Planet's Course Across the Face of the Sun. A Grand Sight From the Observatory' "Many of the residents of San Francisco were noticed yesterday with a piece of smoked glass to their eye, looking curiously at the sun, between the hours of about sunrise and noon, during which time Venus was visible; and even under these disadvantages without the aid of a suitable telescope, it was still a grand and beautiful spectacle. All who missed a view of the transit of Venus are to be commiserated, for should they live to be 100 years old the chance will not come again occur."

**December 7** New York Times. "Across the Sun's Face. *Crowds viewing the rare phenomenon - smoked glass and telescopes in demand; 10 cents a sight. Telescopes at every corner etc; 2000 people viewed it at the Berkeley School on Madison Ave.* 

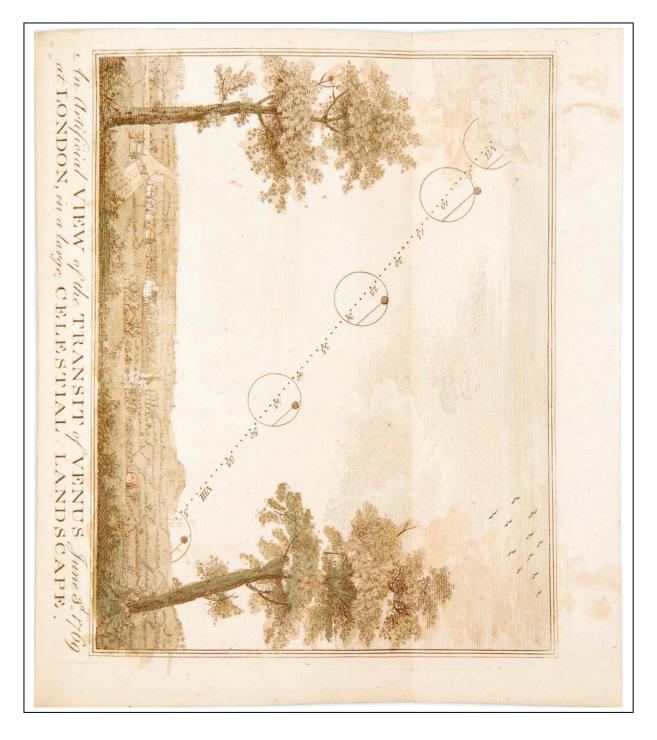
In the following pages you will find reproductions of a few of the actual newspaper articles that provide us with more insight to the way in which the coverage of the Transit of Venus changed as time passed.

## Appendix D A Gallery of Transit Imagery



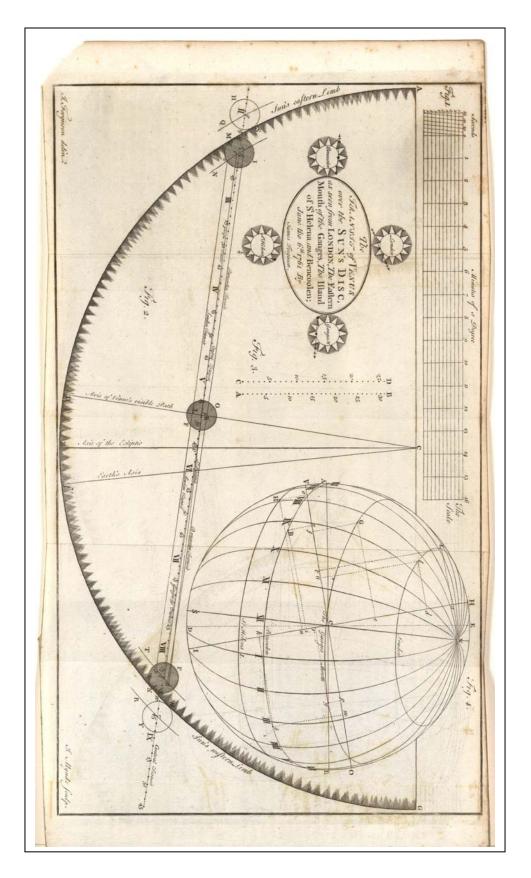


Galileo's drawing of the rings of Saturn and the phases of Venus ca 1609.



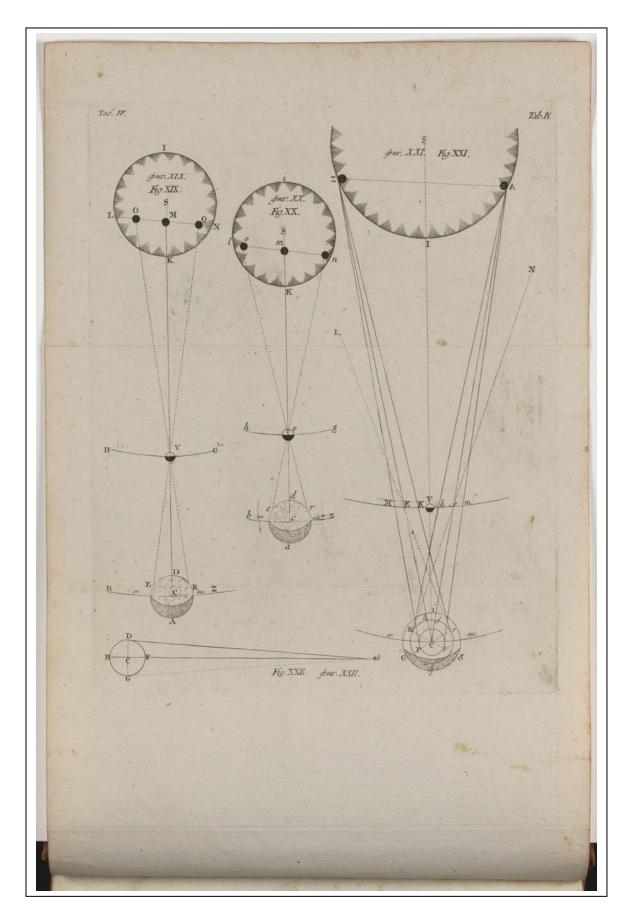
This is an image of an illustration of the 1769 Transit of Venus created by Benjamin Martin. This exquisite illustration shows the part of the 1769 transit of Venus that could be seen from London. Benjamin Martin, an eighteenth century British astronomy popularizer, highlighted the significance of the transits in 1761 and 1769. Such texts demonstrate the public's interest in calculating the size of the solar system and the astronomical mathematics that made this possible.

Institutions of Astronomical Calculations Containing a Survey of the Solar System, Benjamin Martin, London, 1773 (QB 42 .M36 1773) (Courtesy of the Adler Planetarium and Astronomy Museum; Chicago IL)

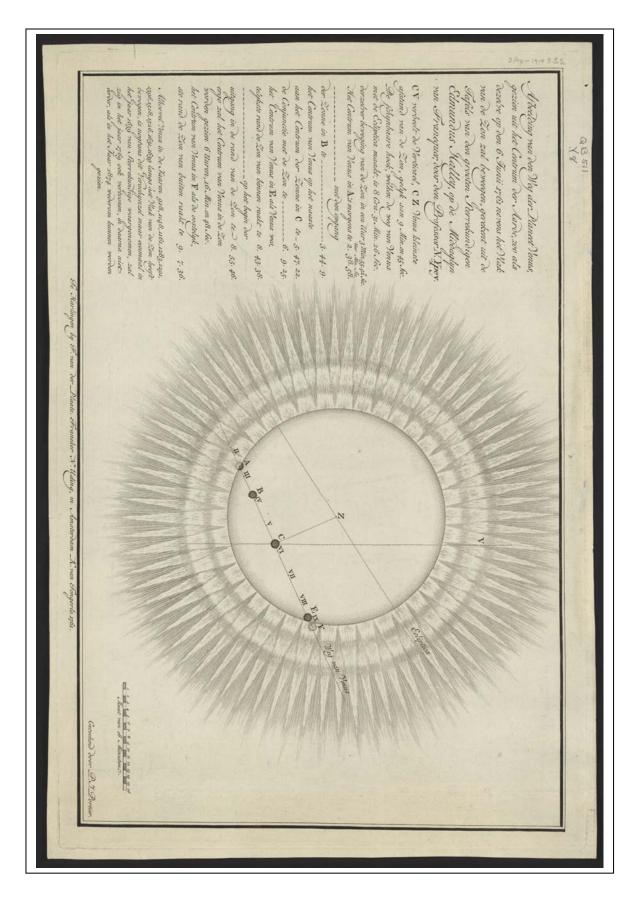


James Ferguson's drawing of the Transit of Venus June 6, 1761 as viewed from London. (*Astronomy explained upon Sir Isaac Newton's Principle...* London, c. 1764-1773; Edinburgh, c. 1811-1841 (edited by Brewster)

## Space Math

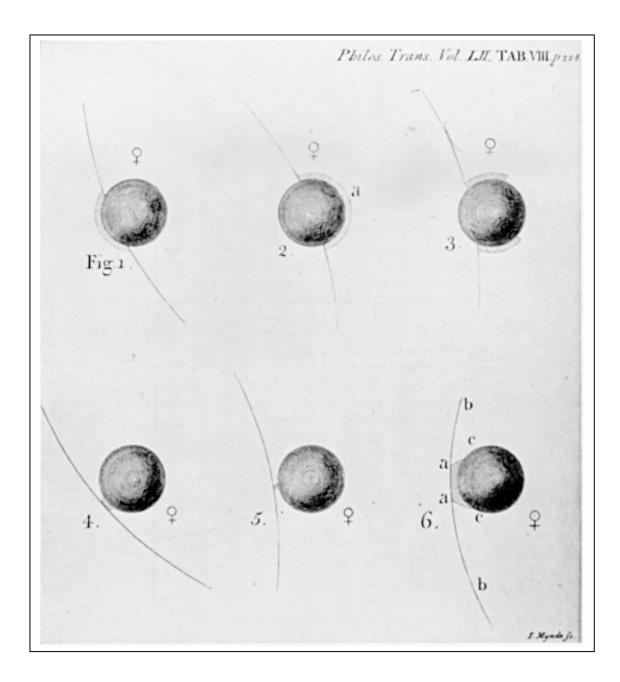


Transit of Venus Christian Mayer Expositio de transitu veneris ante discum solis Saint Petersburg, 1769

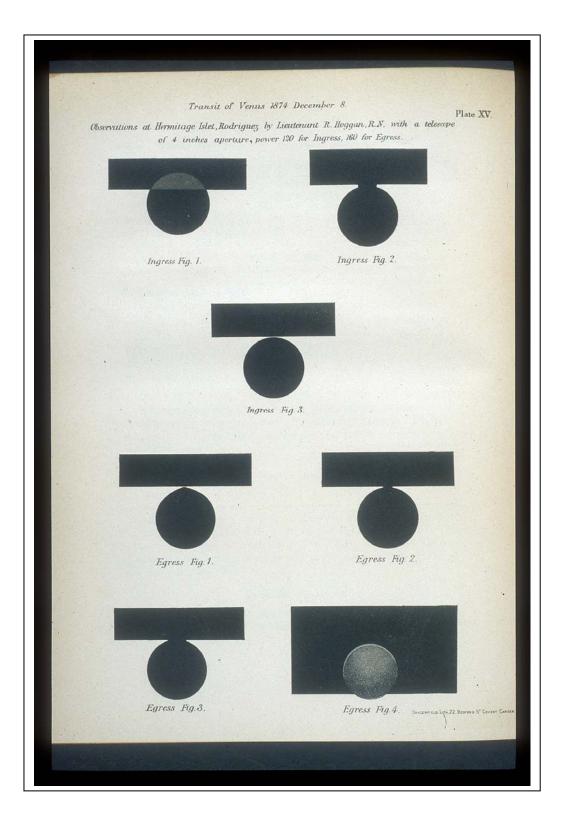


Sketch of the 1761 transit as seen by Nicholas Ypey. (Library of Congress). The coronal details on the sun are not based on actual observations but added for artistic impact.

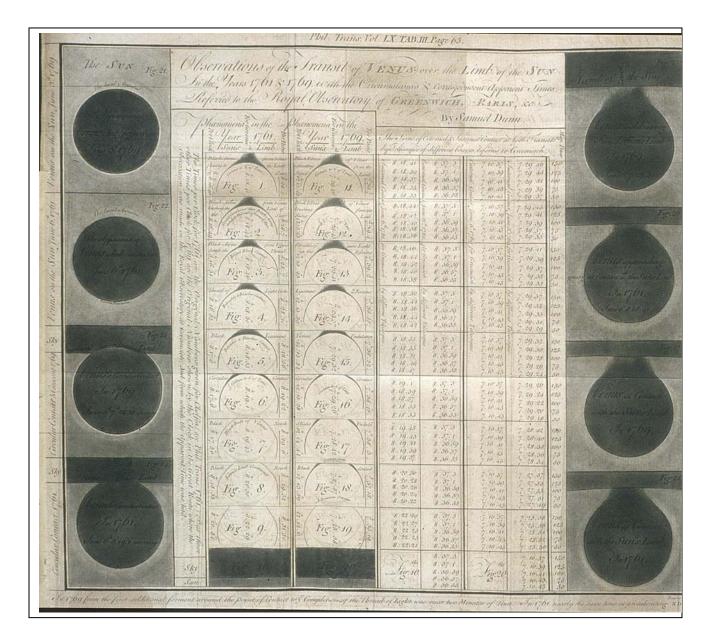
## Space Math



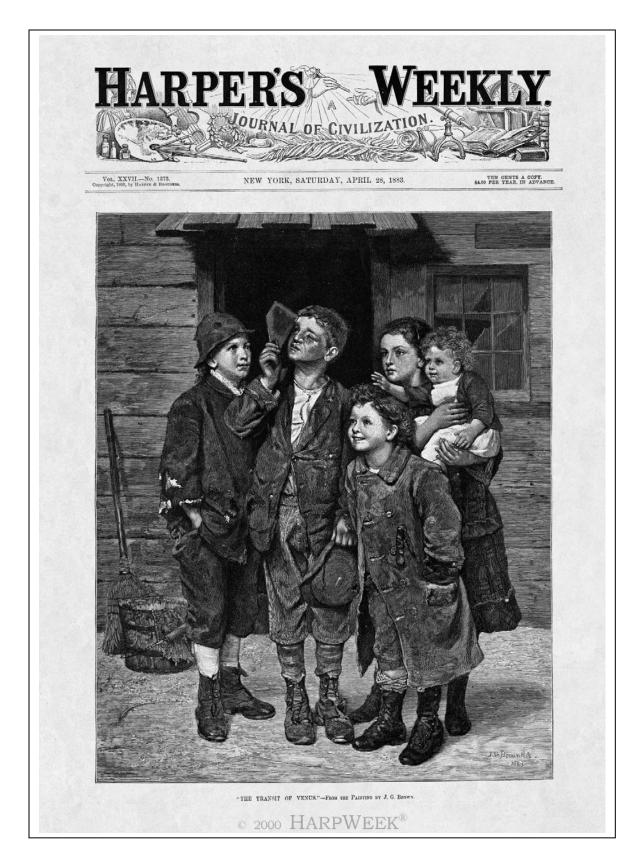
This is a sequence of observations of the 1761 Transit made by Tobin Bergmann. It shows the 'black drop' effect in panels 4, 5 and 6. published in the Philosophical Transactions 52, part 1 (1761), 227–228, also reproduced in Kragh, H. The Moon that Wasn't: The Saga of Venus' Spurious Satellite. Birkhäuser, 2008. P. 41.



Observations of the disk of Venus during the December 6, 1874 Transit of Venus made by amateur astronomer R. Hoggan using a 4inch telescope from the Hermitage Islet, Rodriguez.



Detailed drawings of the Transit of Venus in 1761 and 1769 by astronomer Samuel Dunn at the Royal Greenwich Observatory, published in the Philosophical Transactions, vol. 60, p. 65. Note the Black Drop effect in the right-hand views.



The cover of Harpers Weekly for April 28, 1883 showing children viewing the 1882 Transit of Venus using smoked glass.



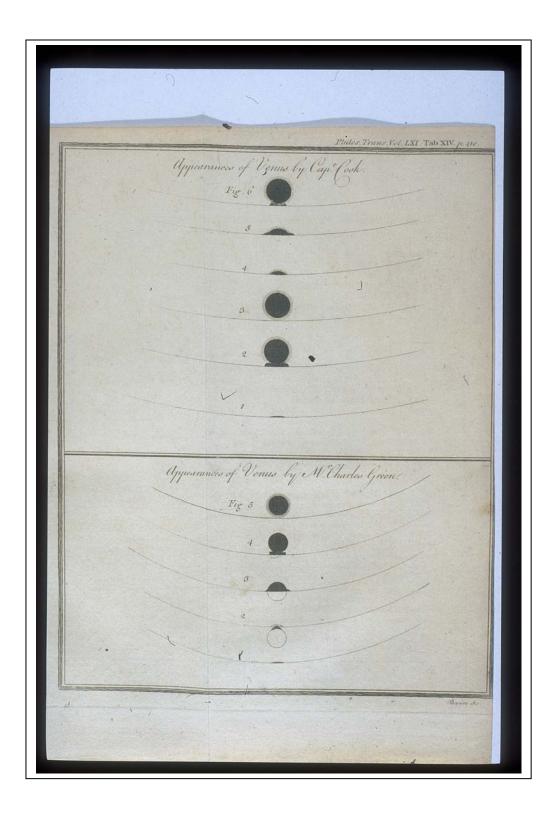
An amusing cartoon of the Transit of Venus of 1882 in Harpers Weekly, The caption reads 'Two successful observations by our artist of the Transit of Venus. The text on the top sun reads 'Venus: How earth stares at me. It makes me feel quite beautiful. Sol: Yes he has always been inquisitive". The lower sun disk reads 'Beauty spot on Old Sol.



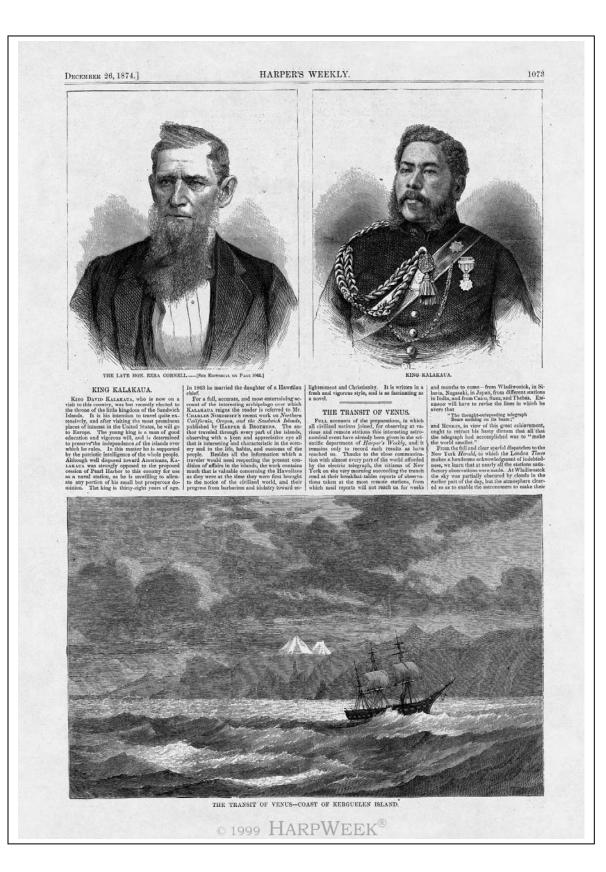
This illustration by James Ferguson shows the locations from which the transit of Venus could be observed on June 6, 1761, and local times at which the transit began and ended. Engraving from *Astronomy explained upon Sir Isaac Newton's Principles...*, 1790.

#### Space Math

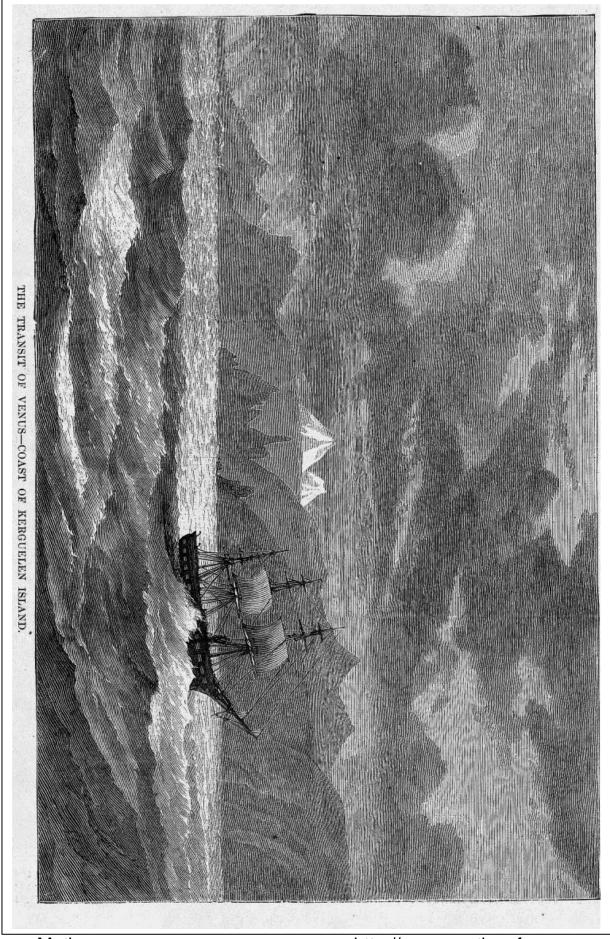
#### http://spacemath.gsfc.nasa.gov



Appearance of the disk of Venus as seen during the Captain Cook expedition of 1874, published in the Philosophical Transactions, Vol. 61, page 410.

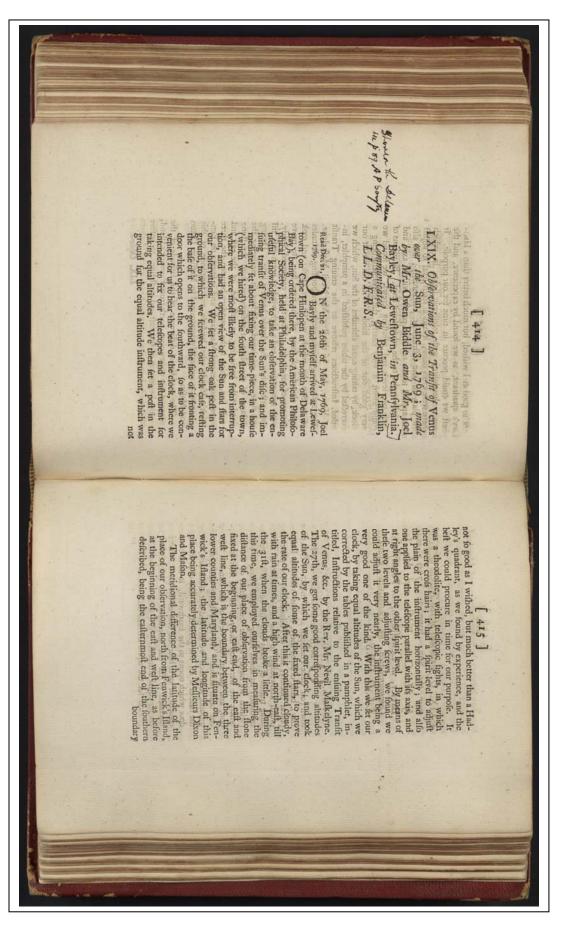


The 1874 Captain Cook voyage to se the Transit of Venus, mentioned in Harpers Weekly, December 26, 1874, including a woodcut of the ship on the High Seas near Kerguelen Island

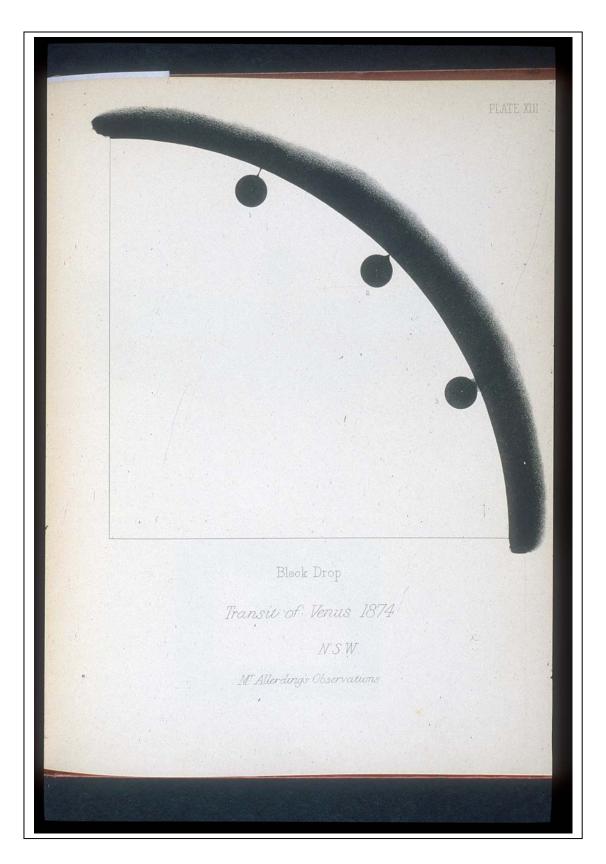


Space Math

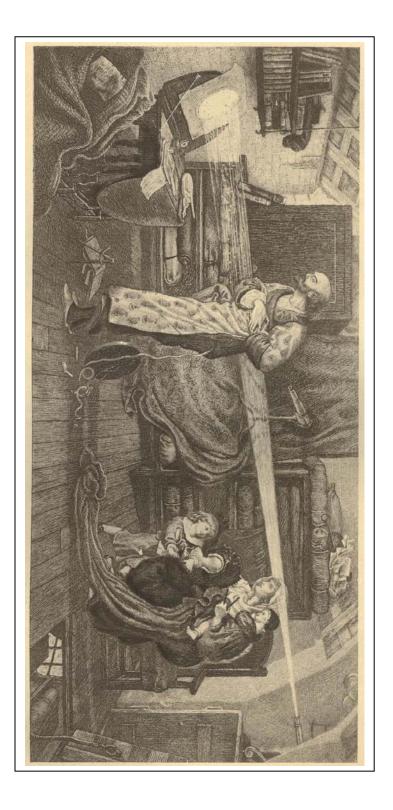
http://spacemath.gsfc.nasa.gov



Benjamin Franklin's article on the Transit of Venus, June 3 1769. (Library of Congress) Space Math http://spacemath.gstc.nasa.gov



Sketches of the Transit of Venus, 1874, in a study of the Black Drop effect. At the US Naval Observatory observed by astronomer Mr. Allerding.

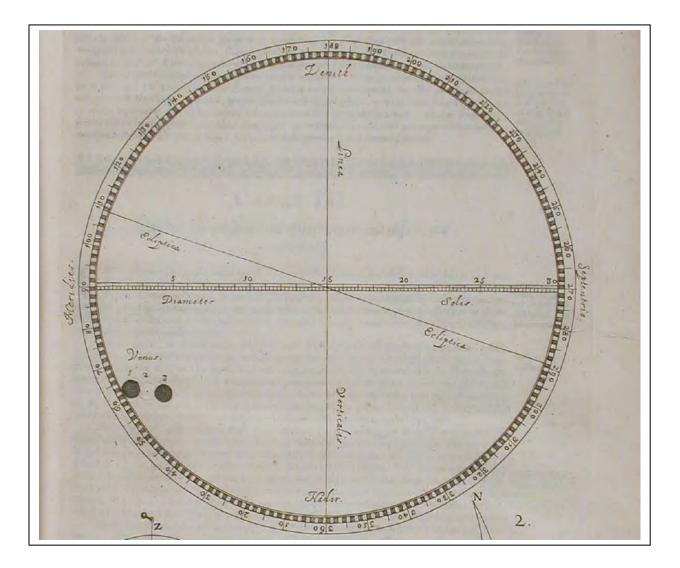


First observation of the transit of Venus by William Crabtree in 1639. Photogravure from an old etching. In "Essays in astronomy" - D. Appleton & company, 1900. Original painting by Ford Madox Brown (1821-1893).

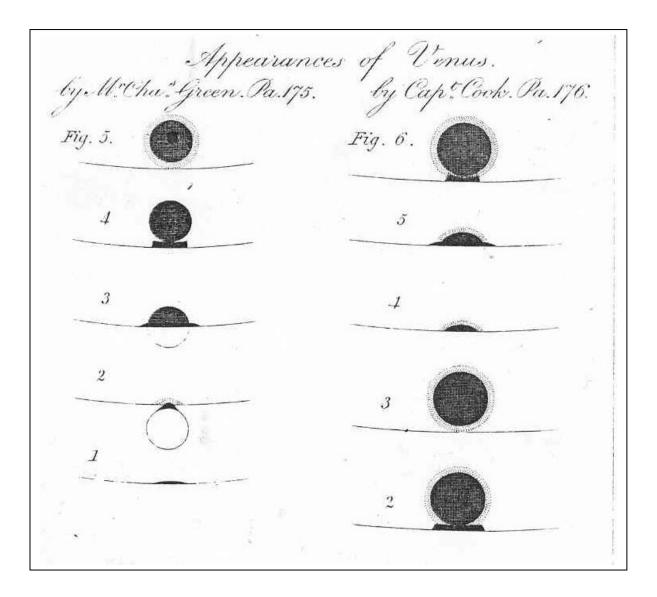
No actual images or paintings exist of this historic moment, and this recreation is much-reproduced in contemporary historical treatments of the event.



Artistic re-creation of Jeremiah Horrocks, who predicted the transit of Venus. Illustration by C. L. Doughty.



Jeremah Horrock's observation of Venus transit across the Sun in 1639. From his work *Venus in sole visa*, printed 1662. published by Jeremiah Horrocks and Jan Hevelius



Drawings of the Transit of Venus from Tahiti by Charles Green. The Royal Society commissioned Charles Green, an astronomer from the Royal Greenwich Observatory, to travel with Captain Cook to the South Seas. They show the characteristic distortion of the image of Venus when near to the limb of the Sun. This made the timing of the exact moment of contact difficult and compromised the results of the observations. (Armagh Observatory 2004 Venus Transit Exhibition http://www.arm.ac.uk/venustransit/exhibit/section-II.html )

Fransit of Venus. 3: of June. 1769. His most sacred Majesty King George the 3. Her most sacred Majesty Queen Charlotte. His Serene Highness Prince Ernest of Mecklenburg-Swelitz. His Serene Highness Prince George of Mecklenburg- Strelitz. altended by. Colonel Desaguliers. This Rev? M. Geo: Wollaston . Stephen Rigand. Justin Vuilliamy. Jeres Sifron, Ben. Williamy John Cuff. Doctor Stephen Demainbray. present. Stis Majesty the King who made his Observation. with a Shorts Reflecting Telescope, magnifying Diameters 170 Firmes, was the first who Jaw the Penumbra of Venus touching the Edge of The Sun's Disk. The Grand Disk. Time (according to civil Rectioning) Was attended to by Stephen Demainbray, appointed to take exact Time by Shelton's Regulator previously regulated by Several Astronomical Observations.

Records of the Transit of Venus observed from Kew by Dr Demainbray and members of the Royal family. They are remarkable for their agreement, with only one second difference between all of the observers, whereas at Greenwich, a similar set of observations yielded a spread of times ranging over one minute. It must have been difficult to disagree with the King. (*Courtesy of Kings College, London*) http://www.arm.ac.uk/venustransit/exhibit/section-II.html

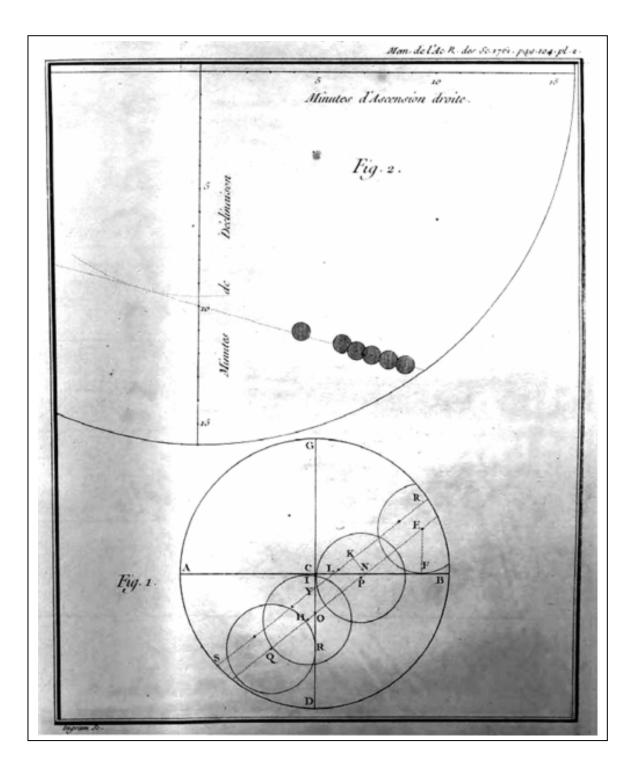
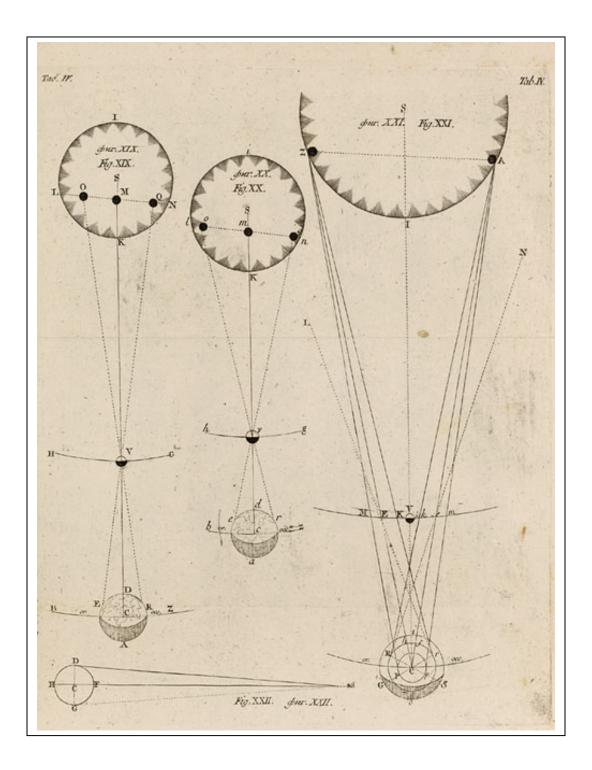
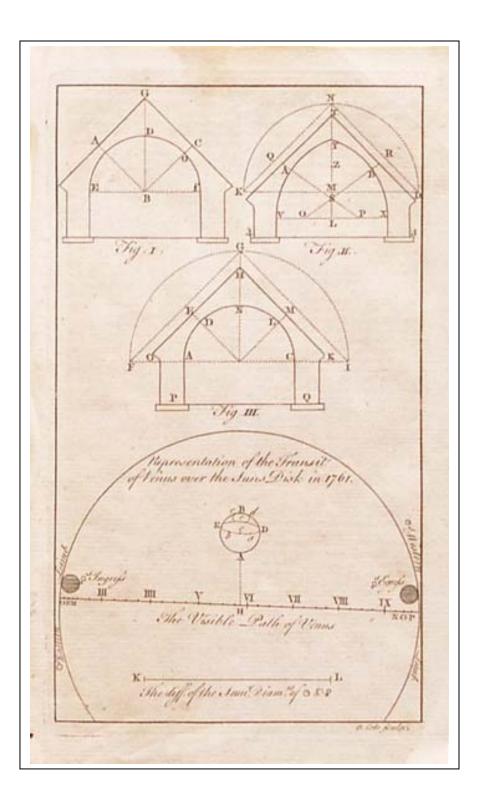


Illustration of the Venus transit of 1761 by Grandjean de Fouchy. Fig. 1 is the Sun's passage over the telescope's field of view, Fig. 2 is the positions of Venus at six different occasions

Grandjean de Fouchy, "Observation du passage de V'enus sur le Soleil," M'emoire de l'Acad'emie Royale des Sciences 63, 1761 (published 1763), 96–104, reproduced in Kragh, H. The Moon that Wasn't: The Saga of Venus' Spurious Satellite. Birkhäuser, 2008. P. 43



The location and path of the Transit of Venus in 1769 depends on your location on Earth. This drawing by Christian Mayer shows how the position of Venus would appear from various vantage points on Earth. (Christian Mayer, *Expositio de transitu veneris ante discum solis* Saint Petersburg, 1769; Adler Planetarium Transit of Venus Exposition)

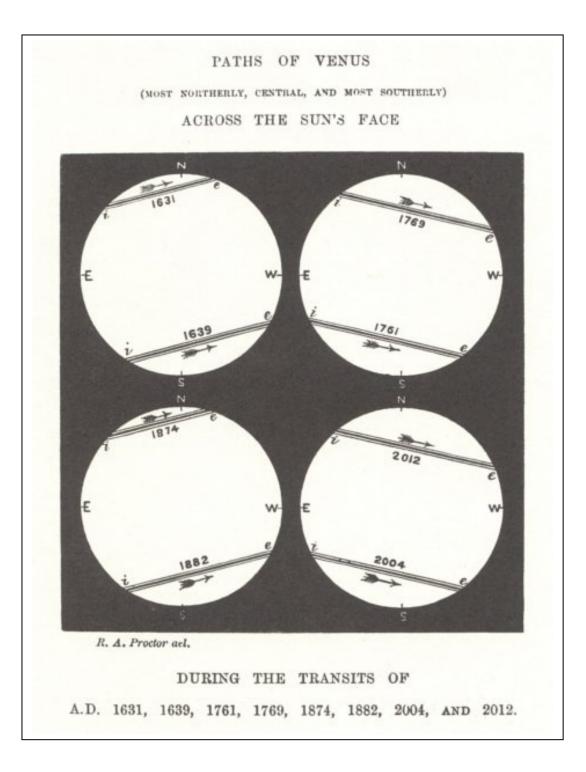


Benjamin Cole "Representation of the Transit of Venus over the Sun's Disk in 1761." London 1761. Figure shows the visible path of Venus and the path of the Sun and the Eastern and Western Limbs of the Arc. Drawn and engraved by Benjamin Cole, a well-known engraver in 18th century London, and published in the *Gentleman's Magazine* in 176161.

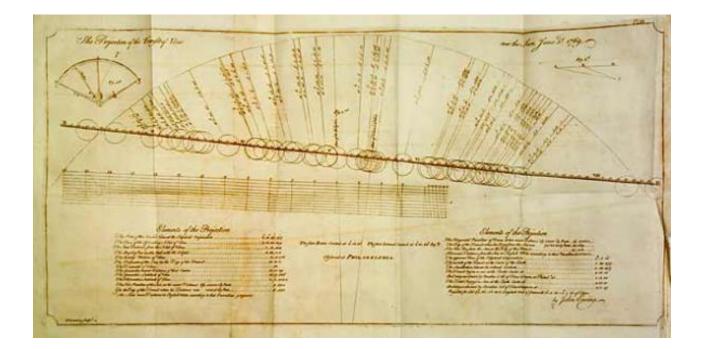
Suns Diameter in Minutes of Dean 10 20 25 Ш mit or visible Path of Venix VIII X H WES' EAS 200

This is a detail from a large folding plate in James Ferguson's *Astronomy explained upon Sir Isaac Newton's Principles*. First published in 1756, Ferguson subsequently added material on the 1761 transit. This is from the edition of 1811, edited by David Brewster.

The full engraving shows the geometry of the transit with the orbits of the Earth and Venus. This detail shows Venus at both ends of its path across the solar disc, at the two moments of internal contact with the limb of the sun. (Museum of the History of Science; University of Oxford)



This illustration in a popular astronomy book by Richard Proctor shows the tracks of the Transit of Venus from 1631 to 2012. It is one of the only known drawings that connects 19th century observations with 21st century observers. Few popular science articles or books bothered to look so far ahead!



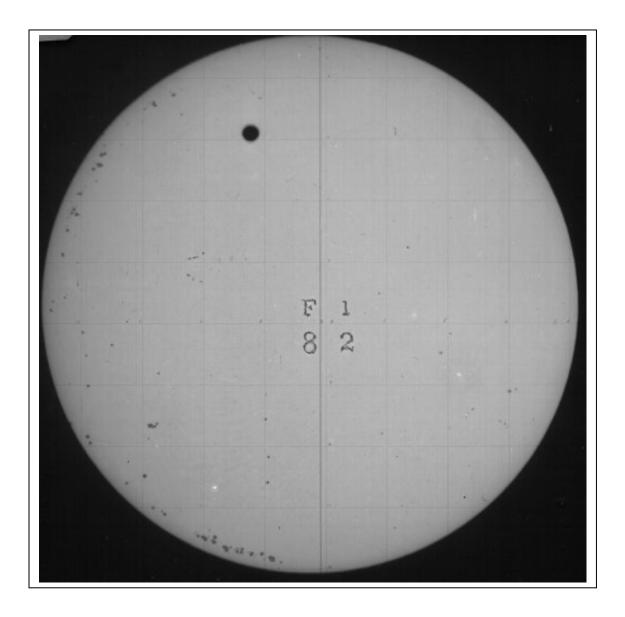
David Rittenhouse's map of the transit of Venus, published in the first volume of the *Transactions* of the American Philosophical Society Philadelphia, 1771. Elected to the American Philosophical Society in 1768, David Rittenhouse led the group of astronomers who observed the transit of Venus the following year. Rittenhouse's calculations of Venus's transit, done with a surveying device of his own design, won him international acclaim after their publication in the Society's *Transactions*, the first scientific journal published in North America.

Transit of & Sal June 3: 1769 50 \_ The first verible apprearance of Fig. I. for the O' Limb, very faint as in Fig. 1. Fig 9 39.20 -- First Internal Contact or thes outer limb of 9 seems to coundide with that of the Ofand appears ar in Fig 2. Fig 3. 10. 20 A Small Thrud oflight Seen below the Penumbra as in fis. 3

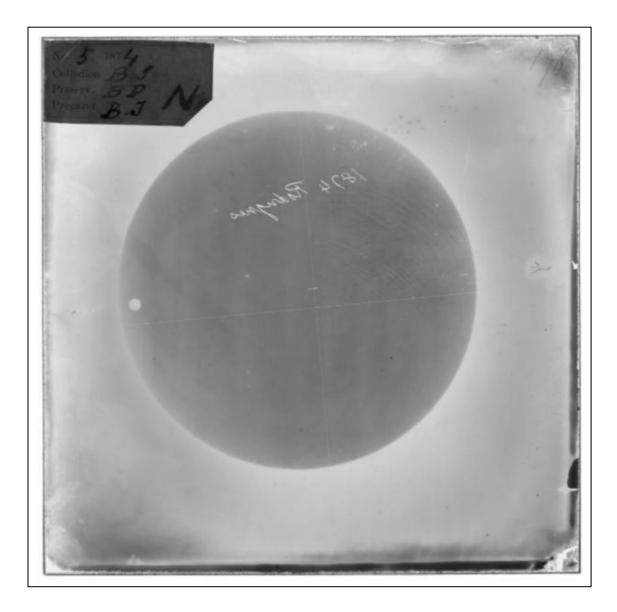
Captain James Cook's remarks on the June 6, 1769 Transit of Venus http://acms.sl.nsw.gov.au/search/simpleSearch.aspx?authority=lcsh&ID=443572



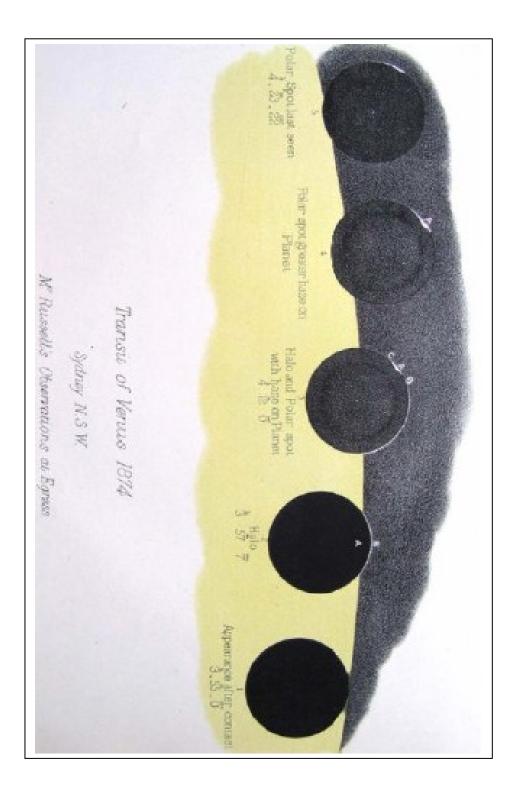
A humorous cartoon by Robert Sayer's *Viewing the Transit of Venus* of 1793, and one of the many satirical prints produced in England in the 18th century. It was printed 24 years after the last Transit of Venus, so the point of the humor, though probably obvious to the public at that time, is largely lost to us today!



An image of one of the 1700 plates taken worldwide during the 1882 transit, showing Venus crossing the disk of the Sun. Also visible is the image of the ruled glass reticle mounted in front of the plate and the vertical line in the centre is that of a thin silver plumb line that hung between the plate and grid. The small dots on the picture were caused by defects in the glass photographic plates.



A glass plate (numbered 5) from the British 1874 expedition to Rodrigues Island. The identification has been scratched on the body of the sun (in reverse here) and the plate also carries a paper label giving details of the photographic process.



A detailed study of the disk of venus during the Transit of Venus of 1874 by astronomer Russel, with particular interest taken in an attempt to find evidence for an atmosphere around Venus.



On the 24th of November 1874 a group of Sydney astronomers landed their pre-fabricated observatory, tents, instruments, bricks and cement at Two-Fold Bay in New South Wales. They were their to observe the Transit of Venus and over the next few days managed to lay down the building piers for the 7.5 inch Merz telescope and erect the wooden observatory we can see here with the numbered planks.

Sitting in the chair is Rev. Scott Sydney Observatory's first astronomer and standing next to him is W. J. MacDonnell next to the 4.5 inch Cooke equatorial. Further along we can see Mr. Watkins next to a 3.5 inch equatorial, Mr. Sharkey a photographer from the Government Printing Office and finally, in the background, the unnamed carpenter who accompanied the expedition.

Photography by John Sharkey, Eden, New South Wales, 1874



This engraving of Matavai Bay in Tahiti shows the bark *Endeavour* anchored in the bay close to Fort Venus, the site from which Captain Cook observed the transit of Venus in 1769.

The engraving is from the bestselling account of Cook's circumnavigation that was published in John Hawkesworth's *An Account of the Voyages Undertaken by the Order of His Present Majesty. For Making Discoveries in the Southern Hemisphere.* This is from a small format edition of 1789, much less lavish and expensive than the original edition of 1773.



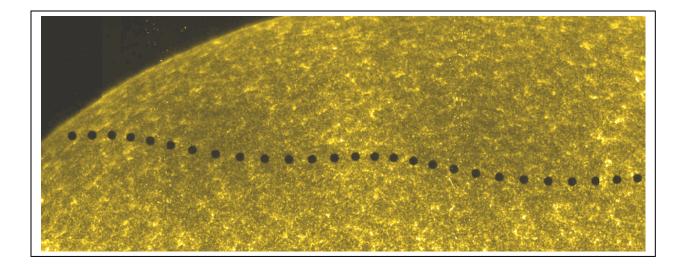




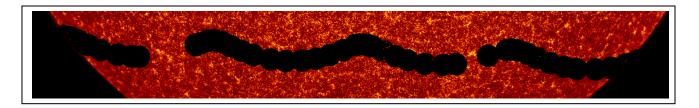
Top) Because of the accuracy of his charts and observations Cook was chosen in 1769 to master the H.M.S. "Endeavour" and travel with an astronomer of the Royal Society, to Tahiti to observe the transit of the planet Venus across the Sun. The "Transit of Venus" was depicted on a Norfolk Island stamp issued in 1969.

Right) A modern-day artist's impression of the observation of the 1769 transit of Venus from Tahiti by Cook and the astronomer Charles Green on a stamp from the Pacific islands of Tuvalu issued in 1979, the bicentenary of Cook's death. Green is presumably intended to be the kneeling figure looking through the telescope.

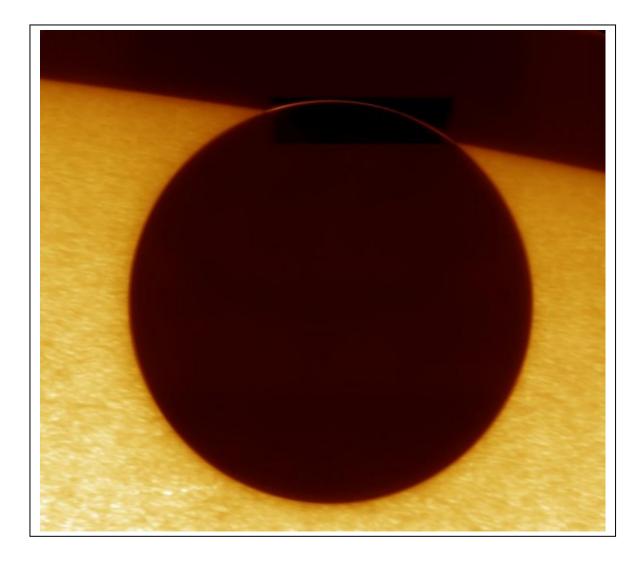
Lower Left) New Zealand issued a set of four stamps to commemorate the bicentenary of Captain James Cook's first voyage to the South Pacific. The lowest-value member of the set, pictured above, depicts Cook himself, the 1769 transit of Venus across the Sun, and an octant (a navigational instrument, forerunner of the sextant) superimposed on the latitude line of 40 degrees.



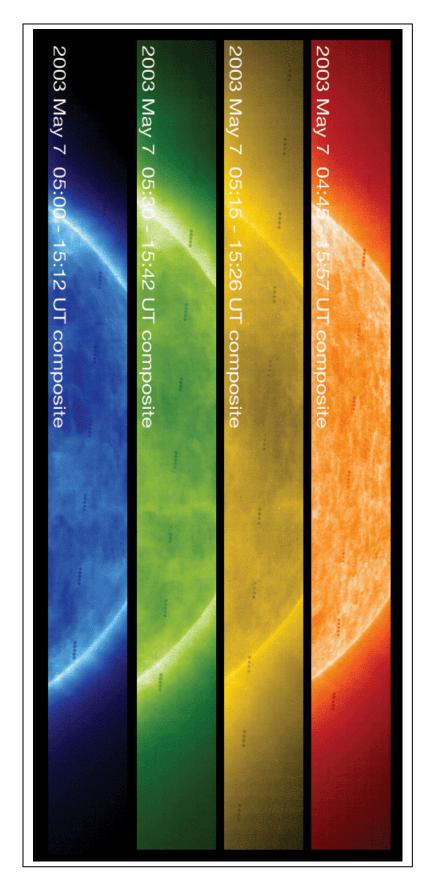
The Transit of Mercury in 2003 was observed by the NASA solar observatory, TRACE in this montage of images. The satellite orbits Earth, so its changing parallax angle of view causes the straight path of the transit to form a wavy line.



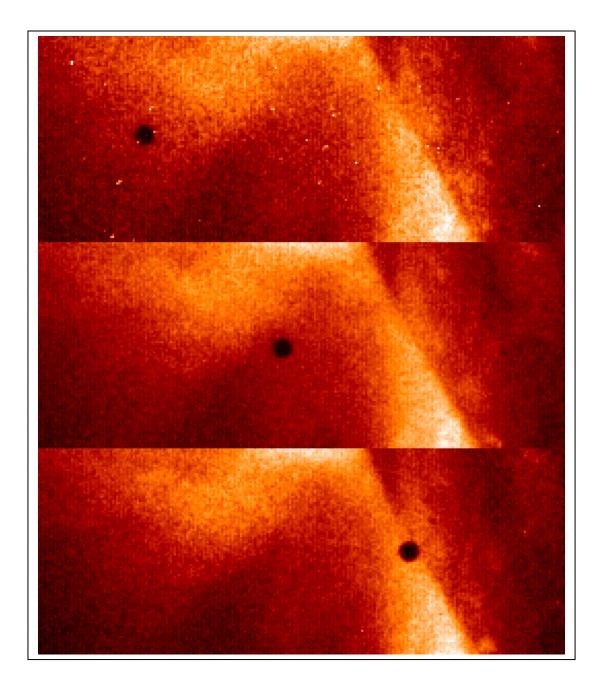
This dramatic photo montage is of the Transit of Venus in 2004 viewed by the TRACE solar observatory in orbit around Earth. The images of the disk of Venus are superimposed to show the wavy passage of Venus across the disk of the sun as a consequence of the parallax effect. The larger diameter of Venus compared to Mercury, along with its closer distance to Earth, makes the disk of Venus appear noticeably larger than the Mercury Transit disks in the 2003 sequence.



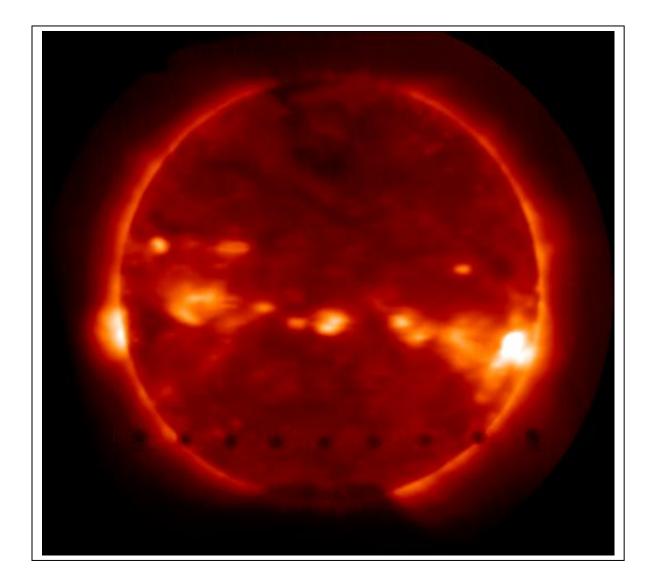
In 2004, the 1-meter, Swedish Vacuum Telescope in La Palma captured this dramatic image of the disk of Venus seen against the turbulent face of the sun. With Venus in transit at the Sun's edge on June 8th, astronomers captured this tantalizing close-up of the bright solar surface and partially silhouetted disk. Enhanced in the sharp picture, a delicate arc of sunlight refracted through the Venusian atmosphere is also visible outlining the planet's edge against the blackness of space. The arc is part of a luminous ring or atmospheric aureole, first noted and offered as evidence that Venus did posses an atmosphere following observations of the planet's 1761 transit.



The Solar and Heliospheric Observatory (SOHO) took a series of exposures of the 2003 Transit of Mercury in this composite of images from 4 of its spectral bands. The disk of mercury appears as four exposures, followed by a gap in time and another series of exposures.

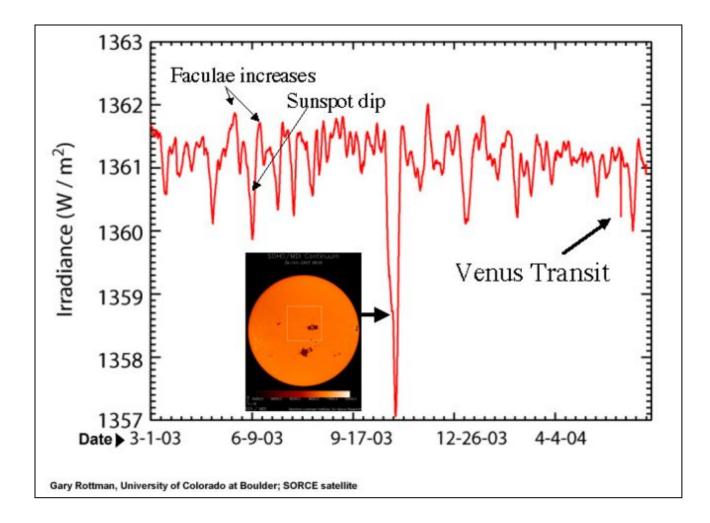


Mercury marched in front of the Sun from 19:12 UT on November 8 to 00:09 UT on November 9, 2007. Mercury was seen as a round-shaped disk on the Sun. The Mercury Transit was observed in Asian countries including Japan in the morning of November 9 local time. The solar space observatory, *Hinode*, which is in a Sun-synchronous orbit around the Earth, observed the event without atmospheric distortion in these x-ray images.



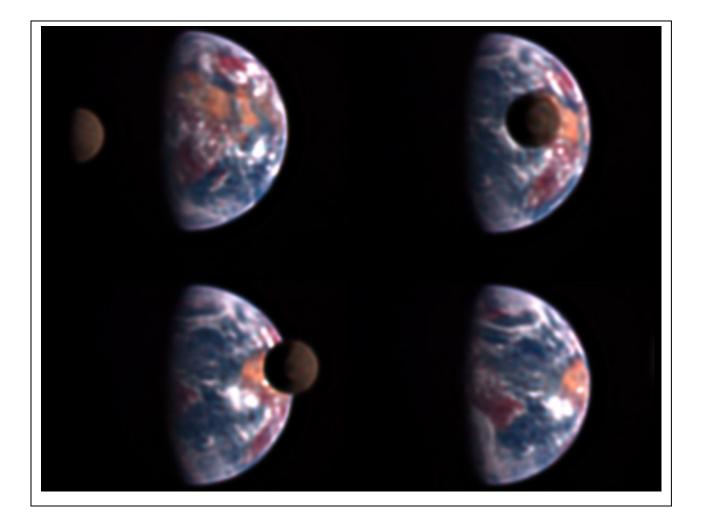
The NOAA Space Environment Center in Boulder, Colo., used the NOAA GOES-12 satellite space weather instrumentation to observe the passage of Venus in front of the sun. This transit of Venus ,which appears as the nine black disks in the lower edge of the sun, is the first in 122 years.

The GOES observation, using its Solar X-ray Imager, is unique because it records the sun's 2 million degree, outer atmosphere in X-rays. This observation is possible only from space, since Earth's atmosphere blocks out X-rays.

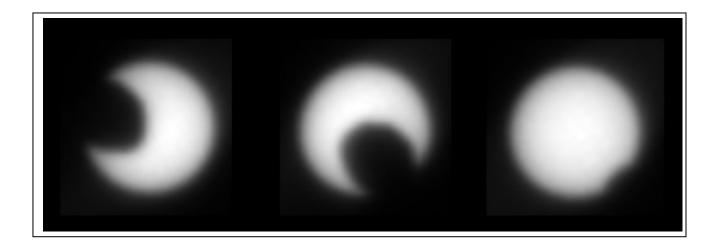


A graph of sunlight shows dips during sunspots, and the increases that surround those dips due to plage (also called faculae). Also shown: decrease in sunlight during the transit of Venus. The result was not a surprise, but since Venus hadn't transited the Sun in more than a century, the effect had never been measured. The drop in sunlight was similar to what happens when a large sunspot crosses the solar surface.

Sunlight reaching Earth is monitored by NASA's Solar Radiation and Climate Experiment (SORCE) satellite. The Venus transit proved to be a good test of instrument sensitivity. "Because of its distance from Earth, Venus appeared to be about the size of a sunspot" on June 8, said Gary Rottman, SORCE Principal Investigator and a scientist at the Laboratory for Atmospheric and Space Physics at the University of Colorado at Boulder.

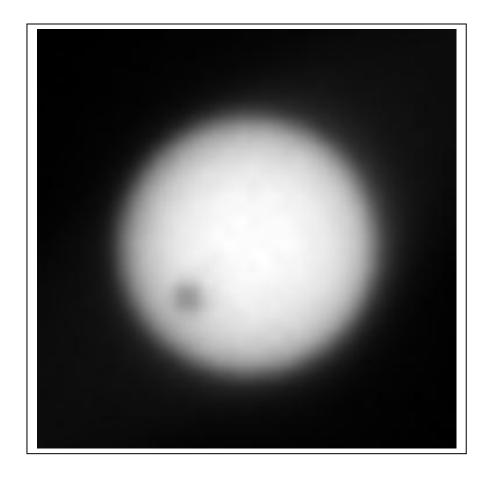


In July 2008, the Deep Impact spacecraft, on its way to a rendezvous with the Comet Hartley-2, captured these images of the Moon transiting the disk of earth from a distance of 31 million miles. These observations allow astronomers to get a feeling for what earth-like planets look like under unusual observing circumstances where details about the surface and environment are not available. (Deep Impact/EPOXI)



This panel illustrates the transit of the martian moon Phobos across the Sun. It is made up of images taken by the Mars Exploration Rover Opportunity on the morning of the 45th martian day, or sol, of its mission. This observation will help refine our knowledge of the orbit and position of Phobos. Other spacecraft may be able to take better images of Phobos using this new information. This event is similar to solar eclipses seen on Earth in which our Moon passes in front of the Sun. The images were taken by the rover's panoramic camera.

Image credit: NASA/JPL/Cornell



15:58:19

15:58:29

15:58:39

15:58:49

15:58:59

15:59:09

15:59:19

15:59:29

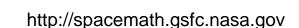
15:59:39

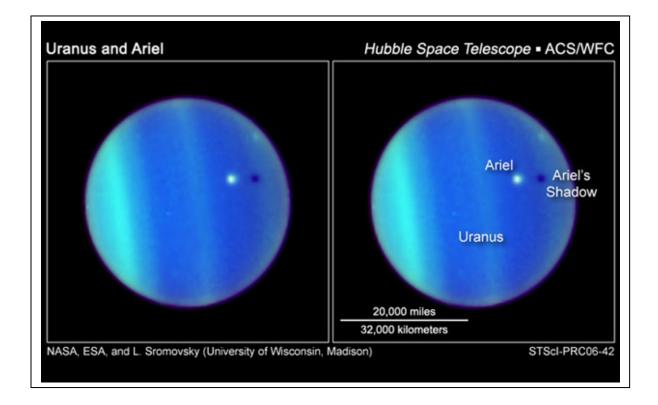
15:59:49

15:59:59

The transit of the martian moon Diemos across the sun as viewed from the surface of mars by the Rover Opportunity on March 4, 2004.

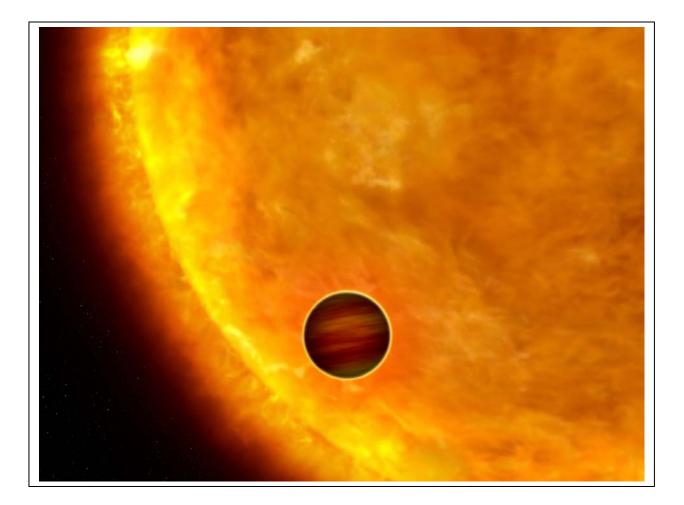
In the photo, the Sun has angular diameter 20.6' while Deimos only has 2.5'. Phobos by contrast usually has an angular diameter of around 12' as seen from Mars. Deimos took a little more than a minute to transit the Sun, passing well off center and moving downward and to the right; more central transits can take up to two minutes from start to end





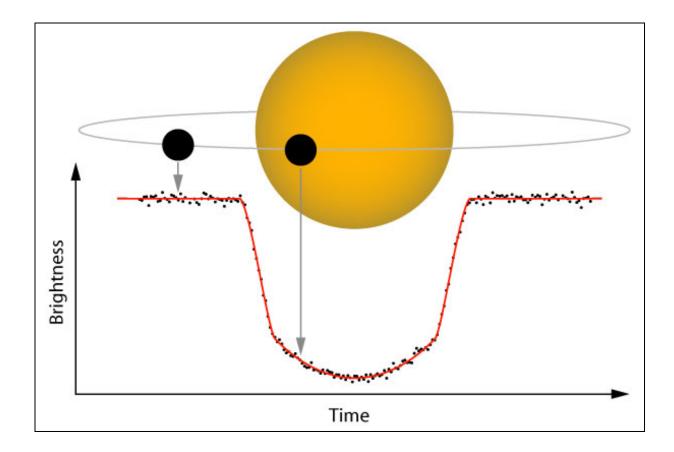
This image is a never-before-seen astronomical alignment of a moon traversing the face of Uranus, and its accompanying shadow. The white dot near the center of Uranus' blue-green disk is the icy moon Ariel. The 700mile-diameter satellite is casting a shadow onto the cloud tops of Uranus. To an observer on Uranus, this would appear as a solar eclipse, where the moon briefly blocks out the Sun as its shadow races across Uranus's cloud tops.

This transit has never been observed before because Uranus is just now approaching its 2007 equinox when the Sun will shine directly over the giant planet's equator. The last time a Uranian equinox occurred, when transits could have been observed, was in 1965. However, telescopes of that era did not have the image sharpness required to view satellite transits on Uranus. When Hubble was launched in 1990, the Sun was shining over Uranus's far northern latitudes. Over the past decade Hubble astronomers have seen the Sun's direct illumination creep toward equatorial latitudes and the moons' orbits approach an edge-on configuration.



Astronomers can detect planets orbiting other stars by the dimming of the star's light as the planet transits across its disk as viewed from Earth.

The above artist rendition shows such a planet, HD209458b which orbits its star every 3.5 days. With an atmospheric temperature of over 2,000 K it is an inhospitable world several times the mass of Jupiter.



A simple geometric model based upon the ratio of the planet's projected area and the area of its star allows astronomers to convert the dimming of the star's light to a precise measure of the diameter of the planet.

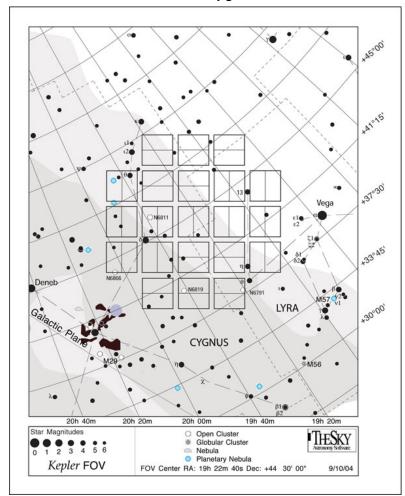
If the star's light dims by 1% or 1/100 of the star's normal brightness, the planet must have a geometric area 1/100 of the star's area so that the planet's diameter is 1/10 that of its star.

The figure above shows a planet passing across its star's surface as viewed from Earth and causing a measurable diminution in the star's brightness.

By measuring the brightness of over 140,000 stars every 30 minutes for 3.5 years, the Kepler Mission can detect exoplanet transits for earth-like planets orbiting within the Habitable Zones of their stars. During its first 40 days in orbit, it has already identified over 700 candidate planets in 2010, but at the scale of our own solar system, most of these planets are inside the orbit of Mercury.

## Appendix E NASA's Kepler Mission

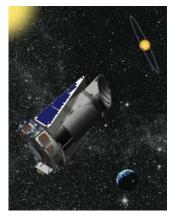
The Kepler Mission is a NASA space observatory designed to discover Earth-like planets orbiting other stars. The spacecraft was launched on March 7, 2009. The mission is named in honor of German astronomer Johannes Kepler. With a planned mission lifetime of at least 3.5 years, Kepler uses a very sensitive instrument called a photometer developed by NASA to continuously monitor the brightness of over 145,000 stars in a fixed field of view towards the constellation Cygnus. The data collected from these observations will be



analyzed to detect periodic brightness changes that indicate the presence of planets orbiting these stars that are transiting the faces of these stars as viewed from Earth.

In this area of the sky the crowded star lanes of the Milky Way are prominent, and among these target stars there are literally thousands that are similar to our own sun in temperature (yellow) and age, making the prospects for discovering one of more planets very high.

The Kepler photometer is so sentitive that it can easily detect the faintest stars in this survey as they change their brightness by only a few parts in ten thousand; enough to catch the transiting disk of a planet about he size of Earth.



# **Additional Mission Information:**

Kepler main website - http://kepler.nasa.gov/

Discovery page - http://kepler.nasa.gov/Mission/discoveries/

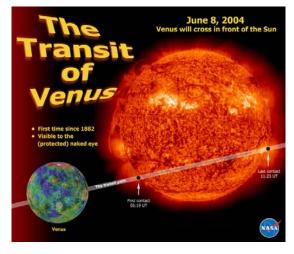
News - http://kepler.nasa.gov/news/nasakeplernews/

Exoplanet Catalog - http://www.planetary.org/exoplanets/

Space Math

http://spacemath.gsfc.nasa.gov

## Appendix F Additional Resources



NASA Sun-Earth Day – The Transit of Venus - This NASA resource, developed for the 2004 transit of Venus contains historical and scientific information about the importance of transits. Also links to webcasts and other multimedia resources. In September, 2011 it will be upgraded for the 2012 transit and provide information about NASA activities

#### http://rst.gsfc.nasa.gov/



Chasing the Goddess of Love Across the Sun - In this web exhibit, you can see historical artifacts that document past transits of Venus. http://www.adlerplanetarium.org/experience/exh ibitions/past/goddessoflove



**Smithsonian Institution Exhibition** on the Transits of Venus. Large resource of historical information and unique books and illustrations. http://www.sil.si.edu/Exhibitions/upcoming.htm



A Search for Habitable Planets

**The Exoplanet Catalog** - Detailed orbital information and properties of over 300 known planets outside our solar system

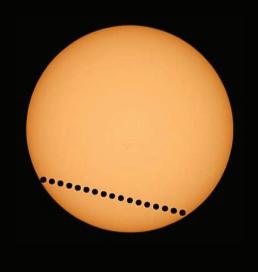
http://www.planetary.org/exoplanets/

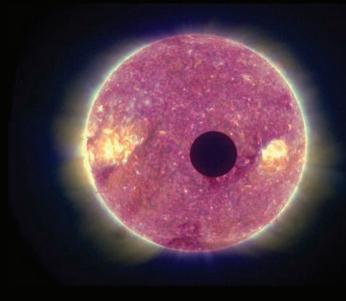
NASA Kepler Mission - Latest exoplanet Discoveries - 1200 planets detected so far! http://kepler.nasa.gov/

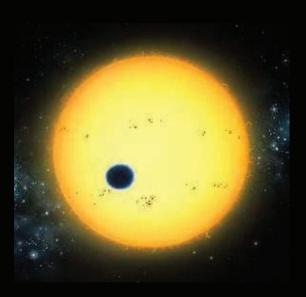
http://spacemath.gsfc.nasa.gov

Space Math

Kepler



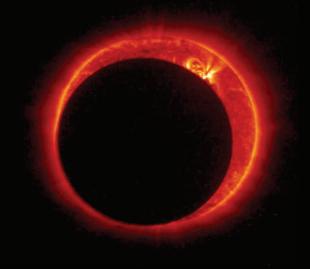






#### **National Aeronautics and Space Administration**

Space Math @ NASA Goddard Spaceflight Center Greenbelt, Maryland 20771 spacemath.gsfc.nasa.gov



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