

# Space Math - IV

This collection of activities is based on a weekly series of space science problems distributed to thousands of teachers during the 2007-2008 school year. They were intended for students looking for additional challenges in the math and physical science curriculum in grades 9 through 12. The problems were created to be authentic glimpses of modern science and engineering issues, often involving actual research data.

The problems were designed to be 'one-pagers' with a Teacher's Guide and Answer Key as a second page. This compact form was deemed very popular by participating teachers.

This booklet was created by the NASA, Hinode satellite program's Education and Public Outreach Project.

For more weekly classroom activities about astronomy and space visit the NASA website,

<http://spacemath.gsfc.nasa.gov>

Add your email address to our mailing list by contacting Dr. Sten Odenwald at

Sten.F.Odenwald@nasa.gov

**Front and back cover credits:** Tycho Crater region (NASA/Orbiter); Ring Current (IMAGE); Helioseismogram (SOHO/MDI); Jewel Box Cluster (NASA/HST)

**Interior Illustrations:** 1) X-ray sun (Hinode XRT); 2) Mercury Transit (Hinode XRT, EIS); Carrington sunspot (MNRAS v.20 p13, 1860); 1947 sunspot (Carnegie Institution of Washington); 5) MMS satellites (NASA/MMS) Reconnection (WWW unattributed); Heliopause (NASA/GSFC/ASD); 7) Sun cross section (NASA/ESA/SOHO); 9) stamp (WWW unattributed); 10) Heiroglyphic math (WWW unattributed); 11) Pleiades (Robert Gendler); 14) CME image (NASA/ESA/SOHO); Benford law (Author); 16) Solar images (NASA/STEREO) parallax sketch (Author); 17) Mira gas tail (NASA/GALEX); XZ Tauri (NASA/HST); 19) Encke tail (NASA/STEREO); 20) Mercury ice (Arecibo Observatory/JPL); 22) Black body curves (WWW unattributed); 23) Moon (NOAO/AURA/NSF), Moon sodium atmosphere (Boston University CSP), Atm composition (Author); 25) solar eclipse (Fred Espenak); 26) NGC-6266 (NASA/Chandra); NGC-6266 photo (DSS/STScI); 27) NGC-7129 image (NASA/Spitzer); 28) Messier-57 (NASA/Spitzer); 29) Tycho (NASA/Lunar Orbiter IV), Denver (NASA/Landsat); 30) Las Vegas (Digital Globe); 31) Bonneville Crater (NASA/JPL/MSSS)

# Table of Contents

Acknowledgments .....	i
Table of Contents.....	ii
Alignment with Standards .....	iii
How to use this Book.....	iv
How fast does the sun spin?.....	1
The Transit of Mercury.....	2
The Oscillation Period of Gaseous Spheres.....	3
Monster Sunspots!.....	4
NASA Juggles Four Satellites at Once.....	5
The Heliopause – A Question of Balance.....	6
A Mathematical Model of the Sun.....	7
A Problem in Satellite Synchrony.....	8
Scientific Notation I.....	9
Scientific Notation II.....	10
Scientific Notation III.....	11
Are You Ready for Sunspot Cycle 24?.....	12
Reality Check!.....	13
CME Kinetic Energy and Mass.....	14
Benford's Law.....	15
An Application of the Pythagorean Theorem.....	16
A Star Sheds a Comet Tail.....	17
XZ Tauri and the Super CME.....	18
Comet Encke Tail Disruption Event.....	19
Is There Ice on Mercury?.....	20
A Trillion Here..A Trillion There.....	21
Measuring Star Temperatures.....	22
The Moon's Atmosphere.....	23
Black Holes – The Event Horizon.....	24
The Last Total Solar Eclipse...Ever!.....	25
Chandra Explores Angular Size.....	26
Spitzer Explores a Dusty Young Star.....	27
Spitzer Explores a Dying Star.....	28
How Big is a Lunar Crater?.....	29
Exploring a City from Space.....	30
Exploring the Mars Spirit Rover from Space.....	31
A Note from the Author.....	32

# Topics and Alignment with Mathematics Standards

Topic	Problem Numbers																																
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31		
Inquiry	X	X	X			X	X								X																		
Technology, rulers						X	X								X		X	X	X	X										X	X	X	
Numbers, patterns, percentages								X	X	X	X		X		X						X		X		X	X		X					
Averages												X																					
Time, distance,speed	X	X			X																				X								
Areas and volumes					X															X							X	X	X	X	X		
Scale drawings	X	X		X													X	X	X	X						X			X	X	X		
Geometry	X	X		X				X								X	X	X	X	X					X	X	X	X	X	X	X		
Scientific Notation			X		X	X	X		X	X	X			X			X	X	X	X			X	X	X		X	X					
Unit Conversions																	X	X	X	X			X				X	X	X	X	X		
Graph or Table Analysis												X		X								X											
Solving for X														X		X																	
Evaluating Fns			X		X	X	X							X								X		X									
Modeling						X	X																										

## Applicable Standards (AAAS Project:2061 Benchmarks).

**(3-5)** - Quantities and shapes can be used to describe objects and events in the world around us. 2C/E1 --- Mathematics is the study of quantity and shape and is useful for describing events and solving practical problems. 2A/E1 [**Relevant problems 6, 7, 13, 15, 17, 18, 19, 20, 13, 21, 29, 30, 31**]

**(6-8)** Mathematicians often represent things with abstract ideas, such as numbers or perfectly straight lines, and then work with those ideas alone. The "things" from which they abstract can be ideas themselves; for example, a proposition about "all equal-sided triangles" or "all odd numbers". 2C/M1 [**Relevant problems 1,2,3,4,5,6,7,8,9,10,11,12,14,16,17,18,19,20,25-31**]

**(9-12)** - Mathematical modeling aids in technological design by simulating how a proposed system might behave. 2B/H1 ---- Mathematics provides a precise language to describe objects and events and the relationships among them. In addition, mathematics provides tools for solving problems, analyzing data, and making logical arguments. 2B/H3 ----- Much of the work of mathematicians involves a modeling cycle, consisting of three steps: (1) using abstractions to represent things or ideas, (2) manipulating the abstractions according to some logical rules, and (3) checking how well the results match the original things or ideas. The actual thinking need not follow this order. 2C/H2 [**Relevant problems 3,5,6,7,14,22,23,24**]



# How to use this book

Teachers continue to look for ways to make math meaningful by providing students with problems and examples demonstrating its applications in everyday life. Space Math offers math applications through one of the strongest motivators-Space. Technology makes it possible for students to *experience* the value of math, instead of just reading about it. Technology is essential to mathematics and science for such purposes as “access to outer space and other remote locations, sample collection and treatment, measurement, data collection and storage, computation, and communication of information.” 3A/M2 authentic assessment tools and examples. The NCTM standards include the statement that "Similarity also can be related to such real-world contexts as photographs, models, projections of pictures" which can be an excellent application for all of the Space Math applications.

This book is designed to be used as a supplement for teaching mathematical topics. The problems can be used to enhance understanding of the mathematical concept, or as a good assessment of student mastery.

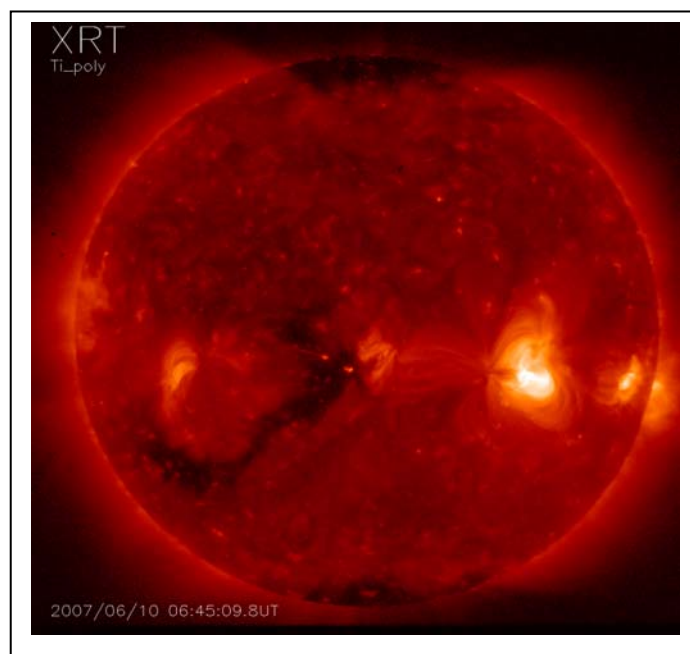
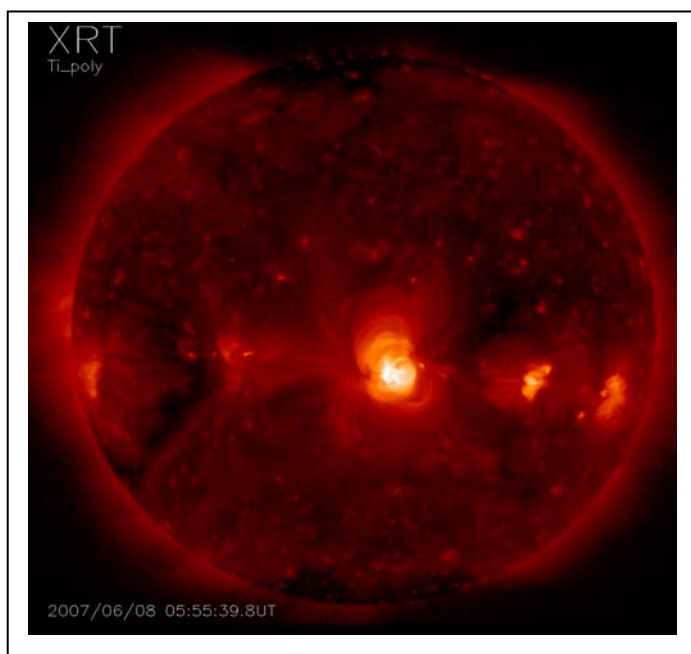
An integrated classroom technique provides a challenge in math and science classrooms, through a more intricate method for using **Space Math IV**. Read the scenario that follows:

Ms. Green decided to pose a new activity using Space Math for her students. She challenged each student team with math problems from the Space Math IV book. She copied each problem for student teams to work on. She decided to have the students develop a factious space craft. Each team was to develop a set of criteria that included reasons for the research, timeline and budget. The student teams had to present their findings and compete for the necessary funding for their space craft. The students were to use the facts available in the Space Math IV book and images taken from the Space Weather Media Viewer, <http://sunearth.gsfc.nasa.gov/spaceweather/FlexApp/bin-debug/index.html#>

**Space Math IV** can be used as a classroom challenge activity, assessment tool, enrichment activity or in a more dynamic method as is explained in the above scenario. It is completely up to the teacher, their preference and allotted time. What it does provide, regardless of how it is used in the classroom, is the need to be proficient in math. It is needed especially in our world of advancing technology and physical science.

# How fast does the sun spin?

1



The sun, like many other celestial bodies, spins around on an axis that passes through its center. The rotation of the sun, together with the turbulent motion of the sun's outer surface, work together to create magnetic forces. These forces give rise to sunspots, prominences, solar flares and ejections of matter from the solar surface.

Astronomers can study the rotation of stars in the sky by using an instrument called a spectroscope. What they have discovered is that the speed of a star's rotation depends on its age and its mass. Young stars rotate faster than old stars, and massive stars tend to rotate faster than low-mass stars. Large stars like supergiants, rotate hardly at all because they are so enormous they reach almost to the orbit of Jupiter. On the other hand, very compact neutron stars rotate 30 times each second and are only 40 kilometers across.

The X-ray telescope on the Hinode satellite creates movies of the rotating sun, and makes it easy to see this motion. A sequence of these images is shown on the left taken on June 8, 2007 (Left); June 10 2007 (Right) at around 06:00 UT.

Although the sun is a sphere, it appears as a flat disk in these pictures when in fact the center of the sun is bulging out of the page at you! We are going to neglect this distortion and estimate how many days it takes the sun to spin once around on its axis.

The radius of the sun is 696,000 kilometers.

**Problem 1** - Using the information provided in the images, calculate the speed of the sun's rotation in kilometers/sec and in miles/hour.

**Problem 2** – About how many days does it take to rotate once at the equator?

**Inquiry Question:** What geometric factor produces the largest uncertainty in your estimate, and can you come up with a method to minimize it to get a more accurate rotation period?

## Answer Key:

**Problem 1** - Using the information provided in the images, calculate the speed of the sun's rotation in kilometers/sec and in miles/hour.

First, from the diameter of the sun's disk, calculate the image scale of each picture in kilometers per millimeter.

$$\text{Diameter} = 76 \text{ mm. so radius} = 38 \text{ mm. Scale} = (696,000 \text{ km})/38 \text{ mm} = 18,400 \text{ km/mm}$$

Then, find the center of the sun disk, and using this as a reference, place the millimeter ruler parallel to the sun's equator, measure the distance to the very bright 'active region' to the right of the center in each picture. Convert the millimeter measure into kilometers using the image scale.

$$\begin{array}{llll} \text{Picture 1: June 8} & \text{distance} = & 4 \text{ mm} & d = 4 \text{ mm} (18,400 \text{ km/mm}) = 74,000 \text{ km} \\ \text{Picture 2; June 10} & \text{distance} = & 22 \text{ mm} & d = 22 \text{ mm} (18,400 \text{ km/mm}) = 404,000 \text{ km} \end{array}$$

Calculate the average distance traveled between June 8 and June 10.

$$\text{Distance} = (404,000 - 74,000) = 330,000 \text{ km}$$

$$\text{Divide this distance by the number of elapsed days (2 days)} \dots\dots\dots 165,000 \text{ km/day}$$

$$\text{Convert this to kilometers per hour} \dots\dots\dots 6,900 \text{ km/hour}$$

$$\text{Convert this to kilometers per second} \dots\dots\dots 1.9 \text{ km/sec}$$

$$\text{Convert this to miles per hour} \dots\dots\dots 4,400 \text{ miles/hour}$$

**Problem 2** – About how many days does it take to rotate once at the equator?

The circumference of the sun is  $2 \pi (696,000 \text{ km}) = 4,400,000 \text{ kilometers}$ .

The equatorial speed is 165,000 km/day so the number of days equals  
 $4,400,000/165,000 = \mathbf{26.6 \text{ days}}$ .

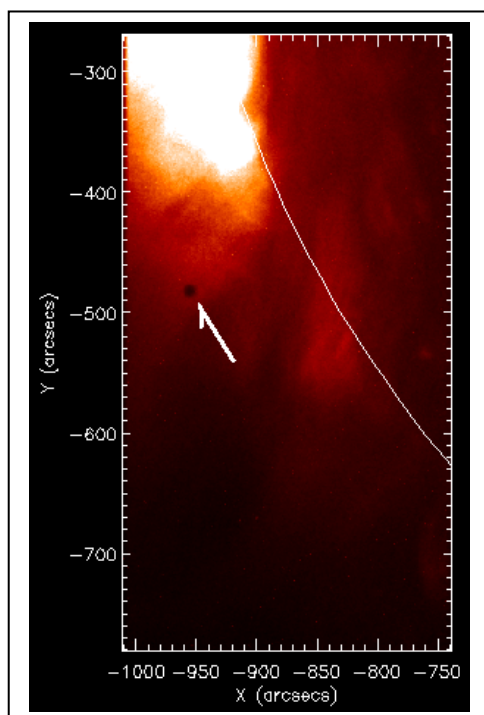
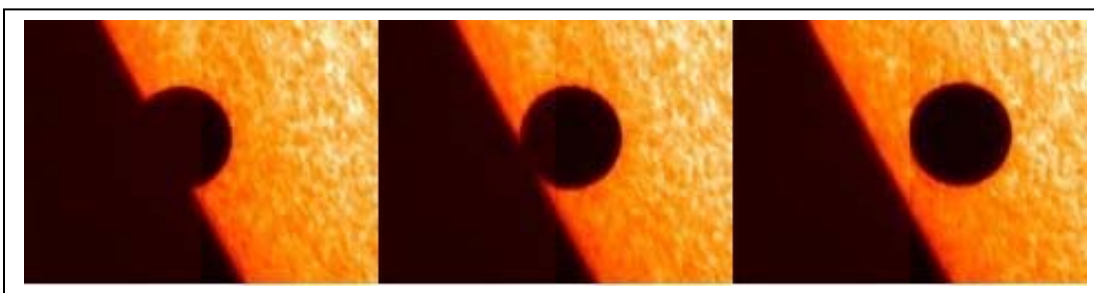
### Inquiry Question:

Because the sun is a sphere, measuring the distance of the spot from the center of the sun on June 10 gives a distorted linear measure due to foreshortening.

The sun has rotated about 20 degrees during the 2 days, so that means a full rotation would take about  $(360/20) \times 2 \text{ days} = 36 \text{ days}$  which is closer to the equatorial speed of the sun of 35 days.

# The Transit of Mercury

2



Every few times a century, the planet Mercury and Earth are lined up in such a way that Mercury passes across the disk of the sun as seen from Earth. The last time this happened was on November 8, 2006, and the next time this will happen will be on May 9, 2016. Since they were first observed in the 1600's, astronomers have studied them intently to learn more about Mercury, and to determine how far the sun is located from Earth. In recent times, astronomers no longer view these transits with much interest since the information that provide can be found by other more direct means. Still, when transits occur, astronomers turn their telescopes, now located in space, to watch the spectacle.

Top) Transit of Mercury obtained with Solar Optical Telescope (SOT) on the Hinode satellite on November 8, 2006. Left) Image obtained with the EUV Imaging Spectrometer (EIS). Solid curve indicates solar limb. The arrow shows the location of Mercury seen against the solar corona.

Problem 1: If the diameter of Mercury as viewed from Earth during the transit was 10 arcseconds, and the diameter of the sun at that time was 1900 arcseconds, what would be the diameter of the circle in the Hinode EIS image in centimeters that would represent the solar disk at this scale?

Problem 2: At the time of the transit, Mercury was about 55 million kilometers from the sun and about 92 million kilometers from Earth. How large, in arcseconds, would Mercury have appeared if it were at the distance of the Sun at this time?

Problem 3: How old will you be when the next Transit of Mercury happens?

Inquiry Problem: Why are transits of Mercury so rare?



### Answer Key:

Problem 1: If the diameter of Mercury as viewed from Earth during the transit was 10 arcseconds, and the diameter of the sun at that time was 1900 arcseconds, what would be the diameter of the circle in the Hinode EIS image in centimeters that would represent the solar disk at this scale?

Answer: The tic marks on the lower image are 10 arcseconds apart. With a millimeter ruler, the separation is about 2 millimeters. The scale of the image is then 5 arcseconds/millimeter. If the solar diameter is 1900 arcseconds, its size on this page would be  $1900 \text{ arcseconds} / (5 \text{ arcseconds/mm}) = 380 \text{ millimeters}$  or 38 centimeters.

Problem 2: At the time of the transit, Mercury was about 55 million kilometers from the sun and about 92 million kilometers from Earth. How large, in arcseconds, would Mercury have appeared if it were at the distance of the Sun at this time?

Answer: The distance to the sun would be  $55 + 92 = 147$  million kilometers. At a distance of 92 million kilometers from Earth, mercury is 10 arcseconds in size, so at 147 million kilometers it would be  $10 \text{ arcseconds} \times (92 \text{ million km} / 147 \text{ million km}) = 6.2 \text{ arcseconds}$ .

Problem 3: How old will you be when the next Transit of Mercury happens?

Answer: If you are 13 years old in 2007, you will be  $13 + (2016 - 2007) = 13 + 9 = 22$  years old, and will be graduating from college!!

Inquiry Problem: Why are transits of Mercury so rare?

Students may use GOOGLE to look up 'transit of Mercury' to find pages that discuss how transits occur.

They should deduce that the circumstances to not re-occur each year because Earth and mercury are on orbits that are tilted with respect to each other.

# The Oscillation Period of Gaseous Spheres

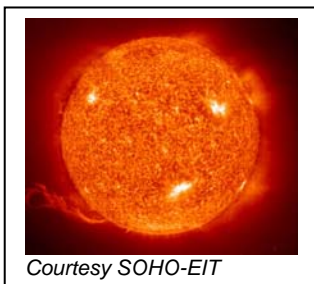
3

Any collection of matter that is governed by the force of gravity has a natural period of oscillation. For example, a simple pendulum such as a playground swing, will move back and forth with a time period in seconds,  $T$ , given by the formula to the right, where  $L$  is the length of the swing in centimeters, and  $g$  is the acceleration of gravity at Earth's surface given by  $980 \text{ cm/sec}^2$ . The period of a swing that is 3 meters long is then  $T = 3.5$  seconds.

$$T = 2\pi \sqrt{\frac{L}{g}}$$

This behavior also applies to any body held together by gravity whether it is a star or a planet. The natural oscillation period of such bodies is given by the formula to the right, where  $D$  is the density of the body in  $\text{grams/cm}^3$  and  $G$  is the Newtonian constant of Gravity  $G = 6.6 \times 10^{-8} \text{ dynes cm}^2 \text{ gm}^2$  and  $T$  is in seconds. For example, the planet Jupiter has a density of about  $D = 1.3 \text{ gm/cm}^3$  so its period,  $T$ , will be about 10,500 seconds or  $T = 3$  hours. From the information below, calculate the natural periods for the various astronomical bodies.

$$T^2 = \frac{3\pi}{G D}$$



Courtesy SOHO-EIT

The sun is about 1.5 million kilometers across, and has an average density of about  $1.5 \text{ grams/cm}^3$

$T =$  \_\_\_\_\_ hours.



Courtesy NASA-Apollo

The Earth has a diameter of about 12,500 kilometers, and has an average density of about  $5.5 \text{ grams/cm}^3$

$T =$  \_\_\_\_\_ minutes.



Courtesy NASA-Dana Berry

A neutron star is about 50 kilometers in diameter, and has an average density of about  $2 \times 10^{14} \text{ grams/cm}^3$

$T =$  \_\_\_\_\_ seconds.

### Answer Key:

The sun is about 1.5 million kilometers across, and has an average density near its surface of about  $1.5 \text{ grams/cm}^3$

$$\begin{aligned} T^2 &= 3 \times (3.141) / (1.5 \times 6.6 \times 10^{-8}) \\ &= 9.5 \times 10^7 \end{aligned}$$

$$\begin{aligned} \text{So } T &= 9,800 \text{ seconds} \\ &= \text{2.7 hours} \end{aligned}$$

The Earth has a diameter of about 12,500 kilometers, and has an average density of about  $5.5 \text{ grams/cm}^3$

$$\begin{aligned} T^2 &= 3 \times (3.141) / (5.5 \times 6.6 \times 10^{-8}) \\ &= 2.5 \times 10^7 \end{aligned}$$

$$\begin{aligned} \text{So } T &= 5,100 \text{ seconds} \\ &= \text{85 minutes} \end{aligned}$$

A neutron star is about 50 kilometers in diameter, and has an average density of about  $2 \times 10^{14} \text{ grams/cm}^3$

$$\begin{aligned} T^2 &= 3 \times (3.141) / (2.0 \times 10^{14} \times 6.6 \times 10^{-8}) \\ &= 7.1 \times 10^{-7} \end{aligned}$$

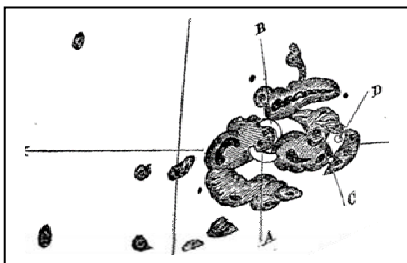
$$\text{So } T = \text{0.00085 seconds}$$

# Monster Sunspots!

Sunspots have been observed for thousands of years because, from time to time, the sun produces spots that are so large they can be seen from Earth with the naked eye...with the proper protection. Ancient observers would look at the sun near sunrise or sunset when Earth's atmosphere provided enough shielding to very briefly look at the sun for a few minutes. Astronomers keep track of these large 'super spots' because they often produce violent solar storms as their magnetic fields become tangled up into complex shapes.

Below are sketches and photographs of some large sunspots that have been observed during the last 150 years. They have been reproduced at scales that make it easy to study their details, but do not show how big they are compared to each other.

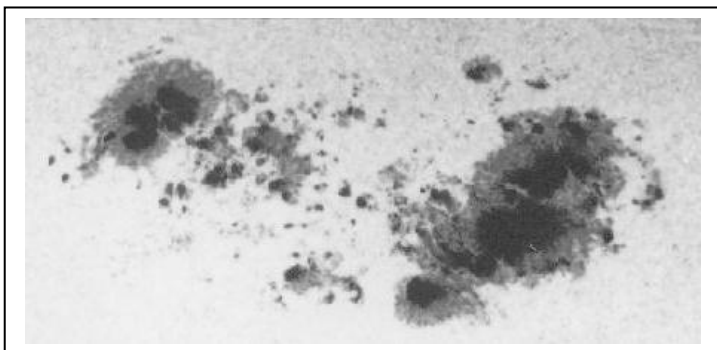
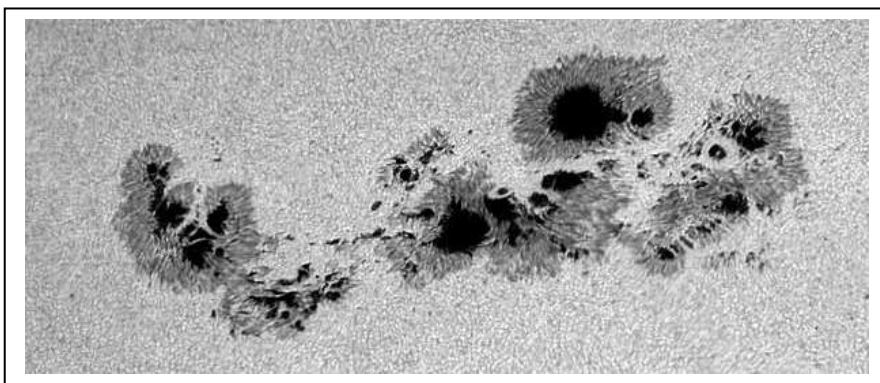
**Problem:** By using a millimeter ruler, use the indicated scales for each image to compute the physical sizes of the three sunspots in kilometers. Can you sort them by their true physical size?



Top is the sunspot drawn by Richard Carrington on August 28, 1859 at a scale of 5,700 kilometers/mm.

Middle is a photograph of a sunspot seen on March 29, 2000 at a scale of 23,500 kilometers/cm.

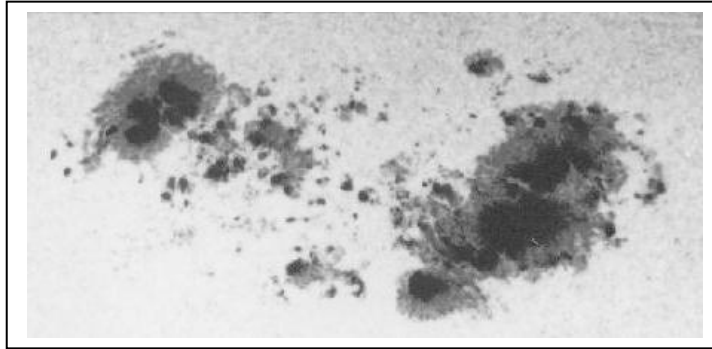
Bottom is a sunspot seen on April 8, 1947 at a scale of 100,000 kilometers/inch



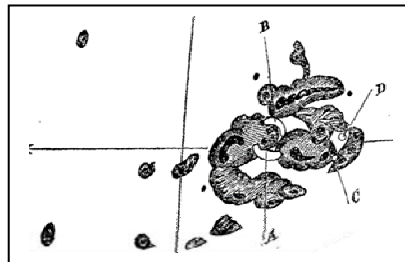


Answer Key:

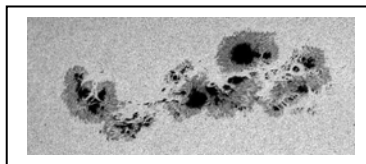
Images ordered from largest to smallest and to scale:



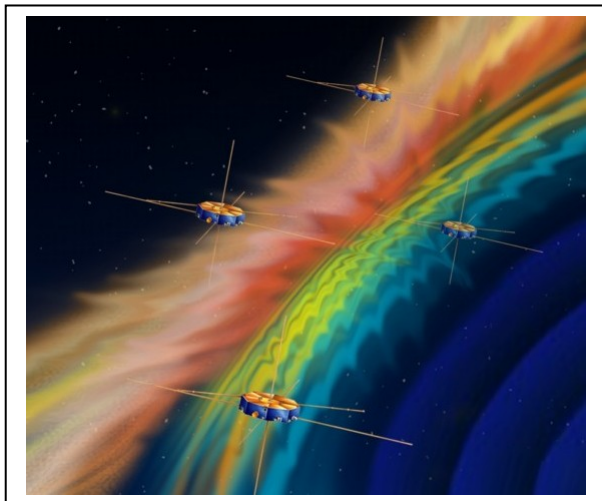
Sunspot seen on April 8, 1947 reproduced at a scale of 100,000 kilometers/inch. The linear extent on the page is 7 centimeters, so the length in inches is  $7 / 2.5 = 2.8$  inches. The true length is then  $2.8 \times 100,000 = \underline{280,000 \text{ kilometers}}$ .



The sunspot drawn by Richard Carrington on August 28, 1859 at a scale of 5,700 kilometers/mm. With a ruler, the distance from the left to the right of the group is about 40 millimeters, so the true length is about  $40 \times 5,700 = \underline{228,000 \text{ kilometers}}$ .



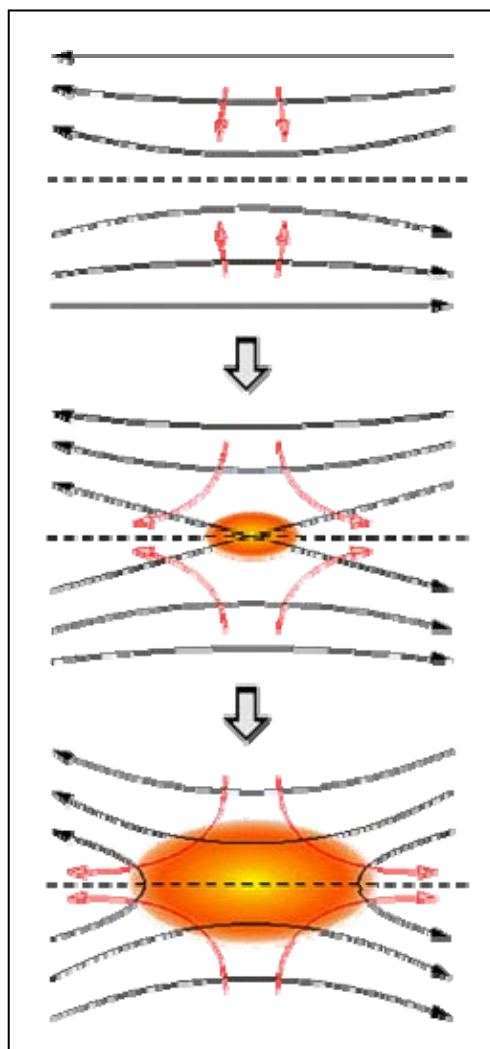
A photograph of a sunspot seen on March 29, 2000 at a scale of 23,500 kilometers/cm. The length of the spot is 90 millimeters or 9 centimeters. The true length is then 211,500 kilometers.



In 2013, NASA will launch the Magnetospheric Multi-Scale (MMS) satellite constellation; a set of four identical satellites that will study the 3-dimensional properties of Earth's magnetosphere. The spacecraft will travel in a tetrahedral formation with inter-spacecraft distances that will vary from ten kilometers to tens of thousands of kilometers.

At the center of the investigations is magnetic reconnection. This process, as shown in the sequence of figures below, breaks and reconnects magnetic lines of force. This converts stored magnetic energy into kinetic energy that causes trapped particles (the expanding spot in the figure) to also gain energy. MMS scientists hope that they will witness a few of these events inside the tetrahedral volume.

MMS will measure the physical properties of the reconnection process including the factors that control it, such as its distribution in space, and its minute-by-minute changes in time. This will help scientists test various theories of how this important process is triggered, evolves in time, and how magnetic energy is transferred into particle motion.



**Problem 1:** The volume of a regular tetrahedron with an edge length of  $A$  is given by the formula:

$$V = \frac{1}{12} A^3 (2)^{1/2}$$

What will be the volume of the MMS constellation: A) at its minimum size of  $A = 10$  km? B) at its maximum size of  $A = 10,000$  km?

**Problem 2:** The speed of the reconnection event is predicted to be about 500 kilometers/sec. About how long will it take disturbances to travel across the region of space being studied by MMS at its largest and its smallest satellite configuration?

**Problem 3:** Magnetic reconnection process (see the figure to left) releases stored magnetic energy (shaded region) which can then be transformed into kinetic energy in the motion of trapped, charged particles. The amount of stored magnetic energy is given by the formula:

$$E = \frac{1}{8\pi} B^2 \times V \quad \text{ergs}$$

Where  $B$  is the magnetic field strength in Gauss, and  $V$  is the volume of the magnetic field in cubic centimeters. If  $B = 0.0002$  Gauss, how much energy could be stored within the tetrahedral volumes of the largest and smallest configurations of MMS?

## Answer Key:

**Problem 1:** The volume of a regular tetrahedron with an edge length of A is given by the formula:

$$V = \frac{1}{12} A^3 (2)^{1/2}$$

What will be the volume of the MMS constellation:

- A) at its minimum size of A = 10 km?  
 B) at its maximum size of A = 10,000 km?

Answer = 118 cubic kilometers

Answer = 118 billion cubic kilometers

**Problem 2:** The speed of the reconnection event is predicted to be about 500 kilometers/sec. About how long will it take disturbances to travel across the region of space being studied by MMS at its largest and its smallest satellite configuration?

Answer: Largest: 10,000 km / (500 km/sec) = 20 seconds.  
 Smallest 10 km / (500 km/sec) = 0.02 seconds.

**Problem 3:** Magnetic reconnection releases stored magnetic energy which can then be transformed into kinetic energy in the motion of trapped, charged particles. The amount of stored magnetic energy is given by the formula:

$$E = \frac{1}{8\pi} B^2 \times V \quad \text{ergs}$$

Where B is the magnetic field strength in Gauss, and V is the volume of the magnetic field in cubic centimeters. If B = 0.0002 Gauss, how much energy could be stored within the tetrahedral volumes of the largest and smallest configurations of MMS?

Largest volume:

$$V = 118 \text{ billion km}^3 \times (1.0 \times 10^{15} \text{ cm}^3/\text{km}^3) = 1.2 \times 10^{26} \text{ cm}^3$$

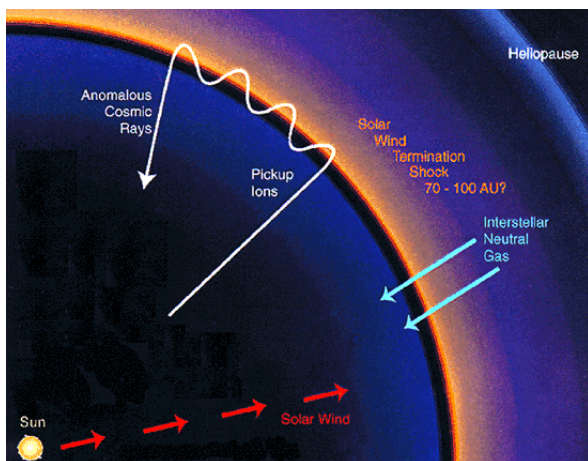
$$E = 0.039 \times (0.0002)^2 \times 1.2 \times 10^{26} \text{ cm}^3 = \underline{1.9 \times 10^{17} \text{ ergs}}$$

Smallest volume:

$$V = 118 \text{ km}^3 \times (1.0 \times 10^{15} \text{ cm}^3/\text{km}^3) = 1.2 \times 10^{17} \text{ cm}^3$$

$$E = 0.039 \times (0.0002)^2 \times 1.2 \times 10^{17} \text{ cm}^3 = \underline{1.9 \times 10^8 \text{ ergs}}$$

# The Heliopause...a question of balance!



Somewhere out beyond Pluto the particles flowing out from the sun in the 'solar wind' encounter a dilute gas called the interstellar medium (ISM). The sun is traveling through space at a speed of about 26 km/sec, and so from the sun's frame of reference, the ISM appears to be flowing past the solar system at this same speed. The solar wind can be thought of as a gas that exerts a pressure on the ISM, and similarly, the ISM exerts a pressure on the solar wind by virtue of its temperature and density.

The distance to the Heliopause is determined by the balance of pressure between the outflowing solar wind and the incoming ISM.

A relationship between these pressures yields the approximate formula shown to the right, where:

$V$  = speed of the solar wind in cm/sec,  
 $R$  = distance from the sun in AU.  
 $D$  = solar wind density at 1AU in particles/cc,  
 $M$  = the mass of a typical ISM atom in grams,  
 $n$  = density of the ISM  
 $T$  = temperature (Kelvins), of the ISM,

$k$  = Boltzmann's constant  $1.38 \times 10^{-16}$  ergs/degree.

Solar Wind Pressure = ISM thermal pressure

$$M V^2 \frac{D}{R^2} = n k T$$

For example, let's select:

$V = 400 \text{ km/sec}$  so that  $V = 4 \times 10^7 \text{ cm/sec}$

$M = \text{mass of hydrogen atom} = 1.6 \times 10^{-24} \text{ grams}$

$T = 100,000 \text{ Kelvins.}$

$D = 5 \text{ particles/cm}^3$

$n = 0.01 \text{ particles/cm}^3$

Then:  $(1.6 \times 10^{-24}) \times (4 \times 10^7)^2 \times (5.0) = (0.01) \times (1.38 \times 10^{-16}) \times (100,000) \times R^2$

And so:  $R = (1.3 \times 10^{-8} / 1.38 \times 10^{-13})^{1/2}$

= 306 AU to the heliopause from the sun. Note the distance to Pluto is about 40 AU!

## Inquiry Problem:

Data from the Voyager spacecraft suggest that the Bow Shock lies just beyond a distance of 100 AU from the sun, but probably not more than 200 AU. With the help of an Excel Spreadsheet, enter the bow shock distance formula into one of the columns. Can you find a range of distances to the heliopause that is consistent with plausible solar wind properties for  $V$  between 400 – 800 km/sec;  $T$  between 50,000 – 200,000 K and  $D$  between 2 – 20 particles/cm<sup>3</sup>?



Answer Key: A sample Excel page.

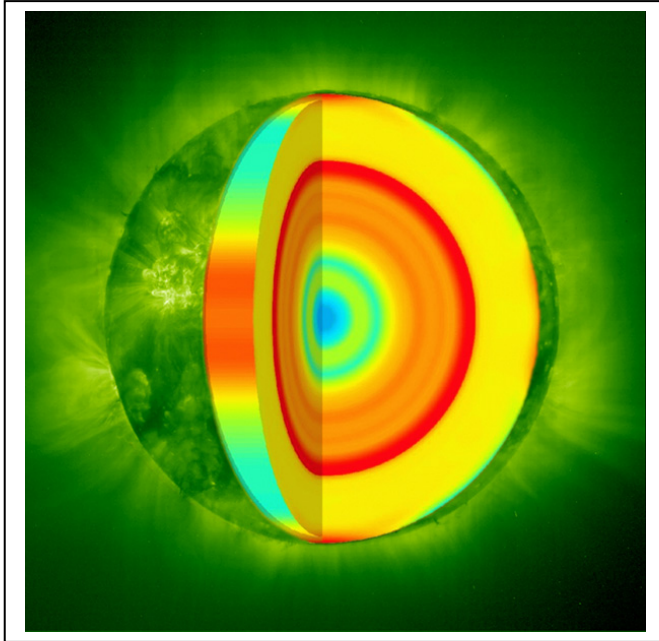
The screenshot shows the Microsoft Excel interface. The Formula Bar at the top displays the formula for cell B10:  $R = \sqrt{(B4 \cdot B7 \cdot (B3 \cdot 100000)^2) / (B6 \cdot B8 \cdot B5)}$ . The spreadsheet contains the following data:

	A	B	C	D	E	F
1	Variable	Value	Units			
2						
3	V=	400	km/s			
4	M=	1.60E-24	gms			
5	T=	100000	kelvins			
6	n=	0.01	particles/cm3			
7	D=	10	particles/cm3			
8	k=	1.38E-16	ergs/degree			
9						
10	R=	431	AU			
11						

The MS-Excel Formula Bar contains the solution for R in Astronomical Units (AU) as a function of the input variables defined in Column A. The values adopted for the example are shown in Column B with the indicated units in Column C. Note that the value for V in km/s has to be converted into cm/s which is why the formula shows  $B3 \cdot 100000$ .

The resulting value for R is shown in Cell B10.

By varying V, n, D and T students can see how the distance to the heliopause varies. It is suggested that students do an on-line literature search to find out what values for these quantities are commonly cited by researchers. They should always cite the reference using proper citation formats, when creating a report on this project. **Values between 120 and 200 AU are reasonable, but students should be encouraged to do a GOOGLE search on the 'Heliopause distance' to explore other published estimates.**



Once an astronomer knows the radius and mass of a planet or star, one of the first things they might do is to try to figure out what the inside looks like.

Gravity would tend to compress a large body so that its deep interior was at a higher density than its surface layers. Imagine that the sun consisted of two regions, one of high density (the core shown in red) and a second of low density (the outer layers shown in yellow). All we know about the sun is its radius (696,000 km) and its mass ( $2.0 \times 10^{30}$  kilograms). What we would like to estimate is how dense these two regions would be, in order for the final object to have the volume of the sun and its total observed mass. We can use the formula for the volume of a sphere, and the relationship between density, volume and mass to create such a model.

Suppose the inner zone has a radius of 417,000 km.  
The volume of this 'core' is

$$V_c = \frac{4}{3} \pi (417,000,000 \text{ m})^3 = 3.0 \times 10^{26} \text{ m}^3$$

The volume of the entire sun is

$$V^* = \frac{4}{3} \pi (696,000,000 \text{ m})^3 = 1.4 \times 10^{27} \text{ m}^3$$

So the outer shell zone has a volume of  
 $V_s = V^* - V_c$

$$\text{So, } V_s = 1.1 \times 10^{27} \text{ km}^3$$

## Mass = Density x Volume,

so if we estimate the density of the sun's core as  $D_c = 100,000 \text{ kg/m}^3$ , and the outer shell as  $D_s = 10,000 \text{ kg/m}^3$ , the masses of the two parts are:

$$M_c = 100,000 \times (3.0 \times 10^{26} \text{ m}^3) = 3.0 \times 10^{31} \text{ kg}$$

$$M_s = 10,000 \times (1.1 \times 10^{27} \text{ m}^3) = 1.1 \times 10^{31} \text{ kg}$$

So the total solar mass in our model would be:

$$M^* = M_c + M_s = 4.1 \times 10^{31} \text{ kg}$$

This is about 20 times more massive than the sun actually is!

## Inquiry Problem.

With the help of an Excel spreadsheet, program the spreadsheet so that you can adjust the radius of the core and calculate the volume of the core and shell zones. Then create a formula so that you can enter various choices for the densities of the core and shell zones and then sum-up the total mass of your mathematical model for the sun.

What are some possible ranges for the core radius, and zone densities (in grams per cubic centimeter) that give about the right mass for the modeled sun? How do these compare with what you find from searching the web literature? How could you improve your mathematical model to make it more accurate? Show how you would apply this method to modeling the interior of the Earth, or the planet Jupiter?

## Inquiry Problem.

With the help of an Excel spreadsheet, program the spreadsheet so that you can adjust the radius of the core and calculate the volume of the core and shell zones. Then create a formula so that you can enter various choices for the densities of the core and shell zones and then sum-up the total mass of your mathematical model for the sun.

Notes: The big challenge for students who haven't used the volume formula is how to work with it for large objects...requiring the use of scientific notation as well as learning how to use the volume formula. Students will program a spreadsheet in whatever way works best for them, although it is very helpful to lay out the page in an orderly manner with appropriate column labels. Students, for example, can select ranges for the quantities, and then generate several hundred models by just copying the formula into several hundred rows. Those familiar with spreadsheets will know how to do this, and it saves entering each number by hand...which would just as easily have been done with a calculator and does not take advantage of spreadsheet techniques.

What are some possible ranges for the core radius, and zone densities (in grams per cubic centimeter) that give about the right mass for the modeled sun? How do these compare with what you find from searching the web literature? How could you improve your mathematical model to make it more accurate? Show how you would apply this method to modeling the interior of the Earth, or the planet Jupiter?

Notes: Most solar models suggest a core density near 160 grams/cc, and an outer 'convection zone' density of about 0.1 grams/cc. The average value for the shell radius is about 0.7 x the solar radius. Students will encounter, using GOOGLE, many pages on the solar interior, but the best key words are things like 'solar core density' and 'convection zone density'. These kinds of interior models are best improved by 1) using more shells to divide the interior of the body, and 2) developing a mathematical model of how the density should change from zone to zone. Does the density increase linearly as you go from the surface to the core, or does it follow some other mathematical function? Students may elect to also test various models for the interior density change such as

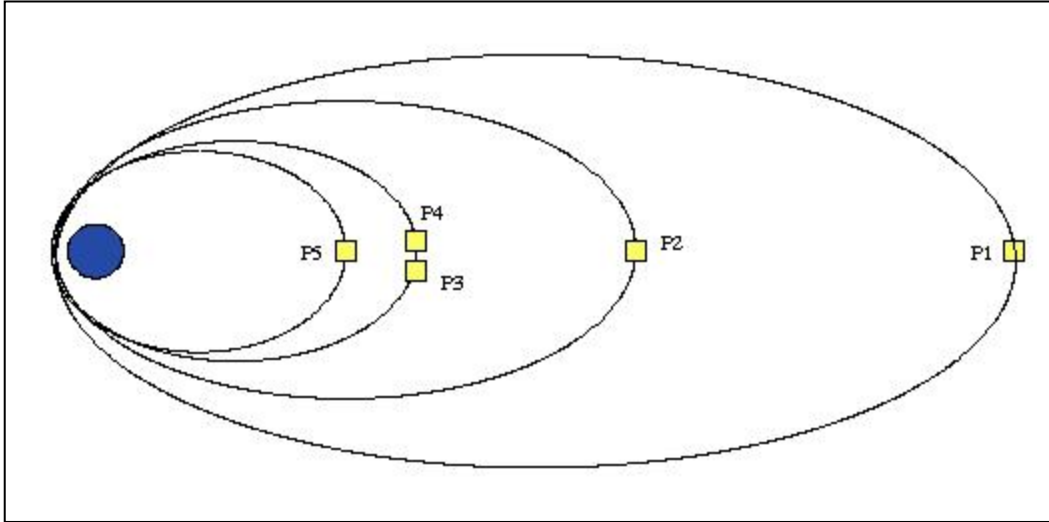
$$D = \frac{100 \text{ gm/cc}}{R} \quad \text{or even} \quad 100 \text{ gm/cc} \times e^{-R/R^*}$$

This method can be applied to any spherical body (planet, asteroid, etc) for which you know a radius and mass ahead of time.

Alternately, if you know a planet's mass and its composition (this gives an average density), then you can estimate its radius!

# A Problem in Satellite Synchrony

8



The THEMIS program has launched five satellites that will orbit earth at different distances to study Earth's magnetic field. The goal is to have them line up a few times each year so that they can study how magnetic storms are triggered and evolve in time.

The following problems will be easier to solve if you use a spreadsheet, but they CAN be worked out by hand by finding the Least Common Multiples. Can you answer them both ways?

**Problem 1** - Suppose you had two satellites orbiting Earth in the same plane. The closer-in satellite had a period of 4 hours, and the second satellite had a period of 8 hours. Looking down at Earth from the North Pole, suppose you started the satellites together at the same longitude, say 100 degrees West. How many hours later would they both be together again?

**Problem 2** - Suppose the faster satellite had a period of 120 minutes, and the slower satellite had a period of 17 hours. After how many hours would they come together again in longitude?

**Problem 3** - Suppose you had three satellites with periods of 2 hours, 4 hours and 6 hours. Using the Least Common Multiple, find how long would it take for them to return to their original line-up at the same longitude.

**Problem 4** - The THEMIS mission uses five satellites with periods of about 19 hours, 24 hours, 24 hours, 48 hours, and 96 hours, how many hours would it take for them all to get together again at the same longitude? Give your answer in days.



## Answer Key:

**Problem 1** - Suppose you had two satellites orbiting Earth. The closer-in satellite had a period of 4 hours, and the second satellite had a period of 8 hours. Looking down at Earth from the North Pole, suppose you started the satellites together at the same longitude, say 100 degrees West. How many hours later would they both be together again?

Answer: Create a time line and mark of the times, starting from 00:00, when the satellites will return to West 100.

Satellite A 00:00.....04:00.....08:00.....12:00.....16:00.....20:00.....24:00...etc

Satellite B 00:00.....08:00.....16:00.....24:00...etc

Another method: If you know the periods of the two satellites, look for the Lowest Common Multiple between them, which in this case is 8 hours.

**Problem 2** - Suppose the faster satellite had a period of 2 hours, and the slower satellite had a period of 17 hours. After how many hours would they come together again in longitude?

In hours:

Sat A: 0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, ..., 30, 32, 34, 36, 38, .....50, 52, 54, .....66, 68, 70, ....

Sat B: 0, 17, 34, 51, ..... 68, ....

Answer: Students can use the spreadsheet approach, or sketch out a timeline. The satellite periods, in hours, are 2 hours and 17 hours. The LCM, in this case, is 34 hours. So every 34 hours, the satellites will return to the same longitude they started at in synchrony.

Teachers; another, more algebraic method is as follows:

The angular speed of the two satellites is A:  $360 \text{ degrees}/2 \text{ hours} = 180 \text{ degrees/hour}$  and B:  $360 \text{ degrees}/17 \text{ hours} = 21.176 \text{ degrees/hour}$ . As time goes on, the speed with which the faster one pulls ahead of the slower one is given by  $V = 180 \text{ degrees/hr} - 21.176 \text{ degrees/hour} = 158.824 \text{ degrees/hour}$ . How much time does it take for the angular distance between the satellites to equal exactly 360 degrees or an exact integer multiple (360, 720, 1080, etc) ?

$$32\text{hr} \times 21.176 \text{ d/hr} = 677.63\text{d} ,$$

$$34\text{hr} \times 21.176 \text{ d/hr} = 719.98 \text{ d.}$$

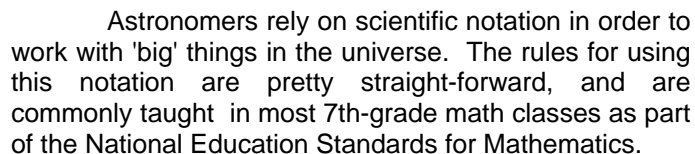
So, after 34 hours, the angular distance between the two satellites is 720 degrees, which is  $2 \times 360$ , so the satellites meet up at the same longitude they started from.

**Problem 3:** Suppose you had three satellites with periods of 2 hours, 4 hours and 6 hours. How long would it take for them to return to their original line-up at the same longitude?

Answer: The prime factors are 2,  $2 \times 2$  and  $2 \times 3$  so the LCM is  $2 \times 2 \times 3 = 12$ , so after 12 hours the satellites will come together again.

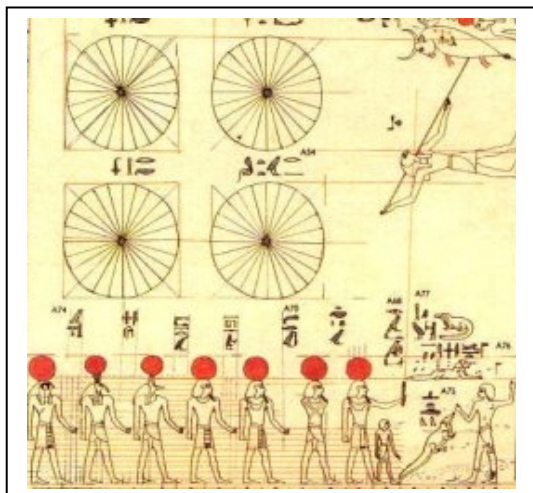
**Problem 4** - The THEMIS mission uses five satellites with periods of about 19 hours, 24 hours, 24 hours, 48 hours, and 96 hours, how many hours would it take for them all to get together again at the same longitude? Give your answer in days.

Answer: Find the prime factors 19: 19, 24:  $2 \times 2 \times 2 \times 3$ , 48:  $2 \times 2 \times 2 \times 2 \times 3$ , 96:  $2 \times 2 \times 2 \times 2 \times 2 \times 3$   
42: then  $19 \times 2 \times 2 \times 2 \times 2 \times 3 = 1824 \text{ hours}$ . Then to check:  $1824/19 = 96$ ,  $1824/24 = 76$ ,  $1824/48 = 38$ ,  $1824/96 = 19$ . So, it will take 1,824 hours or 76 days!



<http://spacemath.gsfc.nasa.gov>





Astronomers rely on scientific notation in order to work with 'big' things in the universe. The rules for using this notation are pretty straightforward, and are commonly taught in most 7th-grade math classes as part of the National Education Standards for Mathematics.

The following problems involve the addition and subtraction of numbers expressed in Scientific Notation. For example:

$$\begin{aligned}
 1.34 \times 10^8 + 4.5 \times 10^6 &= 134.0 \times 10^6 + 4.5 \times 10^6 \\
 &= (134.0 + 4.5) \times 10^6 \\
 &= 138.5 \times 10^6 \\
 &= 1.385 \times 10^8
 \end{aligned}$$

- 1)  $1.34 \times 10^{14} + 1.3 \times 10^{12} =$
- 2)  $9.7821 \times 10^{-17} + 3.14 \times 10^{-18} =$
- 3)  $4.29754 \times 10^3 + 1.34 \times 10^2 =$
- 4)  $7.523 \times 10^{25} - 6.32 \times 10^{22} + 1.34 \times 10^{24} =$
- 5)  $6.5 \times 10^{-67} - 3.1 \times 10^{-65} =$
- 6)  $3.872 \times 10^{11} - 2.874 \times 10^{13} =$
- 7)  $8.713 \times 10^{-15} + 8.713 \times 10^{-17} =$
- 8)  $1.245 \times 10^2 - 5.1 \times 10^{-1} =$
- 9)  $3.64567 \times 10^{137} - 4.305 \times 10^{135} + 1.856 \times 10^{136} =$
- 10)  $1.765 \times 10^4 - 3.492 \times 10^2 + 3.159 \times 10^{-1} =$

Answer Key:

$$1) \quad 1.34 \times 10^{14} + 1.3 \times 10^{12} = (134 + 1.3) \times 10^{12} = \mathbf{1.353 \times 10^{14}}$$

$$2) \quad 9.7821 \times 10^{-17} + 3.14 \times 10^{-18} = (97.821 + 3.14) \times 10^{-18} = \mathbf{1.00961 \times 10^{-16}}$$

$$3) \quad 4.29754 \times 10^3 + 1.34 \times 10^2 = (42.9754 + 1.34) \times 10^2 = \mathbf{4.43154 \times 10^3}$$

$$4) \quad 7.523 \times 10^{25} - 6.32 \times 10^{22} + 1.34 \times 10^{24} = (7523 - 6.32 + 134) \times 10^{22} = \mathbf{7.65068 \times 10^{25}}$$

$$5) \quad 6.5 \times 10^{-67} - 3.1 \times 10^{-65} = (6.5 - 310) \times 10^{-67} = \mathbf{-3.035 \times 10^{-65}}$$

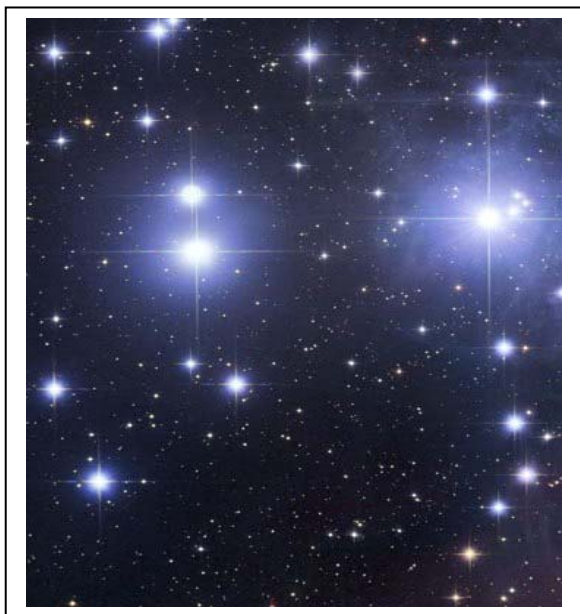
$$6) \quad 3.872 \times 10^{11} - 2.874 \times 10^{13} = (3.872 - 287.4) \times 10^{11} = \mathbf{2.83528 \times 10^{13}}$$

$$7) \quad 8.713 \times 10^{-15} + 8.713 \times 10^{-17} = (871.3 + 8.713) \times 10^{-17} = \mathbf{8.80013 \times 10^{-15}}$$

$$8) \quad 1.245 \times 10^2 - 5.1 \times 10^{-1} = (1245.0 - 5.1) \times 10^{-1} = \mathbf{1.2399 \times 10^2}$$

$$9) \quad 3.64567 \times 10^{137} - 4.305 \times 10^{135} + 1.856 \times 10^{136} = (364.567 - 4.305 + 18.56) \times 10^{135} = \mathbf{3.78822 \times 10^{137}}$$

$$10) \quad 1.765 \times 10^4 - 3.492 \times 10^2 + 3.159 \times 10^{-1} = (17650.0 - 3492 + 3.159) \times 10^{-1} = \mathbf{1.4161159 \times 10^4}$$



The following problems involve the multiplication and division of numbers expressed in Scientific Notation. Report all answers to two significant figures. For example:

$$1.34 \times 10^8 \times 4.5 \times 10^6 = (1.34 \times 4.5) \times 10^{(8+6)} \\ = 6.03 \times 10^{14}$$

To 2 significant figures this becomes...  $6.0 \times 10^{14}$

$$3.45 \times 10^{-5} / 2.1 \times 10^6 = (3.45/2.1) \times 10^{(-5 - (6))} \\ = 1.643 \times 10^{-11}$$

To 2 significant figures this becomes...  $1.6 \times 10^{-11}$

- 1) Number of nuclear particles in the sun:  $2.0 \times 10^{33}$  grams /  $1.7 \times 10^{-24}$  grams/particle
- 2) Number of stars in the visible universe:  $2.0 \times 10^{11}$  stars/galaxy x  $8.0 \times 10^{10}$  galaxies
- 3) Age of universe in seconds:  $1.4 \times 10^{10}$  years x  $3.156 \times 10^7$  seconds/year
- 4) Number of electron orbits in one year:  $(3.1 \times 10^7 \text{ seconds/year}) / (2.4 \times 10^{-24} \text{ seconds/orbit})$
- 5) Energy carried by visible light:  $(6.6 \times 10^{-27} \text{ ergs/cycle}) \times 5 \times 10^{14} \text{ cycles}$
- 6) Lengthening of Earth day in 1 billion years:  $(1.0 \times 10^9 \text{ years}) \times 1.5 \times 10^{-5} \text{ sec/year}$
- 7) Tons of TNT needed to make crater 100 km across:  $4.0 \times 10^{13} \times (1.0 \times 10^{15}) / (4.2 \times 10^{16})$
- 8) Average density of the Sun:  $1.9 \times 10^{33} \text{ grams} / 1.4 \times 10^{33} \text{ cm}^3$
- 9) Number of sun-like stars within 300 light years:  $(2.0 \times 10^{-3} \text{ stars}) \times 4.0 \times 10^6 \text{ cubic light-ys}$
- 10) Density of the Orion Nebula:  $(3.0 \times 10^2 \times 2.0 \times 10^{33} \text{ grams}) / (5.4 \times 10^{56} \text{ cm}^3)$



## Answer Key:

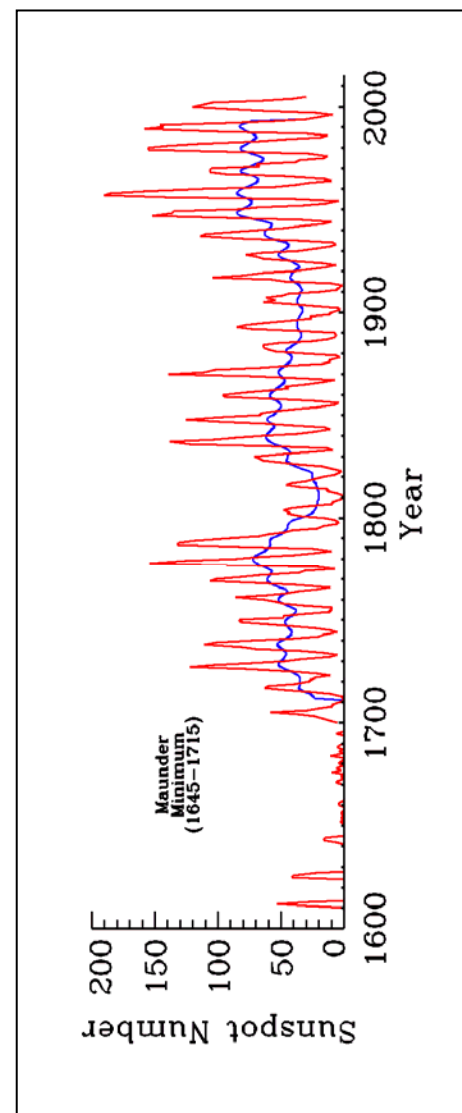
- 1) Number of nuclear particles in the sun:  $2.0 \times 10^{33}$  grams /  $1.7 \times 10^{-24}$  grams/particle  
 **$1.2 \times 10^{57}$  particles (protons and neutrons)**
- 2) Number of stars in the visible universe:  $2.0 \times 10^{11}$  stars/galaxy x  $8.0 \times 10^{10}$  galaxies  
 **$1.6 \times 10^{22}$  stars**
- 3) Age of universe in seconds:  $1.4 \times 10^{10}$  years x  $3.156 \times 10^7$  seconds/year  
 **$4.4 \times 10^{17}$  seconds**
- 4) Number of electron orbits in one year:  $(3.1 \times 10^7 \text{ seconds/year}) / (2.4 \times 10^{-24} \text{ seconds/orbit})$   
 **$1.3 \times 10^{31}$  orbits of the electron around the nucleus**
- 5) Energy carried by visible light:  $(6.6 \times 10^{-27} \text{ ergs/cycle}) \times 5 \times 10^{14} \text{ cycles}$   
 **$3.3 \times 10^{-12}$  ergs**
- 6) Lengthening of Earth day in 1 billion years:  $(1.0 \times 10^9 \text{ years}) \times 1.5 \times 10^{-5} \text{ sec/year}$   
 **$1.5 \times 10^4$  seconds or 4.2 hours longer**
- 7) Tons of TNT needed to make crater 100 km across:  $4.0 \times 10^{13} \times (1.0 \times 10^{15}) / (4.2 \times 10^{16})$   
 **$9.5 \times 10^{11}$  tons of TNT (equals 950,000 hydrogen bombs!)**
- 8) Average density of the Sun:  $1.9 \times 10^{33} \text{ grams} / 1.4 \times 10^{33} \text{ cm}^3$   
**1.4 grams/cm<sup>3</sup>**
- 9) Number of sun-like stars within 300 light years:  $(2.0 \times 10^{-3} \text{ stars}) \times 4.0 \times 10^6 \text{ cubic light-lys}$   
 **$8.0 \times 10^3$  stars like the sun.**
- 10) Density of the Orion Nebula:  $(3.0 \times 10^2 \times 2.0 \times 10^{33} \text{ grams}) / (5.4 \times 10^{56} \text{ cm}^3)$   
 **$1.1 \times 10^{-21}$  grams/cm<sup>3</sup>**

# Are you ready for Sunspot Cycle 24?

In 2007, the best estimates and observations suggested that we had just entered sunspot minimum, and that the next solar activity cycle might begin during the first few months of 2008. In May, 2007, solar physicist Dr. William Pesnell at the NASA, Goddard Spaceflight Center tabulated all of the current predictions for when the next sunspot cycle (2008-2019) will reach its peak. These predictions, reported by many other solar scientists are shown in the table below.

Table: Current Predictions for the Next Sunspot Maximum

Author	Prediction Year	Spots	Year	Method Used
Horstman	2005	185	2010.5	Last 5 cycles
Thompson	2006	180		Precursor
Tsirulnik	1997	180	2014	Global Max
Podladchikova	2006	174		Integral SSN
Dikpati	2006	167		Dynamo Model
Hathaway	2006	160		AA Index
Pesnell	2006	160	2010.6	Cycle 24 = Cycle 22
Maris & Onicia	2006	145	2009.9	Neural Network Forecast
Hathaway	2004	145	2010	Meridional Circulation
Gholipour	2005	145	2011.5	Spectral Analysis
Chopra & Davis	2006	140	2012.5	Disturbed Day Analysis
Kennewell	2006	130		H-alpha synoptic charts
Tritakis	2006	133	2009.5	Statistics of Rz
Tlatov	2006	130		H-alpha Charts
Nevanlinna	2007	124		AA at solar minimum
Kim	2004	122	2010.9	Cycle parameter study
Pesnell	2006	120	2010	Cycle 24 = Cycle 23
Tlatov	2006	115		Unipolar region size
Tlatov	2006	115		Large Scale magnetic field
Prochasta	2006	119		Average of Cycles 1 to 23
De Meyer	2003	110		Transfer function model
Euler & Smith	2006	122	2011.2	McNish-Lincoln Model
Hiremath	2007	110	2012	Autoregressive Model
Tlatov	2006	110		Magnetic Moments
Lantos	2006	108	2011	Even/Odd cycle pattern
Kane	1999	105	2010.5	Spectral Components
Pesnell	2006	101	2012.6	Linear Prediction
Wang	2002	101	2012.3	Solar Cycle Statistics
Roth	2006	89	2011.1	Moving averages
Duhau	2003	87		Sunspot Maxima and AA
Baranovski	2006	80	2012	Non-Linear Dynamo model
Schatten	2005	80	2012	Polar Field Precursor
Choudhuri	2007	80		Flux Transport Dynamo
Javariah	2007	74		Low-Lat. Spot Groups
Svalgaard	2005	70		Polar magnetic field
Kontor	2006	70	2012.9	Statistical extrapolation
Badalyan	2001	50	2010.5	Coronal Line
Cliverd	2006	38		Atmospheric Radiocarbon
Maris	2004	50		Flare energy during Cycle 23



**Problem 1:** What is the average year for the predicted sunspot maximum?

**Problem 2:** What is the average prediction for the total number of sunspots during the next sunspot maximum?

**Problem 3:** Which scientist has offered the most predictions? Do they show any trends?

**Problem 4:** What is the average prediction for the total sunspots during each prediction year from 2003 to 2006?

**Problem 5:** As we get closer to sunspot minimum in 2008, have the predictions for the peak sunspots become larger, smaller, or remain unchanged on average?

**Problem 6:** Which methods give the most different prediction for the peak sunspot number, compared to the average of the predictions made during 2006?

## Answer Key:

**Problem 1:** What is the average year for the predicted sunspot maximum?

**Answer:** There are 21 predictions, with an average year of 2011.3.  
This corresponds to about March, 2011.

**Problem 2:** What is the average prediction for the total number of sunspots during the next sunspot maximum?

**Answer:** The average of the 39 estimates in column 3 is 116 sunspots at sunspot maximum.

**Problem 3:** Which scientist has offered the most predictions? Do they show any trends?

**Answer:** Tlatov has offered 4 predictions, all made in the year 2006. The predicted numbers were 130, 115, 115 and 110. There does not seem to be a significant trend towards larger or smaller predictions by this scientist. The median value is 115 and the mode is also 115.

**Problem 4:** What is the average prediction for the total sunspots during each prediction year from 2003 to 2006?

**Answer:** Group the predictions according to the prediction year and then find the average for that year.

2003: 110, 87 average = 98

2004: 145, 122, 50 average= 106

2005: 185, 145, 80, 70 average= 120

2006: 180, 174, 167, 160, 160, 145, 140, 130, 133, 130, 120,  
115, 115, 119, 122, 110, 108, 101, 89, 80, 70, 38 average= 123

**Problem 5:** As we get closer to sunspot minimum in 2008, have the predictions for the peak sunspots become larger, smaller, or remain unchanged on average?

**Answer:** Based on the answer to problem 4, it appears that the predictions have tended to get larger, increasing from about 98 to 123 between 2003 and 2006.

**Problem 6:** Which methods give the most different prediction for the peak sunspot number, compared to the average of the predictions made during 2006?

**Answer:** Cliverd's Atmospheric Radiocarbon Method (38 spots), Badalyan's Coronal Line Method (50 spots), and Maris's Flare Energy during Cycle 23 (50 spots) seem to be the farthest from the average predictions that have been made by other forecasting methods.

Scientific research is not done in a vacuum. Usually there are lots of other behind-the-scenes issues that an astronomer has to deal with while doing the exciting tasks that go along with research and discovery. Here are a few 'real-world' problems that an astronomer might have to deal with, and that involve nothing more than simple addition, subtraction, multiplication and division..honest!!

**Problem 1:** Suppose an astronomer makes \$75,000 each year (before paying State and Federal taxes). He works 40-hours a week, and gets the same 2-week vacation and 10-holidays off as everyone else, for a total of 1840 hours of work each year. Suppose that if he works for the Federal Government, he is charged 1.6 times his salary to cover his benefits, the use of his office, lab, utilities, and support staff. Suppose that another astronomer works at Company A which charges 1.8 times his salary to cover the same expenses. Suppose a third astronomer works at a university that charges 1.4 times his salary.

- A) What is his take-home hourly pay rate in dollars/hour?
- B) To bring home \$75,000, how much money does he have to get from his grants to cover his salary plus the various benefits and other work expenses charged by his employer?
- C) If he only received \$75,000 each year for a research grant, how many hours each year would he be able to actually work and get paid?

**Problem 2:** A research satellite has a full compliment of experiments, which generate 40,000 bits of data every second (1 byte = 8 bits). The satellite orbits Earth once every 16 hours, but this data can be downloaded only once every orbit for 20 minutes at a time, when the tracking station is free to receive the satellite's transmissions. If the tracking stations can receive a downlink data stream with a bandwidth of 1.3 megabits/second,

- A) How large a 'hard drive' will the satellite need to temporarily store the data before the next data transmission time arrives?
- B) How much data will have been lost each orbit because it could not be downloaded?

**Problem 3:** NASA's total budget during 1969, the year of the Apollo 11 moon landing, was \$4 billion dollars. The Federal budget (Income Tax revenues) that year was \$187 billion. In 2006, the Federal budget included Income Tax revenues of \$1.1 Trillion, and NASA's budget was \$16.2 billion.

- A) What percentage of the 1969 Income Taxes were used for space exploration (Apollo 11 moon landings etc)?
- B) If there were 76 million tax payers in 1969, how much would each payer be paying for NASA's budget that year?
- C) What percentage of the 2006 Income Tax is used for space exploration?
- D) If there were 136 million tax payers that year, what was the average amount each payer paid for NASA's space exploration programs?
- E) Since 1969, the value of the dollar has 'inflated' so that 1 dollar in 1969 now equals about \$5.60. In view of this, what is the actual contribution to space exploration in 2006 compared to 1969 in problem 3(D)?

## Answer Key:

### **Problem 1:** A) What is his take-home hourly pay rate in dollars/hour?

Answer =  $\$75,000/1840 \text{ hrs} = \$40.76 \text{ an hour}$

B) To bring home \$75,000, how much money does he have to get from his grants to cover his salary plus the various benefits and other work expenses charged by his employer?

Answer: Government =  $\$75,000 \times 1.6 = \$120,000$

Company A =  $\$75,000 \times 1.8 = \$135,000$

University =  $\$75,000 \times 1.4 = \$105,000$

C) If he only received \$75,000 each year for a research grant, how many hours each year would he be able to actually work and get paid?

Answer: Government =  $\$75,000 / (1.6 \times \$40.76) = 1,150$  out of 1,840 hours in a full work year = 63%

Company A =  $\$75,000 / (1.8 \times \$40.76) = 1,022$  hours or 56% of a full work year.

University =  $\$75,000 / (1.4 \times \$40.76) = 1,314$  hours or 71 % of a full work year.

### **Problem 2:** A) How large a 'hard drive' will the satellite need to temporarily store the data before the next data transmission time arrives?

Answer: The tracking station can download a maximum of 20 minutes  $\times 60 \text{ sec/min} \times 1.3 \text{ million bits/sec} = 1.6 \text{ billion bits of data}$  or  $(1.6 \text{ billion}/8) = 200 \text{ megabytes of data}$  at the downlink time. The hard drive or Mass Storage Unit (MSU) has to be at least this big.

B) How much data will have been lost each orbit because it could not be downloaded?

Answer: During each 16-hour orbit, the satellite generates  $16 \text{ hours} \times (3600 \text{ seconds/hour}) \times (40,000 \text{ bits/sec}) = 2.3 \text{ billion bits of information}$ , or  $(2.3 \text{ billion bits}/8) = 287 \text{ megaBytes of data}$ . But the tracking station can only handle 200 megabytes of data, so 87 megabytes of data have to be lost each orbit to make room for the next-orbit's data.

### **Problem 3:** A) What percentage of the 1969 Income Taxes were used for space exploration (Apollo 11 moon landings etc)?

Answer:  $100\% \times (4 \text{ billion} / 187 \text{ billion}) = 2.1\%$

B) If there were 76 million tax payers in 1969, how much would each payer be paying for NASA's budget that year?

Answer:  $\$187 \text{ billion} / 76 \text{ million} = \$2,460 \text{ per tax payer for the full federal income tax.}$

$\$2,460 \times 2.1/100 = \$52 \text{ dollars per tax payer.}$

C) What percentage of the 2006 Income Tax is used for space exploration?

Answer:  $100\% \times (16.2 \text{ billion} / 1.1 \text{ trillion}) = 1.4 \%$

D) If there were 136 million tax payers that year, what was the average amount each payer paid for NASA's space exploration programs?

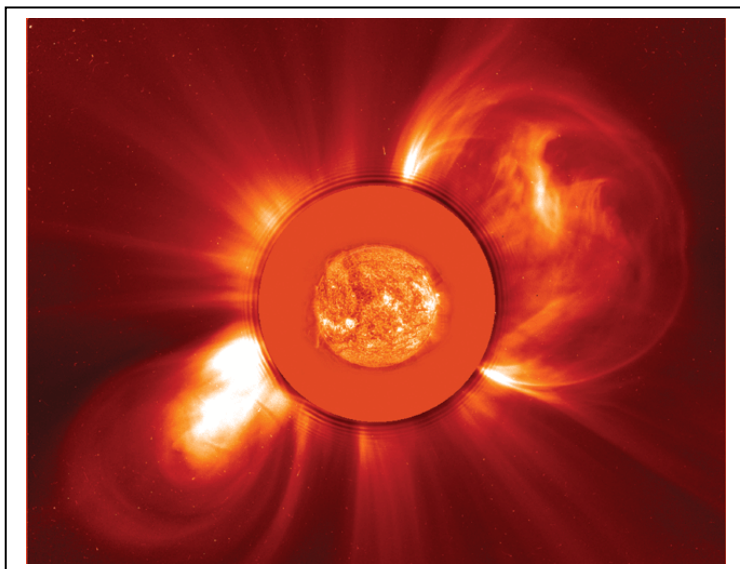
Answer:  $\$1.1 \text{ trillion} / 136 \text{ million} = \$8,088 \text{ per tax payer.}$

$\$8,088 \times 1.4/100 = \$113 \text{ per tax payer.}$

E) Since 1969, the value of the dollar has 'inflated' so that 1 dollar in 1969 now equals about \$5.60. In view of this, what is the actual contribution to space exploration in 2006 compared to 1969 in problem 3(D)?

Answer: Today NASA spends  $\$16.2/5.6 = \$2.9 \text{ billion dollars}$ , and the average tax payer spends  $\$113/5.6 = \$20 \text{ dollars for space exploration by NASA.}$

# CME Kinetic Energy and Mass



Kinetic energy is the energy that a body has by virtue of its mass and speed. Mathematically, it is expressed as one-half of the product of the mass of the object (in kilograms), times the square of the objects speed (in meters/sec).

$$\text{K.E.} = 0.5 m V^2$$

Between October 1996 and May 2006, the SOHO satellite detected and catalogued 11,031 coronal mass ejections like the one seen in the figure to the left. There was enough data available to determine the properties for 2,131 events. The table below gives values for ten of these CMEs.

Date	Speed (km/s)	K.E. (Joules)	Mass (kilograms)
4/8/1996		$1.1 \times 10^{20}$	$2.2 \times 10^9$
8/22/2000	388	$1.3 \times 10^{22}$	
6/10/2001	731	$8.2 \times 10^{23}$	
1/18/2002	64		$2.6 \times 10^{10}$
5/16/2002	1,310		$7.8 \times 10^{10}$
10/7/2002		$7.8 \times 10^{21}$	$3.0 \times 10^{10}$
1/24/2003	387	$9.1 \times 10^{18}$	
10/31/2003	2,198	$1.6 \times 10^{24}$	
11/2/2003		$9.3 \times 10^{25}$	$4.5 \times 10^{13}$
11/10/2004	3,387		$9.6 \times 10^{12}$

Problem 1: Complete the table by determining the value of the missing entries using the formula for Kinetic Energy.

Problem 2: What is the minimum and maximum range for the observed kinetic energies for the 10 CMEs? The largest hydrogen bomb ever tested was the Tsar Bomba in 1961 and was equivalent to 50 megatons of TNT. It had a yield of  $5 \times 10^{23}$  Joules. What is the equivalent yield for the largest CME in megatons, and 'Tsar Bombas'?

Problem 3: What are the equivalent masses of the smallest and largest CMEs in metric tons?

Problem 4: Compare the mass of the largest CME to the mass of a small mountain. Assume that the mountain can be represented as a cone with a volume given by  $\frac{1}{3} \pi R^2 H$  where R is the base radius and H is the height in meters, and assume the density of rock is 3 grams/cm<sup>3</sup>.



Date	Speed (km/s)	K.E. (Joules)	Mass (kilograms)
4/8/1996	<b>316</b>	$1.1 \times 10^{20}$	$2.2 \times 10^9$
8/22/2000	388	$1.3 \times 10^{22}$	<b><math>1.7 \times 10^{11}</math></b>
6/10/2001	731	$8.2 \times 10^{23}$	<b><math>3.1 \times 10^{12}</math></b>
1/18/2002	64	<b><math>5.3 \times 10^{19}</math></b>	$2.6 \times 10^{10}$
5/16/2002	1,310	<b><math>6.7 \times 10^{22}</math></b>	$7.8 \times 10^{10}$
10/7/2002	<b>721</b>	$7.8 \times 10^{21}$	$3.0 \times 10^{10}$
1/24/2003	387	$9.1 \times 10^{18}$	<b><math>1.2 \times 10^8</math></b>
10/31/2003	2,198	$1.6 \times 10^{24}$	<b><math>6.6 \times 10^{11}</math></b>
11/2/2003	<b>2,033</b>	$9.3 \times 10^{25}$	$4.5 \times 10^{13}$
11/10/2004	3,387	<b><math>5.5 \times 10^{25}</math></b>	$9.6 \times 10^{12}$

Problem 1: Complete the table by determining the value of the missing entries using the formula for Kinetic Energy.

**Answer: See above answers in red.**

Problem 2: What is the minimum and maximum range for the observed kinetic energies for the 10 CMEs?

**Answer: Maximum =  $9.3 \times 10^{25}$  Joules. Minimum =  $5.3 \times 10^{19}$  Joules**

The largest hydrogen bomb ever tested was the *Tsar Bomba* in 1961 and was equivalent to 50 megatons of TNT. It had a yield of  $5 \times 10^{23}$  Joules. What is the equivalent yield for the largest CME in megatons, and 'Tsar Bombas'?

**Answer: The CME on November 2, 2003 was equal to**

**$(9.3 \times 10^{25} / 5 \times 10^{23}) = 186$  Tsar Bombas,**

**and an equivalent TNT yield of  $186 \times 50$  megatons = 9,300 megatons!**

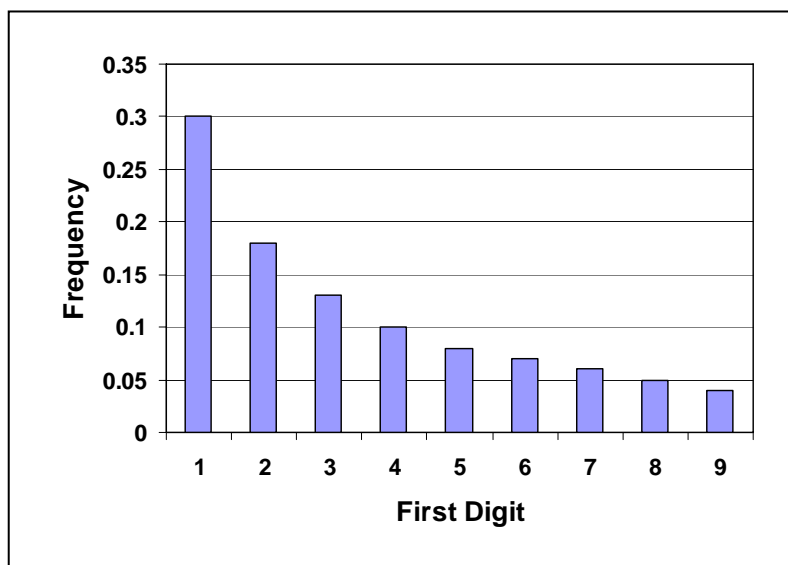
Problem 3: What are the equivalent masses of the smallest and largest CMEs in metric tons?

**Answer: One metric ton is 1,000 kilograms. The smallest mass was for the January 24, 2003 CME with about 120,000 tons. The largest mass was for the November 2, 2003 'Halloween Storm' with about 45 billion metric tons.**

Problem 4: Compare the mass of the largest CME to the mass of a small mountain. Assume that the mountain can be represented as a cone with a volume given by  $V = 1/3 \pi R^2 H$  where R is the base radius and H is the height in meters, and assume the density of rock is  $3 \text{ grams/cm}^3$ .

**Answer: One possibility is for a mountain with a base radius of  $R = 1$  kilometers, and a height of 50 meters. The cone volume is  $0.33 \times 3.14 \times (1000)^2 \times 50 = 5.2 \times 10^7$  cubic meters. The rock density of  $3 \text{ gm/cm}^3$  converted into kg per cubic meters is  $0.003 \text{ kg}/(.01)^3 = 3000 \text{ kg/m}^3$ . This yields a mountain with a mass of 156 billion tons, which is close to the largest CME mass in the table. So CMEs, though impressive in size, carry no more mass than a small hill on Earth!**

# Benford's Law



One remarkable feature of numbers used in the real world (measured in physical units like dollars, meters, grams etc) is that the first leading digit of a collection of these numbers is not random, although the numbers, taken together, may seem so. For example, in the sequence \$123.43, \$327.30, \$457.67, \$1327.03 the first digits are 1, 3, 4 and 1. We could plot this in a histogram so that '1' has a frequency of 2, '3' has a frequency of 1 and '4' has a frequency of 1. When tallied for an entire tax return with several hundred numbers, the frequency table follows Benford's Law.

The Internal Revenue Service uses Benford's Law to identify tax cheats! For example, if we take all the numbers (in dollar units) you enter in a tax form and count up the frequency of 1's, 2's, 3's ... 9's, we will get a histogram that shows that 1's appear 30% of the time, 2s appear 17% of the time, 3s appear 12% of the time, and so on.

American astronomer Simon Newcomb noticed that in logarithm books (used at that time to perform calculations), the earlier pages (which contained numbers that started with 1) were much more worn than the other pages.

What is interesting about this law is that, for example, if you made one measurement in inches and computed the first-digit frequency, you would get the same first-digit histogram if you then changed the units to meters or any other unit of measurement!

## Does the universe obey this law too?

Below are some archives of data online. Tally the first digit frequencies and test if Benford's Law holds.

Problem 1: **Sunspot Numbers** (Pure numbers: no units) from the National Geophysical Data Center:  
<http://www.ngdc.noaa.gov/stp/SOLAR/ftpsunspotnumber.html#american>

Problem 2: **Solar Wind Magnetism**: NASA ACE satellite measurements of the solar wind.  
<http://www.swpc.noaa.gov/ftpmenu/lists/ace2.html>

Open a file with 'mag' as part of the file name such as [200712\\_ace\\_mag\\_1h.txt](#)  
 Look at Column 11 labeled 'Bt' which are the measurements of the total solar wind magnetic field strength in units of nanoTeslas.

Problem 3: **Depth of latest earthquakes** in kilometers from the USGS:  
[http://earthquake.usgs.gov/eqcenter/recenteqsww/Quakes/quakes\\_all.html](http://earthquake.usgs.gov/eqcenter/recenteqsww/Quakes/quakes_all.html)

Inquiry Questions: Can you find any patterns in the frequency histograms for the above problems? Why don't the digits appear with about the same frequency? Can you think of other astronomical data that you can use to further test your hunches, or refine any patterns you may have found?

## Answer Key:

Problem 1: **Sunspot Numbers** (Pure numbers: no units) from the National Geophysical Data Center:

<http://www.ngdc.noaa.gov/stp/SOLAR/ftpsunspotnumber.html#american>

### **Answer:**

Here are some sample numbers for the month of April 1998

53, 55, 48, 47, 57, 61, 91, 97, 103, 102, 93, 75, 61, 60, 63, 56, 34, 26, 30, 31, 31, 24, 17, 19, 14, 13, 13, 27, 30, 39

The leading digits are 5,5,4,4,5,6,9,9,1,1,9,7,6,6,6,5,3,2,3,3,3,2,1,1,1,1,1,2,3,3

Ordered from 1 to 9 by frequency: 7,3,6,2,4,4,1,0,3

Note: There are significantly more 1s.

Problem 2: **Solar Wind Magnetism**: NASA ACE satellite measurements of the solar wind.

<http://www.swpc.noaa.gov/ftpmenu/lists/ace2.html>

Open a file with 'mag' as part of the file name such as [200712\\_ace\\_mag\\_1h.txt](#)

Look at Column 11 labeled 'Bt' which are the measurements of the total solar wind magnetic field strength in units of nanoTeslas.

### **Answer:**

Sample numbers from the above file are: 4.7, 5.4, 4.7, 1.6, 1.6, 2.2, 2.7, 1.9, 2.5, 3.9, 3.7, 3.6, 3.7, 3.0, 3.1, 2.4, 2.7, 2.6, 1.2, 3.1, 4.2, 3.6, 3.6, 3.6, 1.1, 2.8, 2.1, 3.4, 3.4, 2.4, 2.2, 1.7, 0.4, 2.7, 2.8

The leading digits are: 4,5,4,1,1,2,2,1,2,3,3,3,3,3,2,2,2,1,3,1,2,3,3,3,1,2,2,3,3,2,2,1,4,2,2

Ordered from 1 to 9 by frequency: 7,13,12,3,1,0,0,0,0

Note: this data is bounded by a maximum value near 6, so it does not show Benford's Law.

Problem 3: **Depth of latest earthquakes** in kilometers from the USGS:

[http://earthquake.usgs.gov/eqcenter/recenteqsww/Quakes/quakes\\_all.html](http://earthquake.usgs.gov/eqcenter/recenteqsww/Quakes/quakes_all.html)

### **Answer:**

Sample depths: 10.0, 36.5, 12.5, 64.6, 3.9, 10.2, 10.0, 10.0, 98.4, 3.5, 500.7, 92.7, 81.6, 22.4, 2.0, 46.0, 53.3, 100.0, 10.0, 49.1, 57.0

The leading digits are: 1,3,1,6,3,1,1,1,9,3,5,9,8,2,2,4,5,1,1,4,5

Ordered from 1 to 9 by frequency: 7,2,3,2,3,1,0,1,2

Note: this data does show Benford's Law because it is not apparently bounded.

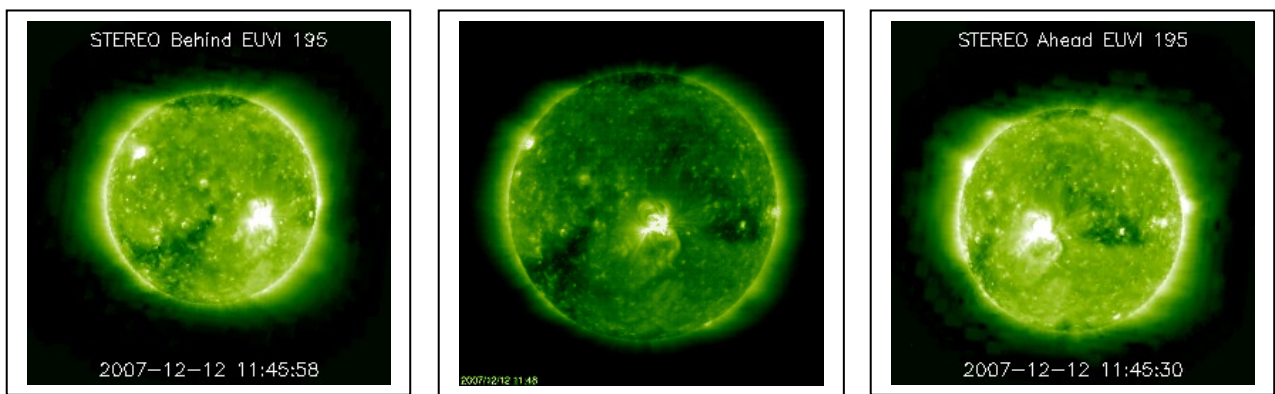
**Inquiry Questions:** Students should discover that numbers that are limited in some way, such as the height of your students (all about the same height) will not show Benford's law. Only numbers that are not in some way bounded above or below in value (such as solar wind magnetism, which has a rather narrow range of values near 5.0 nTeslas). Also, street address and other pure numbers do not follow Benford's Law, only numbers related to unbounded, measured parameters with physical units (temperature, density, mass, energy, length, time, etc) will work.

# An Application of the Parallax Effect

16

Two NASA, STEREO satellites take images of the sun and its surroundings from two separate vantage points along Earth's orbit. From these two locations, one located ahead of the Earth, and the other located behind the Earth along its orbit, they can create stereo images of the 3-dimensional locations of coronal mass ejections (CMEs) and storms on or near the solar surface.

The three images below, taken on December 12, 2007, combine the data from the two STEREO satellites (left and right) taken from these two locations, with the single image taken by the SOHO satellite located half-way between the two STEREO satellites (middle). Notice that there is a large storm event, called Active Region 978, located on the sun. The changing location of AR978 with respect to the SOHO image shows the perspective change seen from the STEREO satellites. You can experience the same *Parallax Effect* by holding your thumb at arms length, and looking at it, first with the left eye, then with the right eye. The location of your thumb will shift in relation to background objects in the room.

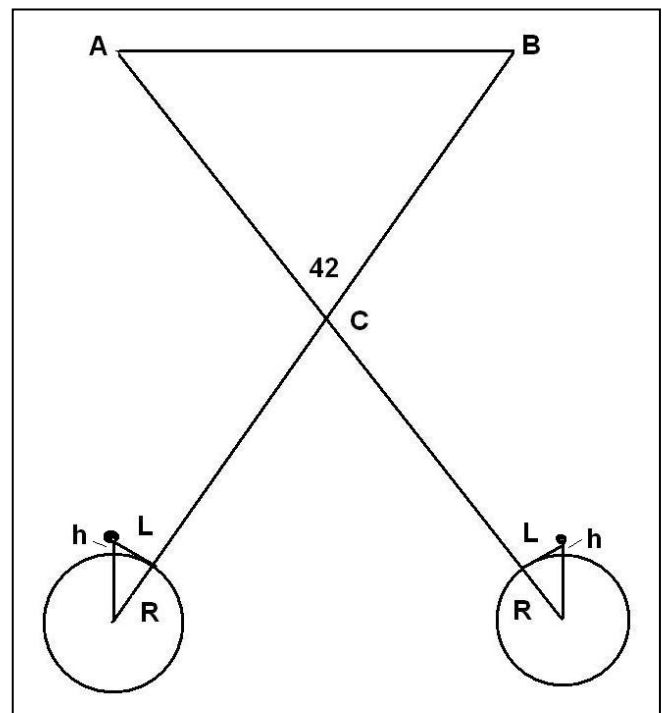


The diagram to the right shows the relevant parallax geometry for the two satellites A and B, separated by an angle of 42 degrees as seen from the sun. The diagram lengths are not drawn to scale. The radius of the sun is 696,000 km.

**Problem 1:** With a millimeter ruler, determine the scale of each image in km/mm. How many kilometers did AR978 shift from the center position (SOHO location for AR) between the two STEREO images? This is the average measure of 'L' in the diagram.

**Problem 2:** Using the Pythagorean Theorem, determine the equation for the height,  $h$ , in terms of  $R$  and  $L$ . Assume the relevant triangle is a right triangle.

**Problem 3:** How high ( $h$ ) above the sun's surface, called the photosphere, was the AR978 viewed by STEREO and SOHO on December 12, 2007?



## Answer Key

**Problem 1:** With a millimeter ruler, determine the scale of each image in km/mm. How many kilometers did AR978 shift from the center position (SOHO location for AR) between the two STEREO images? This is the measure of 'L' in the diagram.

**Answer:**

STEREO-Left image, sun diameter = 28 mm, actual = 1,392,000 km, so the scale is

$$1392000 \text{ km} / 28\text{mm} = \mathbf{49,700 \text{ km/mm}}$$

SOHO-center sun diameter = 36 mm, so the scale is

$$1392000 \text{ km} / 36\text{mm} = \mathbf{38,700 \text{ km/mm}}$$

STEREO-right sun diameter = 29 mm, so the scale is

$$1392000 \text{ km} / 29 \text{ mm} = \mathbf{48,000 \text{ km/mm}}$$

Taking the location of the SOHO image for AR978 as the reference, the left-hand image shows that AR978 is about 5 mm to the right of the SOHO location which equals 5 mm x 49,700 km/mm = 248,000 km. From the right-hand STEREO image, we see that AR978 is about 5 mm to the left of the SOHO position or 5 mm x 48,000 km/mm = 240,000 km.

**The average is 244,000 kilometers.**

**Problem 2:** Using the Pythagorean Theorem, determine the equation for the height, h, in terms of R and L.

**Answer:** 
$$(R + h)^2 = R^2 + L^2$$

$$h = (R^2 + L^2)^{1/2} - R$$

**Problem 3:** How high (h) above the sun's surface, called the photosphere, was the AR978 viewed by STEREO and SOHO on December 12, 2007?

**Answer:** 
$$h = ((244,000)^2 + (696,000)^2)^{1/2} - 696,000$$

$$h = 737,500 - 696,000$$

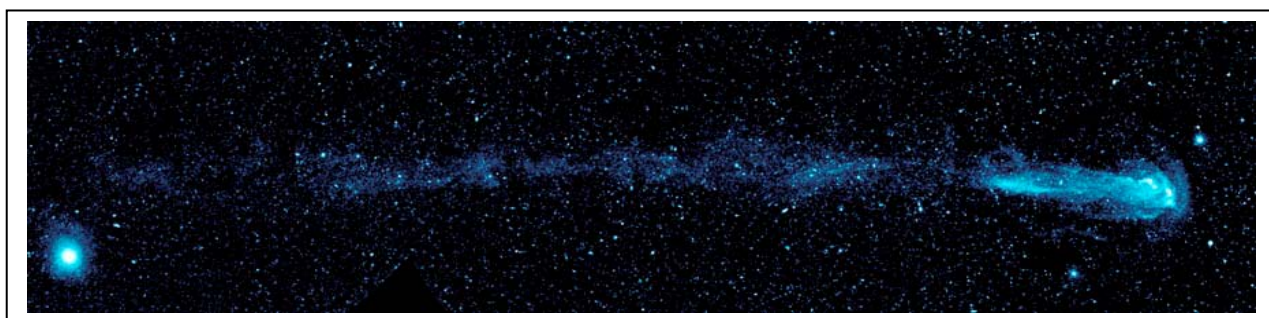
$$h = 41,500 \text{ kilometers}$$

## A Star Sheds a Comet Tail!

For 400 years, astronomers have carefully watched the star Mira in the constellation Cetus the Whale, made famous for its variable brightness. This red giant star is very old, and it is in its last few million years of life before evolving onto a planetary nebula and white dwarf 'combo'. Our own sun will reach a similar stage in about 7 billion years. While astronomers thought they knew quite a lot about this star after decades of satellite and ground-based telescopic study, the NASA, Galaxy Evolution Explorer (GALEX) satellite uncovered a surprising new twist to this famous star. Using its sensitive ultraviolet telescope while routinely mapping the light from stars across the entire sky, GALEX detected faint light from an enormous tail of dust stretching over 13 light years from Mira.

Red giant and red supergiant stars are known to be factories for producing dust grains in their cooling outer atmospheres where temperatures can reach a chilling 1,500 K - enough for molecules like silicon dioxide to condense out like raindrops, solidifying into small dust grains. These are then driven away from the star by radiation pressure. Some stars form spherical shells that can be so dusty that the star literally fades from optical view behind a dense obscuring wall of dust. But Mira does something different. It seems to be ejecting large quantities of oxygen, carbon and nitrogen!

Mira is known to be a fast-moving star traveling at a speed of 130 km/sec (or about 300,000 miles per hour). As its atmosphere sheds these gases in vast clouds, these clouds are left behind the star as it plows through the gases in the Interstellar Medium. Mira sheds about 10 Earth masses worth of dust and gas every ten years. The distance to Mira is 350 light years, and at this distance, the picture below spans an area 15 light years long and 4 light years wide.



Problem 1 - From the speed of Mira, how long did it take for it to travel 13 light years?

Problem 2 - What is the scale of the image in units of light years per centimeter?

Problem 3 - From your answer to Problem 1, what is the scale of the gas tail in thousands of years per centimeter, rounded to the nearest thousands?

Problem 4 - Did Mira emit gas in a steady rate during the period of time estimated in Problem 1?

Problem 5 - Can you create a chronology for Mira that tells about the sequence of events, in time, of its history of ejecting gas during the time that the dust tail was produced?



Problem 1 - From the speed of Mira, how long did it take for it to travel 13 light years?

**Answer:** First, a light year is the distance light travels in one year at a speed of 300,000 km/sec. There are 31 million seconds in a year, so  $(3 \times 10^5 \text{ k/s}) \times (3.1 \times 10^7 \text{ sec/yr}) = 9.3 \times 10^{12}$  kilometers. The speed of Mira is 350 kilometers/s, so the time required is just

$$\begin{aligned} T &= (13 \text{ light years} \times 9.3 \times 10^{12} \text{ km/LY}) / 130 \text{ km/sec} \\ &= 9.3 \times 10^{11} \text{ seconds or} \\ &= 30,000 \text{ years.} \end{aligned}$$

Problem 2 - What is the scale of the image in units of light years per centimeter?

**Answer:** 14.5 centimeters = 13 Light years so  $13 \text{ LY} / 14.5 \text{ cm} = 0.9 \text{ light years/cm}$

Problem 3 - From your answer to Problem 1, what is the scale of the gas tail in thousands of years per centimeter, rounded to the nearest thousands?

**Answer:** 30,000 years / 14.5 centimeters = **2000 years/cm**

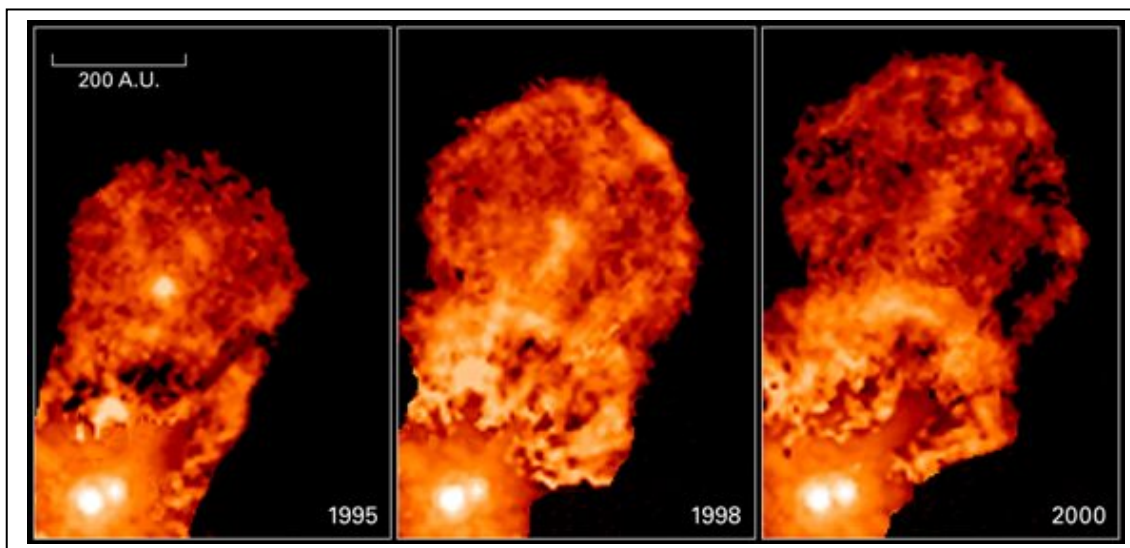
Problem 4 - Did Mira emit gas in a steady rate during the period of time estimated in Problem 1?

**Answer: No, because the gas ejected by the star during the last 30,000 years is clumpy with many gaps. This means there were periods when the star was ejecting gas, and periods where this process seems to have temporarily stopped.**

Problem 5 - Can you create a chronology for Mira that tells about the sequence of events in its history of ejecting gas during the time that the tail was produced?

**Answer: The current period of gas loss seems to extend back to about 3000 years ago. There is then a gap of about 1,000 years when there seemed to be little gas production. Then about 4,000 years ago and extending back to about 23,000 years ago, the gas production was occurring but at a reduced level from recent times (more intense during the last 3,000 years). There is then another gap between about 23,000 years ago and 27,000 years ago with reduced gas production, followed by an earlier brief phase of gas production extending from about 27,000 to 30,000 years ago.**

**Note: Some of the fading of the tail may be due to the interaction of the gas with the interstellar medium which may be trying to dissipate the tail. Some of this interaction may also be partly responsible for the detailed clumpiness in the tail, so it is probably not a good idea to analyze the tail structure finer than centimeter differences.**



These pictures were taken by the Hubble Space Telescope between 1995 and 2000. They show a time sequence that captures the explosion of matter from the star XZ Tauri in the constellation Taurus. The star is located 450 light years from the sun. This star is less than one million years old, and is probably similar to what our own sun was like at the same age.

Today, our sun still ejects gas in events called Coronal Mass Ejections (CMEs) but involve far less matter ejected into space.

**Problem 1** - Using a millimeter ruler, and the fact that 1 A.U. = 147 million kilometers what is the scale of these images in kilometers per millimeter?

**Problem 2** - A) How many millimeters did the XZ Tauri cloud travel between 1995 and 2000? B) How many kilometers did it travel?

**Problem 3** - What was the average speed of the cloud between 1995 - 2000 in; A) kilometers per day; B) kilometers per hour? C) kilometers per second?

## Extra for Experts:

**Problem 4** - The estimated total mass of this 'round' cloud was  $1.1 \times 10^{25}$  grams. Assuming it filled a spherical volume with a volume  $V = \frac{4}{3} \pi R^3$  A) What was the volume of this cloud in 1998 in cubic centimeters? B) What was the density of this cloud in grams per cubic centimeter? C) If one hydrogen atom has a mass of  $1.6 \times 10^{-24}$  grams, how many hydrogen atoms per cubic centimeter were present in the star's interplanetary space?

## Answer Key:

Problem 1 - Using a millimeter ruler, and the fact that 1 A.U. = 147 million kilometers what is the scale of these images in kilometers per millimeter?

Answer - The 200 AU image scale is 17.5 millimeters long.  $200 \text{ AU} = 200 \times 147$  million kilometers, so the scale is  $200 \times 147$  million km/17.5 mm = 1,680 million km/mm

Problem 2 - A) How many millimeters did the XZ Tauri cloud travel between 1995 and 2000? B) How many kilometers did it travel?

Answer A) Depending on how you measure, the cloud traveled about 12 millimeters. B)  $12 \text{ millimeters} \times 1,680 \text{ million km/mm} = 20,160 \text{ million kilometers}$ .

Problem 3 - What was the average speed of the cloud between 1995 - 2000 in; A) kilometers per day; B) kilometers per hour? C) kilometers per second?

Answer: A)  $2000-1995=$ Five years =  $365 \times 5 = 1825$  days, so it traveled  $20,160$  million km/ $1825$  days =  $11.0$  million km/day. B)  $11.0$  million km /  $24$  hours =  $458,000$  km/hour. C)  $458,000 \text{ km}/3600 \text{ seconds} = 127 \text{ kilometers/second}$ .

## Extra for Experts:

Problem 4 - The estimated total mass of this 'round' cloud was  $1.1 \times 10^{25}$  grams. Assuming it filled a spherical volume

A) What was the volume of this cloud in 1998 in cubic centimeters?

Answer - The diameter of the cloud in the middle image is 38 mm or  $38 \times 1,680$  million km =  $63,840$  million km. The radius is  $31,920$  million km. This equals a radius of  $31,920 \times 10^6 \text{ km} \times 1.0 \times 10^5 \text{ cm/km} = 3.2 \times 10^{15} \text{ cm}$ . The volume of a sphere is  $V = 4/3 \pi R^3$ , so the volume of the cloud is  $4/3 \pi (3.2 \times 10^{15} \text{ cm})^3 = 1.4 \times 10^{47}$  cubic centimeters.

B) What was the density of this cloud in grams per cubic centimeter?

Answer: Density = Mass / volume =  $1.1 \times 10^{25} \text{ grams} / 1.4 \times 10^{47} \text{ cubic centimeters} = 7.8 \times 10^{-23} \text{ grams/cc}$

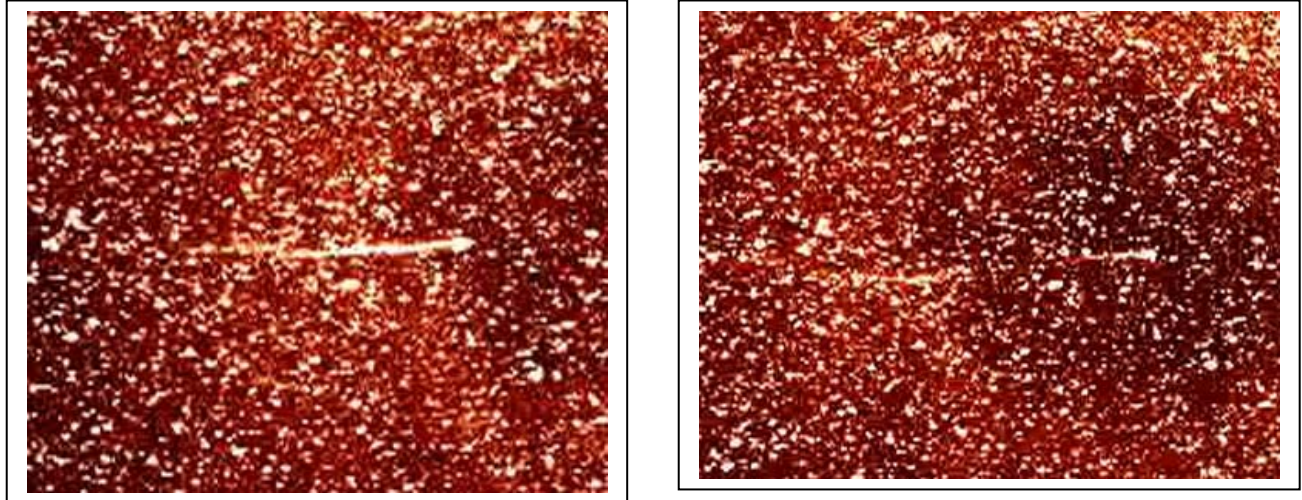
C) If one hydrogen atom has a mass of  $1.6 \times 10^{-24}$  grams, how many hydrogen atoms per cubic centimeter were present in the star's interplanetary space?

Answer =  $7.8 \times 10^{-23} \text{ grams/cc} / (1.6 \times 10^{-24} \text{ grams}) = 50 \text{ hydrogen atoms/cc}$ .

# The Comet Encke Tail Disruption Event

19

On April 20, 2007, NASA's STEREO satellite witnessed a rare solar system event. The Comet Encke had just passed inside the orbit of Venus and was at a distance of 114 million kilometers from STEREO-A, when a Coronal Mass Ejection occurred on the sun. The cloud of magnetized gas passed over the comet's tail at 18:50 UT, and moments later caused the tail of the comet to break into two. The two images below show two images from the tail breakup sequence. The left image was taken at 18:10 UT and the right image was taken at 20:50 UT. Each image subtends an angular size of 6.4 degrees x 5.3 degrees. For comparison, the Full Moon would correspond to a circle with a diameter of 0.5 degrees.



**Problem 1** - What is the scale of the images in arcminutes per millimeter? (1 degree=60 arcminutes)

**Problem 2** - How many seconds elapsed between the time the two images were taken by the STEREO-A satellite?

**Problem 3** - The left image shows the comet with an intact tail. The right image shows the tail separated from the head of the comet (the right-most bright feature along the comet's horizontal axis which we will call Point A), and flowing to the left. Meanwhile, you can see that the comet has already begun to reform a new tail. Carefully examine the right-hand image and identify the right-most end of the ejected tail (Call it Point B). Note that star images do not move, and are more nearly point-like than the tail gases. How far, in millimeters, is Point B from Point A?

**Problem 4** - From the image scale, convert your answer to Problem 3 into arcminutes.

**Problem 5** - The distance of the comet was 114 million kilometers, and at that distance, one arcminute of angular separation corresponds to 33,000 kilometers. How far did the tail fragment travel between the times of the two images?

**Problem 6** - What was the speed of the tail fragment?

**Problem 7** - If the comet's speed was about 40 km/sec and the CME speed was at least several hundred times faster, based on your answer to Problem 6, was the comet fragment 'left behind' or did the CME carry it off?

## Answer Key:

**Problem 1** - What is the scale of the images in arcminutes per millimeter? (1 degree=60 arcminutes)

**Answer:** horizontally, the image span 6.4 degrees x 60 minutes/degree = 384 arcminutes. The length is 77 millimeters, so the scale is  $384/77 = 5.0$  **arcminutes/mm**

**Problem 2** - How many seconds elapsed between the time the two images were taken by the STEREO-A satellite?

**Answer:** 20:50 - 18:10 = 2 hours and 40 minutes = 160 minutes or **9600 seconds**.

**Problem 3** - The left image shows the comet with an intact tail. The right image shows the tail separated from the head of the comet (the right-most bright feature along the comets horizontal axis which we will call Point A), and flowing to the left. Meanwhile, you can see that the comet has already begun to reform a new tail. Carefully examine the right-hand image and identify the right-most end of the ejected tail (Call it Point B). Note that star images do not move, and are more nearly point-like than the tail gases. How far, in millimeters, is Point B from Point A?

**Answer:** An answer near **17 millimeters** is acceptable, but students may measure from 15 to 20 millimeters as reasonable answers.

**Problem 4** - From the image scale, convert your answer to Problem 3 into arcminutes.

**Answer:** 17 millimeters x 5 arcminutes/mm = **85 arcminutes**.

**Problem 5** - The distance of the comet was 114 million kilometers, and at that distance, one arcminute of angular separation corresponds to 33,000 kilometers. How far did the tail fragment travel between the times of the two images?

**Answer:** 85 arcminutes x 33,000 kilometers/arcminute = **2.8 million kilometers**.

**Problem 6** - What was the speed of the tail fragment?

**Answer:** 2.8 million kilometers/9600 seconds = **292 kilometers/second**.

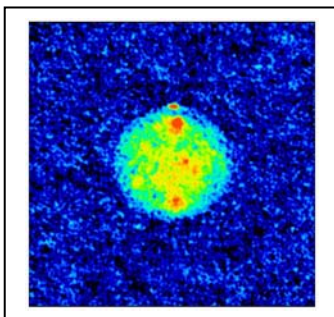
**Problem 7** - If the comet's speed was about 40 km/sec and the CME speed was at least several hundred times faster, based on your answer to Problem 6, was the comet fragment 'left behind' or did the CME carry it off?

**Answer:** **The speed in Problem 6 is much closer to the CME speed than the comet speed, so the fragment was carried off by the CME and not ejected by the comet.**

*This collision was studied in detail by Dr. Angelos Vourlidas and his colleagues at the Naval Research laboratory in Washington, D.C and the Rutherford Laboratory in England. They deduced from a more careful analysis that the CME speed was about 500 km/sec and the solar wind speed was about 420 km/sec. The tail fragment was carried off by the CME. Details can be found in The Astrophysical Journal (Letters), vol. 668, pp L79-L82 which was published on October 10, 2007. A movie of the encounter may be seen at the STEREO web site ( <http://stereo.gsfc.nasa.gov>) in their movie gallery.*



# Is there Ice on Mercury ?



The NASA MESSENGER spacecraft performed its first flyby of Mercury on January 14, 2008. In addition to mapping the entire surface of this planet, one of its goals is to shed new light on the existence of ice under the polar regions of this hot planet. Ice on Mercury? It's not as strange as it seems!

In 1991, Duane Muhleman and her colleagues from Caltech and the Jet Propulsion Laboratory, created the first radar map of Mercury. The image, shown here, contained a stunning surprise. The bright (red) dot at the top of the moon image to the left indicates strong radar reflection at Mercury's North Pole, resembling the strong radar echo seen from the ice-rich polar caps of Mars.



In 1999, astronomer John Harmon at the Arecibo Observatory in Puerto Rico, repeated the 1991 study, this time using the powerful microwave beam of the Arecibo Radio Telescope. The microwave energy reflected from mercury and was detected by the VLA radio telescope array in New Mexico, where a new image was made.

The radio-wavelength image to the left shows Mercury's North Polar Region at very high resolution. The image is 370 kilometers wide by 400 kilometers tall.

All the bright features are believed to be deposits of frozen water ice, at least several meters thick in the permanently shaded floors of the craters.

Reference: Harmon, Perillat and Slade, 2001, Icarus, vol 149, p.1-15

Problem 1 - From the information provided in the essay, what is the scale of the image in kilometers per millimeter?

Problem 2 - Measure the diameters of the craters, in kilometers, and estimate the total surface area covered by the large white patches in A) square kilometers and B) square meters.

Problem 3 - Suppose the icy deposit is mixed into the Mercurian surface to a depth of 10 meters. What is the total volume of the ice within the craters you measured in cubic meters?

Problem 4 - Suppose half of the volume is taken up by rock. What is the total remaining volume of ice?

Problem 5 - The density of ice is 917 kilograms/cubic meter. How many kilograms of ice are present?

Problem 6 - If this ice were 100% water ice, and 3.8 kilograms of water equals 1.0 gallons, how many gallons of water might be locked up in the shadowed craters of Mercury?



## Answer Key:

Problem 1 - From the information provided in the essay, what is the scale of the image in kilometers per millimeter?

Answer; The image is 370 kilometers wide by 400 kilometers tall. The image is 95 millimeters wide by 104 millimeters tall. The scale is therefore about **4.0 kilometers / millimeter**.

Problem 2 - Measure the diameters of the craters, in kilometers, and estimate the total surface area covered by the large white patches in A) square kilometers and B) square meters.

Answer: Students should measure the diameters of at least the 5 large craters that form the row slanted upwards from right to left through the center of the image. Their diameters are about 90 km, 40 km, 30 km, 20 km and 25 km. The area of a circle is  $\pi R^2$ , so the crater areas are 6,400 km<sup>2</sup>, 700 km<sup>2</sup>, 314 km<sup>2</sup> and 490 km<sup>2</sup>. The total area A) in square kilometers is about **7,900 km<sup>2</sup>** or B)  $7,900 \times (1000 \text{ m/km}) \times (1000 \text{ m/km}) = \mathbf{7.9 \times 10^9 \text{ meters}^2}$ . Students may reasonably ask how to estimate the area of partially-filled craters such as the largest one in the image. They may use appropriate percentage estimates. For example, the largest crater is about 1/2 filled (white color in image) so its area can be represented as  $6,400 \times 0.5 = 3,200 \text{ km}^2$ .

Problem 3 - Suppose the icy deposit is mixed into the Mercurian surface to a depth of 10 meters. What is the total volume of the ice within the craters you measured?

Answer: Volume = surface area x height =  $7.9 \times 10^9 \text{ meters}^2 \times 10 \text{ meters} = 7.9 \times 10^{10} \text{ meters}^3$ .

Problem 4 - Suppose half of the volume is taken up by rock. What is the total remaining volume of ice?

Answer;  $7.9 \times 10^{10} \text{ meters}^3 \times 0.5 = \mathbf{8.0 \times 10^{10} \text{ meters}^3}$

Problem 5 - The density of ice is 917 kilograms/cubic meter how many kilograms of ice are present?

Answer:  $8.0 \times 10^{10} \text{ meters}^3 \times 917 \text{ kg/meters}^3 = \mathbf{7.3 \times 10^{13} \text{ kilograms}}$

Problem 6 - If this ice were 100% water ice, and 3.8 kilogram of water equals 1.0 gallons, how many gallons of water might be locked up in the shadowed craters of Mercury?

Answer:  $7.3 \times 10^{13} \text{ kilograms} / 3.8 \text{ kg/gallon} = \mathbf{1.9 \times 10^{13} \text{ gallons or 19 trillion gallons!}}$

# A Trillion Here...A Trillion There

A light-year is nearly 9.5 trillion kilometers. This is an unbelievably large number that is so huge, no one can possibly comprehend just how large it is compared to a walk to the local store, or a drive to Grandma's! Even by astronomical standards, astronomers rarely work with numbers this large. The total number of stars in the Milky Way is 'only' measured in the billions. The distance to the farthest reaches of the visible universe is 'only' 14 billion light years. Let's bring this huge number 'one trillion' down to Earth by looking at the 2006-2007 finances of the United States and the world.

## US Statistics for 134 million taxpayers:

1...Federal Income Tax revenue	\$ 2.2 trillion.
2...Federal Government expenses	\$ 2.6 trillion.
3...The Gross Domestic Product (National wealth)	\$12.9 trillion
4...Consumer debt (credit cards, loans etc)	\$ 2.5 trillion
5...Total Mortgage debt (commercial and residential)	\$ 6.5 trillion
6...Total federal debt (including unfunded programs)	\$59.1 trillion
7...External debt owed to other countries	\$16.3 trillion
8...Total retirement assets (401k, private pensions, etc)	\$16.1 trillion
9...Annual income for US taxpayers	\$ 7.4 trillion

## World Statistics for 6 billion people:

10...Gross World Product	\$65.8 trillion
11...Debt to World banking	\$40.0 trillion
12...Value of all the shares on all the world stock markets	\$66.6 trillion
13...Yearly exports	\$ 6.6 trillion
14... Insurance premiums (life and non-life)	\$ 3.7 trillion
15...Assets of 9.5 million wealthiest individuals.	\$37.2 trillion

Problem 1 - What was the average Gross Domestic Product wealth for each US taxpayer?

Problem 2 - What was the average consumer debt for each US Taxpayer?

Problem 3 - About how much mortgage debt does each US taxpayer have?

Problem 4 - What is the total federal debt for each taxpayer including unfunded programs (like Social Security and Medicaid)?

Problem 5 - What percentage of the taxpayer annual income is paid in the federal Income Tax each year?

Problem 6 - What percentage of the world's wealth (Gross World Product) is the US wealth?

Problem 7 - About how much money does each taxpayer have in retirement assets in the US?

Problem 8 - Comparing the US taxes to the US government expenses, how much debt does the US generate each year?

Problem 9 - What is the average wealth per person in the world?

Problem 10 - What percentage of the world population includes the richest individuals?

Problem 11 - What is the average wealth of the non-richest individuals?

## Answer Key:

### US Statistics for 134 million taxpayers:

1...Federal Income Tax revenue	\$ 2.2 trillion.
2...Federal Government expenses	\$ 2.6 trillion.
3...The Gross Domestic Product (National wealth)	\$12.9 trillion
4...Consumer debt (credit cards, loans etc)	\$ 2.5 trillion
5...Total Mortgage debt (commercial and residential)	\$ 6.5 trillion
6...Total federal debt (including unfunded programs)	\$59.1 trillion
7...External debt owed to other countries	\$16.3 trillion
8...Total retirement assets (401k, private pensions, etc)	\$16.1 trillion
9...Annual income for US taxpayers	\$ 7.4 trillion

### World Statistics for 6 billion people:

10...Gross World Product	\$65.8 trillion
11...Debt to World banking	\$40.0 trillion
12...Value of all the shares on all the world stock markets	\$66.6 trillion
13...Yearly exports	\$ 6.6 trillion
14... Insurance premiums (life and non-life)	\$ 3.7 trillion
15...Assets of 9.5 million wealthiest individuals.	\$37.2 trillion

Problem 1 - What was the average Gross Domestic Product wealth for each US taxpayer?

**Answer: \$12.9 trillion/134 million = \$12,900,000 million/134 million = \$96,300.**

Problem 2 - What was the average consumer debt for each US Taxpayer?

**Answer: \$2.5 trillion/134 million = \$2,500,000 million/134 million = \$18,600**

Problem 3 - About how much mortgage debt does each US taxpayer have?

**Answer: \$6.5 trillion/134 million = \$6,500,000 million/134 million = \$48,500**

Problem 4 - What is the total federal debt for each taxpayer including unfunded programs (like Social Security and Medicaid)?

**Answer: \$59.1 trillion/134 million = \$59,100,000 million/134 million = \$441,000**

Problem 5 - What percentage of the taxpayer annual income is paid in the federal Income Tax each year?

**Answer: (\$2.2 trillion / \$7.4 trillion) x 100% = 29.7%**

Problem 6 - What percentage of the world's wealth (Gross World Product) is the US wealth?

**Answer: ( \$12.9 trillion / \$65.8 trillion) x 100% = 19.6 %**

Problem 7 - About how much money does each taxpayer have in retirement assets in the US?

**Answer: \$16.1 trillion / 134 million = \$16,100,000 million/134 million = \$120,000**

Problem 8 - Comparing the US taxes to the US government expenses, how much debt does the US generate each year?

**Answer: \$2.2 trillion - \$2.6 trillion = \$0.4 trillion or \$400 billion**

Problem 9 - What is the average wealth per person in the world?

**Answer: \$65.8 trillion / 6 billion = \$65,800 billion/6 billion = \$11,000 per person**

Problem 10 - What percentage of the world population includes the richest individuals?

**Answer: ( 9.5 million / 6 billion) x 100% = 0.16 %**

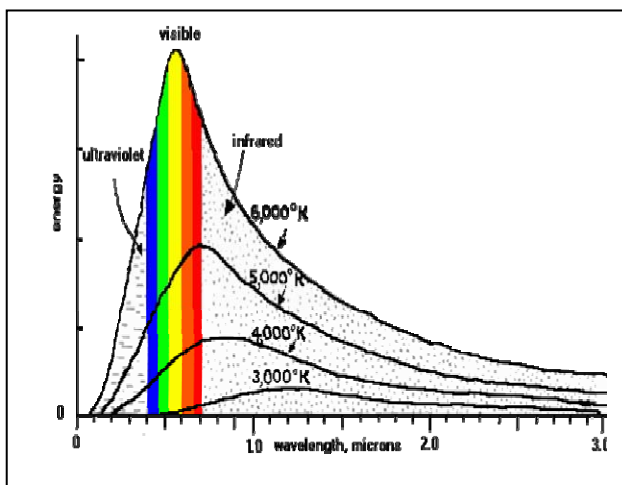
Problem 11 - About what is the average wealth of the non-richest individuals?

**Answer: ( \$65.8 trillion - \$37.2 trillion) / 6 billion people = \$4,700 per person**

# Measuring Star Temperatures

Careful measurements of a star's light spectrum gives astronomers clues about its temperature. For example, incandescent bodies that have a red glow are 'cool' while bodies with a yellow or blue color are 'hot'. This can be made more precise by measuring very carefully exactly how much light a star produces at many different wavelengths.

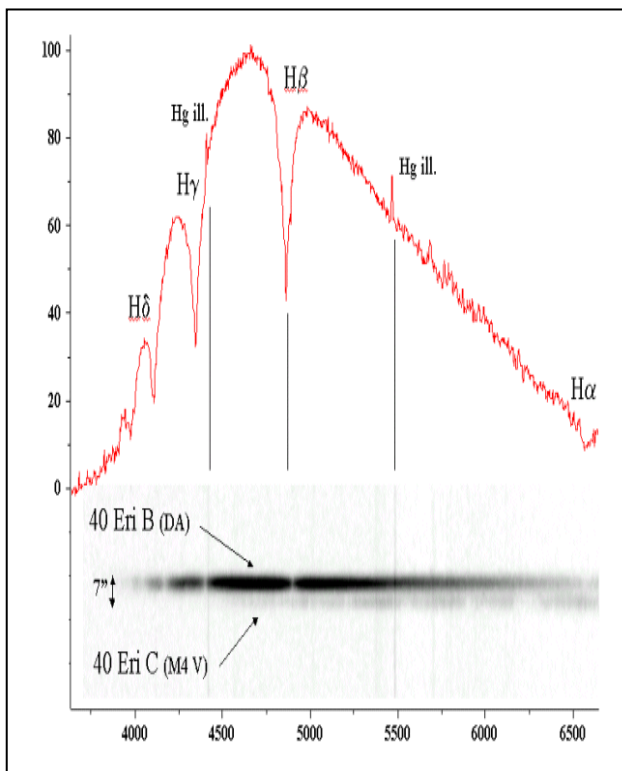
In 1900, physicist Max Planck worked out the mathematical details for how to exactly predict a body's spectrum once its temperature is known. The curve is therefore called a Planck 'black body' curve. It represents the brightness at different wavelengths of the light emitted from a perfectly absorbing 'black' body at a particular temperature.



From the mathematical properties of the Planck Curve, it is possible to determine a relationship between the temperature of the body and the wavelength where most of its light occurs - the peak in the curve. This relationship is called the Wein Displacement Law and looks like this:

$$\text{Temperature} = \frac{2897 \text{ Kelvin}}{\text{Wavelength}}$$

Where the temperature will be in units of Kelvin degrees, and the wavelength will be in units of micrometers.



The lower plot is the spectrum of the white dwarf star 40 Eridani B. The horizontal axis is in units of Angstroms. (10,000 Angstroms = 1 micrometer)

**Problem 1** - Based on the overall shape of the curve, and the wavelength where most of the light is being emitted, use the Wein Displacement Law to determine the temperature of 40 Eridani B.

**Problem 2** - What would be the peak wavelengths of the following stars in A) Angstroms; B) nanometers?

- A) Antares ..... 3,100 K
- B) Zeta Orionis..... 30,000 K
- C) Vega ..... 9,300 K
- D) Regulus..... 13,000 K
- E) Canopus..... 7,300 K
- F) OTS-44 brown dwarf... 2,300 K

Note:

- 1 micron = 10,000 Angstroms
- 1 nanometer = 10 Angstroms

### Answer Key:

**Problem 1** - Based on the overall shape of the curve, and the wavelength where most of the light is being emitted, use the Wein Displacement Law to determine the temperature of the white dwarf star 40 Eridani B.

**Answer:** The peak of the curve is near 4600 Angstroms or 0.46 micrometers. The temperature is  $2897 / 0.46 = 6,280$  K.

**Problem 2** - What would be the peak wavelengths of the following stars in Angstroms:

**Answer:**

A) Antares occurs at	2897/3100	= 0.9 microns or	9000 Angstroms
B) Zeta Orionis is at	2897/30,000	= 0.10 microns or	1000 Angstroms
C) Vega .....	9,300 K	= 0.31 microns or	3100 Angstroms
D) Regulus.....	13,000 K	= 0.22 microns or	2200 Angstroms
E) Canopus.....	7,300 K	= 0.39 microns or	3900 Angstroms
F) OTS-44 brown dwarf...	2,300 K	= 1.2 microns or	12,000 Angstroms

What would be the peak wavelengths of the following stars in nanometers?

**Answer:**

A) Antares:	9000 Angstroms =	900 nanometers.
B) Zeta Orionis:	1000 Angstroms =	100 nanometers
C) Vega:	3100 Angstroms =	310 nanometers.
D) Regulus:	2200 Angstroms =	220 nanometers
E) Canopus:	3900 Angstroms =	390 nanometers
F) OTS-44 brown dwarf:	12,000 Angstroms =	1200 nanometers.

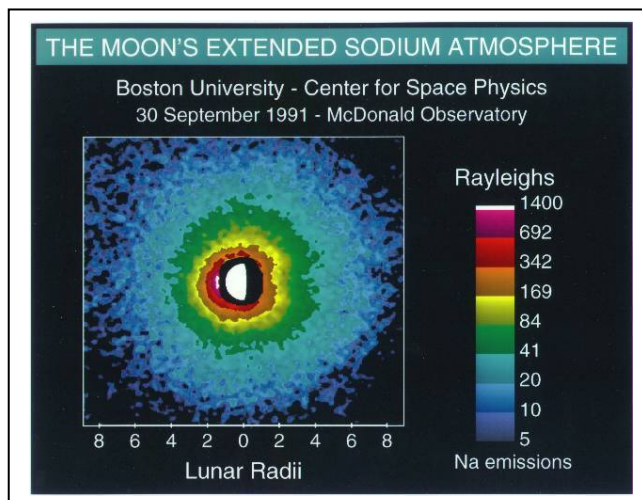


Courtesy: T.A.Rector, I.P.Dell'Antonio  
(NOAO/AURA/NSF)

Experiments performed by Apollo astronauts were able to confirm that the moon does have a very thin atmosphere.

The Moon has an atmosphere, but it is very tenuous. Gases in the lunar atmosphere are easily lost to space. Because of the Moon's low gravity, light atoms such as helium receive sufficient energy from solar heating that they escape in just a few hours. Heavier atoms take longer to escape, but are ultimately ionized by the Sun's ultraviolet radiation, after which they are carried away from the Moon by solar wind.

Because of the rate at which atoms escape from the lunar atmosphere, there must be a continuous source of particles to maintain even a tenuous atmosphere. Sources for the lunar atmosphere include the capture of particles from solar wind and the material released from the impact of comets and meteorites. For some atoms, particularly helium and argon, outgassing from the Moon's interior may also be a source.



Problem 1: The Cold Cathode Ion Gauge instrument used by Apollo 12, 14 and 15 recorded a daytime atmosphere density of 160,000 atoms/cc of hydrogen, helium, neon and argon in equal proportions. What was the density of helium in particles/cc?

Problem 2: The atomic masses of hydrogen, helium, neon and argon are 1.0 AMU, 4.0 AMU, 20 AMU and 36 AMU. If one AMU =  $1.6 \times 10^{-24}$  grams, a) How many grams of hydrogen are in one cm<sup>3</sup> of the moon's atmosphere? B) Helium? C) Neon? D) Argon? E) Total grams from all atoms?

Problem 3: Assume that the atmosphere fills a spherical shell with a radius of 1,738 kilometers, and a thickness of 170 kilometers. What is the volume of this spherical shell in cubic centimeters?

Problem 4. Your answer to Problem 2E is the total density of the lunar atmosphere in grams/cc. If the atmosphere occupies the shell whose volume is given in Problem 3, what is the total mass of the atmosphere in A) grams? B) kilograms? C) metric tons?

## Answer Key:

**Problem 1:** The Cold Cathode Ion Gauge instrument used by Apollo 12, 14 and 15 recorded a daytime atmosphere density of 160,000 atoms/cc of hydrogen, helium, neon and argon in equal proportions. What was the density of helium in particles/cc?

Answer: Each element contributes 1/4 of the total particles so hydrogen = 40,000 particles/cc; helium = 40,000 particles/cc, argon=40,000 particles/cc and argon=40,000 particles/cc

**Problem 2:** The atomic masses of hydrogen, helium, neon and argon are 1.0 AMU, 4.0 AMU, 20 AMU and 36 AMU. If one AMU =  $1.6 \times 10^{-24}$  grams, a) How many grams of hydrogen are in one cm<sup>3</sup> of the moon's atmosphere? B) Helium? C) Neon? D) Argon? E) Total grams from all atoms?

Answer: A) Hydrogen =  $1.0 \times (1.6 \times 10^{-24} \text{ grams}) \times 40,000 \text{ particles} = 6.4 \times 10^{-20} \text{ grams}$

B) Helium =  $4.0 \times (1.6 \times 10^{-24} \text{ grams}) \times 40,000 \text{ particles} = 2.6 \times 10^{-19} \text{ grams}$

C) Neon =  $20.0 \times (1.6 \times 10^{-24} \text{ grams}) \times 40,000 \text{ particles} = 1.3 \times 10^{-18} \text{ grams}$

D) Argon =  $36.0 \times (1.6 \times 10^{-24} \text{ grams}) \times 40,000 \text{ particles} = 2.3 \times 10^{-18} \text{ grams}$

E) Total =  $(0.064 + 0.26 + 1.3 + 2.3) \times 10^{-18} \text{ grams} = \underline{3.9 \times 10^{-18} \text{ grams per cc.}}$

**Problem 3:** Assume that the atmosphere fills a spherical shell with a radius of 1,738 kilometers, and a thickness of 170 kilometers. What is the volume of this spherical shell in cubic centimeters?

Answer: Compute the difference in volume between A sphere with a radius of  $R_i = 1,738 \text{ km}$  and  $R_o = 1,738 + 170 = 1,908 \text{ km}$ .  $V = \frac{4}{3} \pi (1908)^3 - \frac{4}{3} \pi (1738)^3 = 2.909 \times 10^{10} \text{ km}^3 - 2.198 \times 10^{10} \text{ km}^3 = 7.1 \times 10^9 \text{ km}^3$

$$\begin{aligned} \text{Volume} &= 7.1 \times 10^9 \text{ km}^3 \times (10^5 \text{ cm/km}) \times (10^5 \text{ cm/km}) \times (10^5 \text{ cm/km}) \\ &= 7.1 \times 10^{24} \text{ cm}^3 \end{aligned}$$

Note: If you use the 'calculus technique' of approximating the volume as the surface area of the shell with a radius of  $R_i$ , multiplied by the shell thickness of  $h = 170 \text{ km}$ , you will get a slightly different answer of  $6.5 \times 10^9 \text{ km}^3$  or  $6.5 \times 10^{24} \text{ cm}^3$

**Problem 4.** Your answer to Problem 2E is the total density of the lunar atmosphere in grams/cc. If the atmosphere occupies the shell whose volume is given in Problem 3, what is the total mass of the atmosphere in A) grams? B) kilograms?

A) Mass = density x volume =  $(3.9 \times 10^{-18} \text{ gm/cc}) \times 7.1 \times 10^{24} \text{ cm}^3 = 2.8 \times 10^7 \text{ grams}$

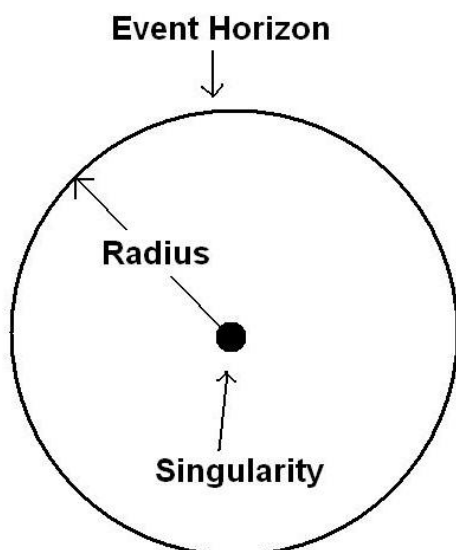
B) Mass =  $2.8 \times 10^7 \text{ grams} \times (1 \text{ kg}/1000 \text{ gms}) = 28,000 \text{ kilograms.}$

C) Mass =  $28,000 \text{ kg} \times (1 \text{ ton} / 1000 \text{ kg}) = 28 \text{ tons.}$

Teacher note: You may want to compare this mass to some other familiar objects. Also, the Apollo 11 landing and take-off rockets ejected about 1 ton of exhaust gases. Have the students discuss the human impact (air pollution!) on the lunar atmosphere from landings and launches.



# Black Holes....The Event Horizon



Black holes are objects that have such intense gravitational fields, they do not allow light to escape from them. They also make it impossible for anything that falls into them to escape, because to do so, they would have to travel at speeds faster than light. **No forms of matter or energy can travel faster than the speed of light, so that is why black holes are so unusual!**

## There are three parts to a simple black hole:

**Event Horizon** - That's the part that we see from the outside. It looks like a black, spherical surface with a very sharp edge in space.

**Interior Space** - This is a complicated region where space and time can get horribly mangled, compressed, stretched, and otherwise a very bad place to travel through.

**Singularity** - That's the place that matter goes when it falls through the event horizon. It's located at the center of the black hole, and it has an enormous density. You will be crushed into quarks long before you get there!

$$\text{Radius} = \frac{2GM}{c^2}$$

where  $c = 3 \times 10^{10} \text{ cm/sec}$   
 $G = 6.67 \times 10^{-8} \text{ dynes cm}^2 / \text{gm}^2$   
 you get

**Radius =  $1.48 \times 10^{-28} M$  centimeters**

Black holes can, in theory, come in any imaginable size. The size of a black hole depends on the amount of mass it contains. It's a very simple formula, especially if the black hole is not rotating. These 'non-rotating' black holes are called Schwarzschild Black Holes.

**Problem 1** - The formula in red gives the Schwarzschild radius of a black hole, in centimeters, in terms of its mass, in grams. From the equation for the radius in terms of the speed of light,  $c$ , and the constant of gravity,  $G$ , verify the formula shown in red.

**Problem 2** - Calculate the Schwarzschild radius, in centimeters, for Earth where  $M = 5.7 \times 10^{27}$  grams. (Express answer to two significant figures)

**Problem 3** - Calculate the Schwarzschild radius, in kilometers, for the sun, where  $M = 1.98 \times 10^{33}$  grams. (Express answer to two significant figures)

**Problem 4** - Calculate the Schwarzschild radius, in kilometers, for the entire Milky Way, with a mass of 250 billion suns. (Express answer to two significant figures)

**Problem 5** - Calculate the Schwarzschild radius, in centimeters, for a black hole with a mass of an average human being with  $M = 60$  kilograms. (Express answer to two significant figures)

Answer Key:

**Problem 1** - The formula in red gives the Schwarzschild radius of a black hole, in centimeters, in terms of its mass, in grams. From the equation for the radius in terms of the speed of light,  $c$ , and the constant of gravity,  $G$ , verify the formula shown in red.

$$\begin{aligned}\text{Answer: Radius} &= 2 \times (6.67 \times 10^{-8}) / (3 \times 10^{10})^2 M \\ &= 1.5 \times 10^{-28} M \quad \text{centimeters}\end{aligned}$$

where  $M$  is the mass of the black hole in grams.

**Problem 2** - Calculate the Schwarzschild radius, in centimeters, for Earth where  $M = 5.7 \times 10^{27}$  grams.

$$\begin{aligned}\text{Answer: } R &= 1.48 \times 10^{-28} (5.7 \times 10^{27}) \text{ centimeters} \\ R &= 0.84 \text{ centimeters !}\end{aligned}$$

**Problem 3** - Calculate the Schwarzschild radius, in kilometers, for the sun, where  $M = 1.98 \times 10^{33}$  grams.

$$\begin{aligned}\text{Answer: } R &= 1.48 \times 10^{-28} (1.98 \times 10^{33}) \text{ centimeters} \\ R &= 293,000 \text{ centimeters} \\ &= 2.9 \text{ kilometers}\end{aligned}$$

**Problem 4** - Calculate the Schwarzschild radius, in kilometers, for the entire Milky Way, with a mass of 250 billion suns.

Answer: If a black hole with the mass of the sun has a radius of 2.93 kilometers, a black hole with 250 billion times the sun's mass will be 250 billion times larger, or

$$R = (2.93 \text{ km} / \text{sun}) \times 250 \text{ billion suns} = 730 \text{ billion kilometers.}$$

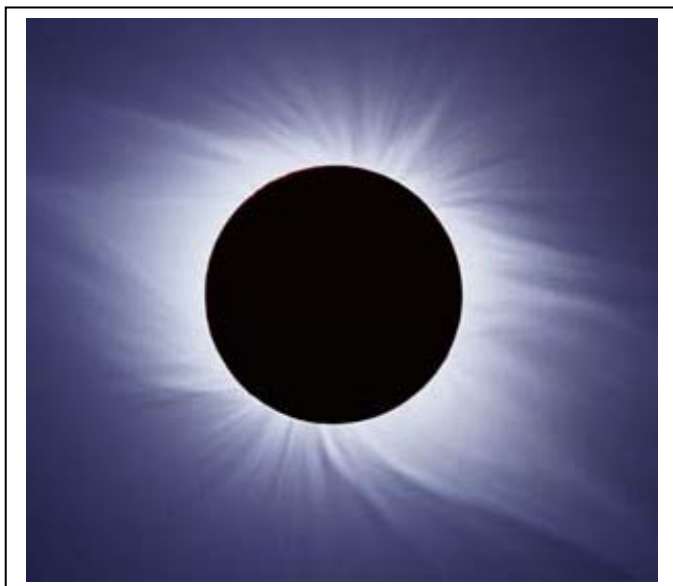
Note: The entire solar system has a radius of about 4.5 billion kilometers!

**Problem 5** - Calculate the Schwarzschild radius, in centimeters, for a black hole with a mass of an average human being with  $M = 60$  kilograms.

$$\begin{aligned}\text{Answer: } R &= 1.48 \times 10^{-28} (60,000) \text{ centimeters} \\ R &= 8.9 \times 10^{-24} \text{ centimeters.}\end{aligned}$$

Note: A proton is only about  $10^{-14}$  centimeters in diameter.

# The Last Total Solar Eclipse...Ever!



Total solar eclipses happen because the angular size of the moon is almost exactly the same as the sun's, despite their vastly different distances and sizes.

The moon has been steadily pulling away from earth over the span of billions of years. There will eventually come a time when these two angular sizes no longer match up. The moon will be too small to cause a total solar eclipse.

When will that happen?

*Image courtesy Fred Espenak*

<http://sunearth.gsfc.nasa.gov/eclipse/eclipse.html>

**Problem 1** - The minimum distance to the moon, called the perigee, is 356,400 kilometers. At that distance, the angular size of the moon from the surface of Earth is 0.559 degrees. Suppose you doubled the distance to the moon. What would its new angular size be, as seen from the surface of Earth?

**Problem 2** - Suppose you increased the moon's distance by 50,000 kilometers. What would the angular size now be?

**Problem 3** - The smallest angular size of the sun occurs near the summer solstice at a distance of 152 million kilometers, when the sun has an angular diameter of 0.525 degrees. How far away, in kilometers, does the moon have to be to match the sun's apparent diameter?

**Problem 4** - How much further away from Earth will the moon be at that time?

**Problem 5** - The moon is moving away from Earth at a rate of 3 centimeters per year. How many years will it take to move 3 kilometer further away?

**Problem 6** - How many years will it take to move the distance from your answer to Problem 4?

**Problem 7** - When will the last Total Solar Eclipse be sighted in the future?

## Answer Key:

**Problem 1** - The minimum distance to the moon, called the perigee, is 356,400 kilometers. At that distance, the angular size of the moon from the surface of Earth is 0.559 degrees. Suppose you doubled the distance to the moon. What would its new angular size be, as seen from the surface of Earth?

Answer: Because objects appear smaller the farther away they are, if you double the distance, the moon will appear half its former size, or  $0.559/2 = 0.279$  degrees across.

**Problem 2** - Suppose you increased the moon's distance by 50,000 kilometers. What would the angular size now be?

Answer: The distance is now 356,400 kilometers + 50,000 kilometers = 406,400 kilometers. The distance has increased by  $406,400/356,400 = 1.14$ , so that means that the angular size has been reduced to  $0.559 / 1.14 = 0.49$  degrees.

**Problem 3** - The smallest angular size of the sun occurs near the summer solstice at a distance of 152 million kilometers, when the sun has an angular diameter of 0.525 degrees. How far away, in kilometers, does the moon have to be to match the sun's apparent diameter?

Answer:  $0.559/0.525 = 1.06$  times further away from Earth or  $356,400 \text{ km} \times 1.06 = 377,800$  kilometers.

**Problem 4** - How much further away from Earth will the moon be at that time?

Answer:  $377,800 \text{ kilometers} - 356,400 \text{ kilometers} = 21,400 \text{ kilometers}$ .

**Problem 5** - The moon is moving away from Earth at a rate of 3 centimeters per year. How many years will it take to move 3 kilometer further away?

Answer:  $(300,000 \text{ centimeters}) / (3 \text{ centimeters} / \text{year}) = 100,000 \text{ years}$ .

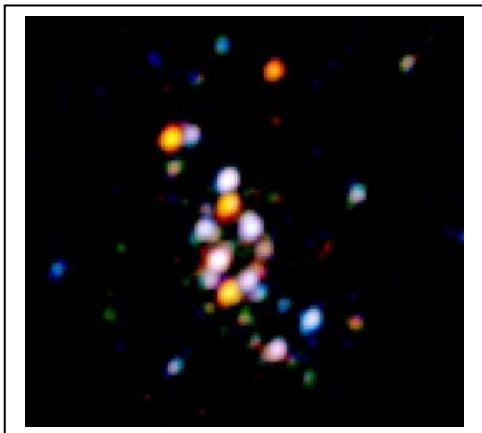
**Problem 6** - How many years will it take to move the distance from your answer to Problem 4?

Answer:  $(21,400 \text{ kilometers} / 3 \text{ kilometers}) \times 100,000 \text{ years} = 713 \text{ million years}$ .

**Problem 7** - When will the last Total Solar Eclipse be sighted in the future?

Answer: About 713 million years from now.

# Chandra Explores Angular Size



The above image of the globular cluster NGC-6266 was taken by the Chandra Satellite and shows only the stars in the cluster that are producing X-rays. Each spot is a binary star in which one of the members is a dense neutron star which is consuming gas from its normal star companion to produce x-rays. The field is 66.8 arcseconds wide.

**Problem 1** - Measure the width of the image in millimeters, and calculate the image scale in arcseconds per millimeter.

**Problem 2** - About what is the smallest separation between stars in the picture in: A) millimeters? B) arcseconds?

**Problem 3** - What is the diameter of the densest part of the x-ray cluster in: A) millimeters? B) arcseconds?

**Problem 4** - The distance to the cluster is 22,500 light years. What is: A) the smallest star separation and B) the diameter of the x-ray cluster?

Astronomers measure the sizes of objects in the sky in terms of their angular size. For instance, the moon appears to be about  $1/2$  degree in diameter. The planet Venus, when it is closest to Earth, appears to be even smaller - only about  $1/60$  of a degree. This small angle is called the arc-minute. There are 60 arcminutes in one degree.

What we really would like to know is, physically, how big something is in kilometers, instead of how big it appears to be in angular measure. To get this information, all we need to know is how far away the object is from us.

The moon is 324,000 kilometers away, and Venus is about 40 million kilometers away from Earth at its closest distance. The following formula lets us convert angles into kilometers:

$$\text{Size} = \text{Distance} \times \text{Diameter in degrees} / 57.3 \text{ degrees}$$

With the data for the Moon and Venus we get

$$\begin{aligned} \text{Moon} &= 324,000 \text{ km} \times (0.5) / 57.296 = 2,800 \text{ km} \\ \text{Venus} &= 40 \text{ million km} \times (1/60) / 57.296 = 11,600 \text{ km.} \end{aligned}$$

For more distant objects, we use a slightly different formula. Instead of an angular size measured in degrees, we use angles measured in arcseconds. There are 60 arcseconds in one arc minute, so one arcsecond is  $1/60 \times 1/60 = 1/3600$  degree, so we get:

$$\text{Diameter} = \text{distance} \times \text{Size} / (3600 \times 57.296)$$

which becomes

$$\text{Diameter} = \text{distance} \times \text{Size} / (206,265)$$

The binary star system P Eridani is 26.6 light years from Earth. Its two stars have a maximum separation of 11.8 arcseconds, which will occur in the year 2048. How far apart will they be if 1 light year (LY) equals 6 trillion kilometers?

$$\text{Separation} = 26.6 \times 11.8 / 206265 = 0.0015 \text{ LY.}$$

This equals  $0.0015 \text{ LY} \times 6 \text{ trillion km/LY} = 9 \text{ billion kilometers}$ , or twice the distance to Pluto!

For more on angular size, and an interactive calculator, visit the NASA, Chandra website:  
[http://chandra.harvard.edu/photo/scale\\_distance.html](http://chandra.harvard.edu/photo/scale_distance.html)

## Answer Key:

**Problem 1** - Measure the width of the image in millimeters, and calculate the image scale in arcseconds per millimeter.

**Answer:** 58 millimeters corresponds to 66.8 arcseconds, so the scale is  $66.8/58 = 1.2 \text{ arcseconds / millimeter}$ .

**Problem 2** - About what is the smallest separation between stars in the picture in A) millimeters? B) arcseconds?

**Answer:** Students answers may vary depending on which stars they choose: A) **About 2 mm** B)  $2 \text{ mm} \times 1.2 \text{ arcsec/mm} = 2.4 \text{ arcseconds}$ .

**Problem 3** - What is the diameter of the densest part of the x-ray cluster in A) millimeters? B) arcseconds?

**Answer:** Students answers may vary: A) **10 millimeters**; B)  $10 \text{ mm} \times 1.2 \text{ arcsec/mm} = 12 \text{ arcseconds}$ .

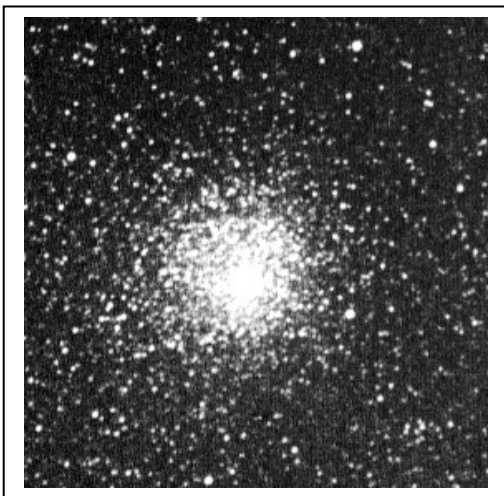
**Problem 4** - The distance to the cluster is 22,500 light years. What is A) the smallest star separation and B) the diameter of the x-ray cluster?

Answer:

A) Separation =  $22,500 \text{ light years} \times (2.4 \text{ arcseconds}) / 206265 = 0.26 \text{ light years}$ .

B) Diameter =  $22,500 \text{ light years} \times (12 \text{ arcseconds}) / 206265 = 1.3 \text{ light years}$ .

Students estimates will differ depending on which star pairs or diameters they measure.



NGC-6266 is also called Messier-62 and is located in the constellation Ophiuchus. Here is a picture of the entire cluster. The diameter of the star cluster in this image is 15 arcminutes. At a distance of 22,500 light years, its diameter is

$$22,500 \text{ LY} \times (15/60)/57.296 = 98 \text{ light years!}$$

The Chandra image is only a very small part of the center of this cluster!



This is an image taken by the Spitzer Space telescope in the infrared part of the electromagnetic spectrum. Instead of seeing the light from stars, it sees mainly the light from heated dust grains in space, glowing with temperatures between 100 to 200 K degrees.

These bright young stars are found in a rosebud-shaped (and rose-colored) nebulosity known as NGC 7129. The star cluster and its associated nebula are located at a distance of 3300 light-years in the constellation Cepheus. A recent census of the cluster reveals the presence of 130 young stars. The stars formed from a massive cloud of gas and dust. Most stars in our Milky Way galaxy, including our own sun, are thought to form in such clusters.

Most of the infrared light comes from a deeply embedded proto-star called FIR-2, which produces 430 times the total power of the sun, with a temperature of about 35 K. We can use this information to estimate the mass of this nebula!

**Problem 1** - A single dust grain is about 0.2 microns in diameter, and has a density of about 2 grams/cm<sup>3</sup>. What is the total mass, in grams, of this dust grain if it has a spherical shape?

**Problem 2** - What is the total power produced by this infrared source if the power from the sun is  $3.8 \times 10^{33}$  ergs/sec?

**Problem 3** - A single dust grain, 0.2 microns in diameter, at a temperature of 35 K, emits about  $7.0 \times 10^{-13}$  ergs/sec of power. How many dust grains are needed to produce the infrared power observed from FIR-2?

**Problem 4** - From your answer to Problem 1 and 3, what is the total mass of dust grains involved in producing the infrared light from FIR-2; A) in grams? B) In units of the sun's mass which is  $1.9 \times 10^{33}$  grams?

**Problem 5** - By mass, the interstellar medium consist of 99% gas and 1% dust grains. If the gas within FIR-2 has the same composition, what is the total mass of the interstellar medium within FIR-2 in Solar Masses?



## Answer Key:

Problem 1 - A single dust grain is about 0.2 microns in diameter, and has a density of about 2 grams/cm<sup>3</sup>. What is the total mass, in grams, of this dust grain if it has a spherical shape?

Answer: Radius = 1.0 microns,  
 Mass = Volume x Density  
 $= \frac{4}{3} \pi R^3 \times \text{Density}$   
 $= \frac{4}{3} \times 3.14 \times (0.1 \times 10^{-4})^3 \times 2.0$   
 $= \mathbf{8.4 \times 10^{-15} \text{ grams}}$

Problem 2 - What is the total power produced by this infrared source if the power from the sun is  $3.8 \times 10^{33}$  ergs/sec?

Answer: Power =  $430 \times 3.8 \times 10^{33}$  ergs/sec =  $\mathbf{1.6 \times 10^{36} \text{ ergs/sec}}$

Problem 3 - A single dust grain, 0.2 microns in diameter, at a temperature of 35 K, emits about  $7.0 \times 10^{-13}$  ergs/sec of power. How many dust grains are needed to produce the infrared power observed from FIR-2?

Answer: Number =  $1.6 \times 10^{36} \text{ ergs/sec} / 7.0 \times 10^{-13} \text{ ergs/sec/dust grain}$   
 $= \mathbf{2.3 \times 10^{48} \text{ dust grains}}$

Problem 4 - From your answer to Problem 1 and 3, what is the total mass of dust grains involved in producing the infrared light from FIR-2; A) in grams? B) in units of the sun's mass which is  $1.9 \times 10^{33}$  grams?

Answer: A)  $8.4 \times 10^{-15} \text{ grams / dust grain} \times 2.3 \times 10^{48} \text{ dust grains} = \mathbf{1.9 \times 10^{34} \text{ grams}}$   
 B)  $1.9 \times 10^{34} \text{ grams} / 1.9 \times 10^{33} \text{ grams} = \mathbf{10.0 \text{ Solar Masses.}}$

Problem 5 - By mass, the interstellar medium consist of 99% gas and 1% dust grains. If the gas within FIR-2 has the same composition, what is the total mass of the interstellar medium within FIR-2 in solar Masses?

Answer -  $100 \times 10.0 = \mathbf{1000 \text{ Solar Masses.}}$

The actual dust mass has been estimated as about 6 solar masses by C. Eiroa (AA 1998, v. 335, p. 243. and Muzerolle, et al, 2004, ApJ Suppl. V. 154, p.379.) The largest uncertainty in the problem is the size of the dust grain and the infrared radiation power emitted by the dust grain. This problem assumed that the dust grain size is typical of what is found for interstellar dust grains. But in the environment of the protostar, dust grain sizes are expected to vary due to grain growth. Also, the reflectivity (albedo) of the dust grain depends on its composition and size in a complex way.



NASA/JPL-Caltech/J. Hora (Harvard-Smithsonian Center for Astrophysics)

NASA's Spitzer Space Telescope finds a delicate flower in the Ring Nebula, as shown in this image. The outer shell of this planetary nebula looks surprisingly similar to the delicate petals of a camellia blossom. A planetary nebula is a shell of material ejected from a dying star. Located about 2,000 light years from Earth in the constellation Lyra, the Ring Nebula is also known as Messier-57 or NGC 6720. The ring is about 6,000 to 8,000 years old based on the speed of its expansion. The Infrared power equals 9.6 times the sun, and the dust temperature is about 64 K.

Problem 1 - A single dust grain is about 0.2 microns in diameter, and has a density of about 3 grams/cm<sup>3</sup>. What is the total mass, in grams, of this dust grain if it has a spherical shape?

Problem 2 - What is the total power produced by this infrared source if the power from the sun is  $3.8 \times 10^{33}$  ergs/sec?

Problem 3 - A single dust grain, 0.2 microns in diameter, at a temperature of 64 K, emits about  $3.75 \times 10^{-10}$  ergs/sec of power. How many dust grains are needed to produce the infrared power observed from the dust grains in the ring of M-57?

Problem 4 - From your answer to Problem 1 and 3, what is the total mass of dust grains involved in producing the infrared light from M-57; A) in grams? B) In units of the sun's mass which is  $1.9 \times 10^{33}$  grams?

Problem 5 - By mass, the interstellar medium consist of 99% gas and 1% dust grains. If the gas within the planetary nebula has the same composition, what is the estimated total mass of the nebular medium inside M-57 in solar masses?

Problem 6: The mass of the white dwarf star left behind is about 1.4 solar masses. What percentage of the mass of the star was lost during its planetary nebula phase?

## Answer Key:

Problem 1 - A single dust grain is about 0.2 microns in diameter, and has a density of about 3 grams/cm<sup>3</sup>. What is the total mass, in grams, of this dust grain if it has a spherical shape?

**Answer;**

$$V = 4/3 (3.14) (0.1 \times 10^{-4})^3 = 4.2 \times 10^{-15} \text{ cm}^3.$$

$$M = 4.2 \times 10^{-15} \text{ cm}^3 \times 3 \text{ grams/cm}^3.$$

$$= 1.3 \times 10^{-14} \text{ grams per dust grain.}$$

Problem 2 - What is the total power produced by this infrared source if the power from the sun is  $3.8 \times 10^{33}$  ergs/sec?

$$\text{Answer: } 9.6 \text{ times the sun} \times 3.8 \times 10^{33} \text{ ergs/sec} = 3.7 \times 10^{34} \text{ ergs/sec}$$

Problem 3 - A single dust grain, 0.2 microns in diameter, at a temperature of 64 K, emits about  $3.75 \times 10^{-10}$  ergs/sec of power. How many dust grains are needed to produce the infrared power observed from the dust drains in the ring of M-57?

$$\text{Answer: } (3.7 \times 10^{34} \text{ ergs/sec}) / (3.75 \times 10^{-10} \text{ ergs/sec}) = 9.7 \times 10^{43} \text{ dust grains.}$$

Problem 4 - From your answer to Problem 1 and 3, what is the total mass of dust grains involved in producing the infrared light from M-57; A) in grams? B) In units of the sun's mass which is  $1.9 \times 10^{33}$  grams?

$$\text{Answer: A) } 1.38 \times 10^{-14} \text{ grams per dust grain} \times 9.7 \times 10^{43} \text{ dust grains} = 1.3 \times 10^{30} \text{ grams}$$

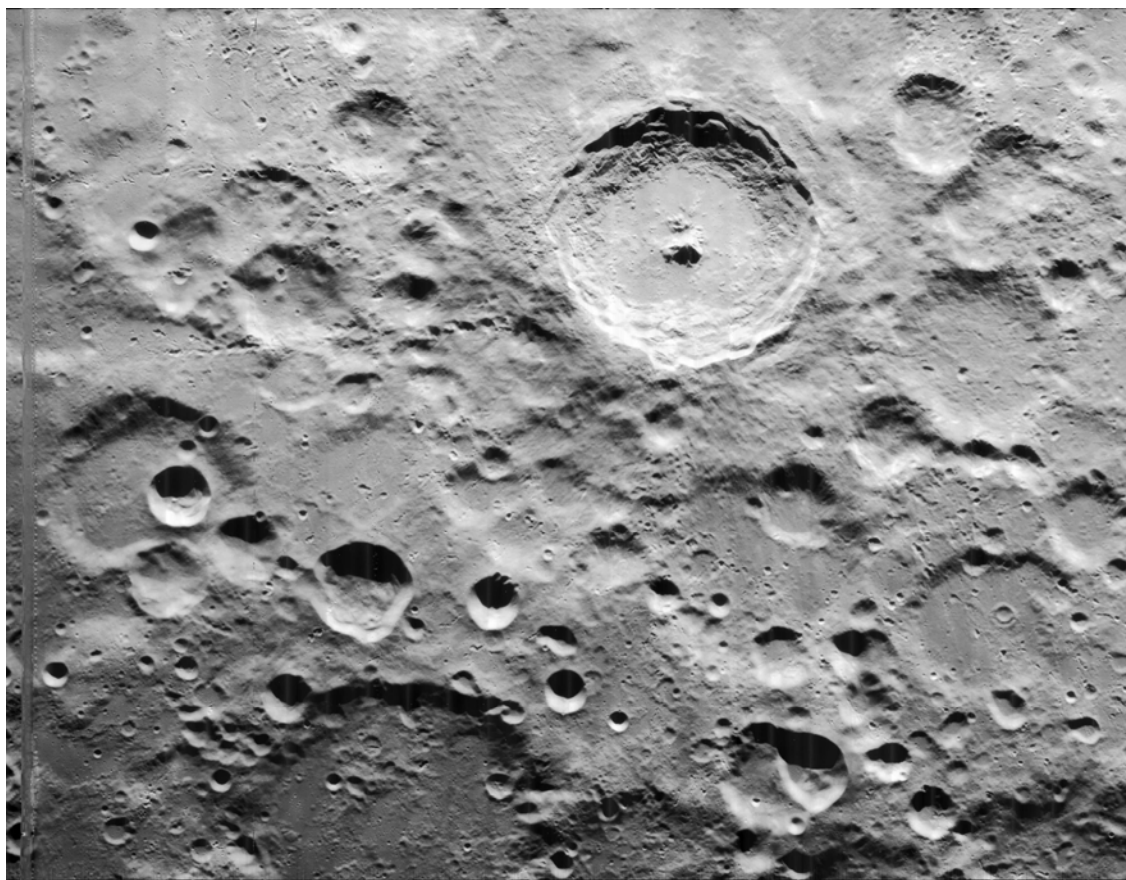
$$\text{B) } 1.3 \times 10^{30} \text{ grams} / 1.9 \times 10^{33} \text{ grams} = 0.0007 \text{ solar masses.}$$

Problem 5 - By mass, the interstellar medium consist of 99% gas and 1% dust grains. If the gas within the planetary nebula has the same composition, what is the estimated total mass of the nebular medium inside M-57 in solar masses?

$$\text{Answer: About } 100 \times 0.0007 \text{ solar masses} = 0.07 \text{ solar masses.}$$

Problem 6: The mass of the white dwarf star left behind is about 1.4 solar masses. What percentage of the mass of the star was lost during its planetary nebula phase?

$$\text{Answer: About } 100\% \times 0.07/1.4 = \text{only } 5\%.$$



This is a NASA image taken by the Lunar Orbiter IV spacecraft as it captured close-up images of the lunar surface in May, 1967. The large crater at the top-center is Tycho. Other images from the Lunar Orbiter spacecrafts can be found at the Lunar Orbiter Photo Gallery (<http://www.lpi.usra.edu/resources/lunarorbiter/>) The satellite was at an altitude of 2,994 kilometers when it took this image, which measures 350 km x 270 km. The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the field of view of the image is 16.6 kilometers x 4.1 kilometers.

Step 1: Measure the width of the lunar image with a metric ruler. How many millimeters long is the image?

Step 2: Read the explanation for the image and note any physical scale information provided. The information in the introduction says that the image is 350 kilometers along its largest dimension.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in kilometers per millimeter. Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in kilometers.

**Problem 1:** What is the diameter of the crater Tycho in kilometers?

**Problem 2:** How large is the smallest feature you can see?

**Problem 3:** How large are some of the smaller hills at the floor of the crater, in meters?

**Problem 4:** About how large are the most common craters in the field?

**Problem 5:** Which crater is about the same size as Denver, which has a diameter of about 25 km?

### Answer Key:

Step 1: Measure the width of the lunar image with a metric ruler. How many millimeters long is the image?

**Answer: 150 millimeters.**

Step 2: Read the explanation for the image and note any physical scale information provided. The information in the introduction says that the image is 350 kilometers along its largest dimension.

**Answer: 350 kilometers.**

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in kilometers per millimeter.

**Answer: 350 kilometers / 150 millimeters = 2.3 kilometers / millimeter.**

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in kilometers.

Problem 1: What is the diameter of the crater Tycho in kilometers?

**Answer: About 35 millimeters x 2.3 km/mm = 80.5 kilometers in diameter.**

Problem 2: How large is the smallest feature you can see?

**Answer: There are many small details in the image, pits, hills, etc, that students can estimate 0.1 to 0.3 millimeters for a physical size of 0.23 to 0.7 kilometers.**

Problem 3: How large are some of the smaller hills at the floor of the crater, in meters?

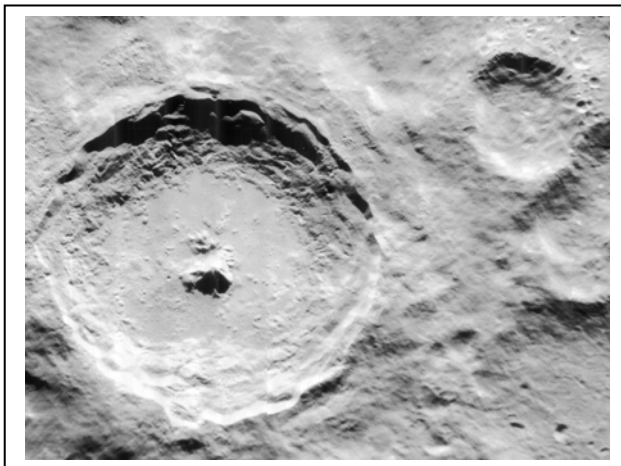
**Answer: These small features are about 0.1 millimeters across or 230 meters in size.**

Problem 4: About how large are the most common craters in the field?

**Answer: The answer may vary a bit, but the small craters that are 0.5 millimeters across are the most common. These have a physical size of about 1 kilometer.**

Problem 5: Which crater is about the same size as Denver, which has a diameter of about 25 km?

**Answer: In order to fit Denver into one of these lunar craters, it will have to appear to be about 25 km x (1 millimeter/2.3 km) = 11 millimeters across. There are three craters just to the right of Tycho that are about this big. Students should not get 'lost' trying to exactly match up their estimate with a precise lunar feature. 'Close-enough' estimates are good enough! See below comparison as a guide.**







This QuickBird Satellite image was taken of downtown Las Vegas Nevada from an altitude of 450 kilometers. Private companies such as Digital Globe (<http://www.digitalglobe.com>) provide images such as this to many different customers around the world. The large building shaped like an upside-down 'Y' is the Bellagio Hotel at the corner of Las Vegas Boulevard and Flamingo Road. The width of the image is 700 meters. The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the field of view of the image is 700 meters wide.

Step 1: Measure the width of the image with a metric ruler. How many millimeters long is the image?

Step 2: Use clues in the image description to determine a physical distance or length. Convert this to meters.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters.

**Problem 1:** How long is each of the three wings of the Bellagio Hotel in meters?

**Problem 2:** What is the length of a car on the street in meters?

**Problem 3:** How wide are the streets entering the main intersection?

**Problem 4:** What is the smallest feature you can see, in meters?

**Problem 5:** What kinds of familiar objects can you identify in this image?



## Answer Key:

This QuickBird Satellite image was taken of downtown Las Vegas Nevada on October 14, 2005 from an altitude of 450 kilometers. Private companies such as Digital Globe (<http://www.digitalglobe.com>) provide images such as this to many different customers around the world. The large building shaped like an upside-down 'Y' is the Bellagio Hotel at the corner of Las Vegas Boulevard and Flamingo Road. The width of the image is 700 meters.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the field of view of the image is 700 meters wide.

Step 1: Measure the width of the image with a metric ruler. How many millimeters long is the image?

Answer: 150 millimeters.

Step 2: Use clues in the image description to determine a physical distance or length. Convert this to meters.

Answer: The information in the introduction says that the image is 700 meters long.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter.

Answer:  $700 \text{ meters} / 150 \text{ millimeters} = 4.7 \text{ meters} / \text{millimeter}$ .

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters.

Problem 1: How long is each of the three wings of the Bellagio Hotel in meters?

Answer: About 25 millimeters on the image or  $25 \text{ mm} \times (4.7 \text{ meters/mm}) = 117.5 \text{ meters}$ .

Problem 2: What is the length of a car on the street in meters?

Answer: About 1 millimeter on the image or  $1 \text{ mm} \times 4.7 \text{ meters/mm} = 4.7 \text{ meters}$ .

Problem 3: How wide are the streets entering the main intersection?

Answer: About 8 millimeters on the image or  $8 \text{ mm} \times 4.7 \text{ meters/mm} = 37 \text{ meters}$ .

Problem 4: What is the smallest feature you can see, in meters?

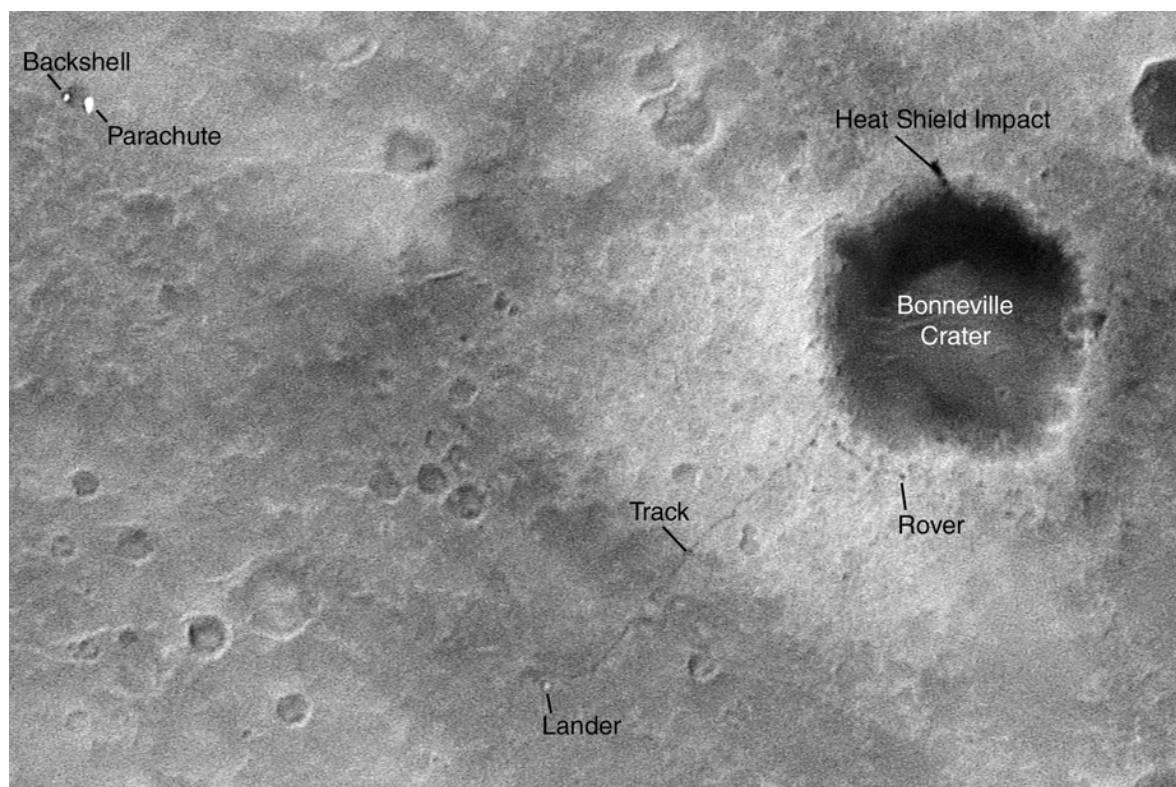
Answer: Some of the small dots on the roof tops are about 0.2 millimeters across which equals 1 meter.

Problem 5: What kinds of familiar objects can you identify in this image?

Answer: Will vary depending on student.

1. Cars, busses
2. swimming pools and reflecting ponds
3. Trees
4. lane dividers
5. Shadows of people walking across the plaza to the Hotel.

Note: Ask the students to use image clues to determine the time of day (morning, afternoon, noon); Whether it is rush-hour or not; Time of year, etc.



This NASA, Mars Orbiter image of the Mars Rover, Spirit, landing area near Bonneville Crater. The width of the image is exactly 895 meters. (Credit: NASA/JPL/MSSS). It shows the various debris left over from the landing, and the track of the Rover leaving the landing site.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the width of the image is 895 meters.

Step 1: Measure the width of the image with a metric ruler. How many millimeters wide is it?

Step 2: Use clues in the image description to determine a physical distance or length. Convert to meters.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to two significant figures.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to two significant figures.

**Problem 1:** About what is the diameter of Bonneville Crater rounded to the nearest ten meters?

**Problem 2:** How wide, in meters, is the track of the Rover?

**Problem 3:** How big is the Rover?

**Problem 4:** How small is the smallest well-defined crater to the nearest meter in size?

**Problem 5:** A boulder is typically 5 meters across or larger. Are there any boulders in this picture?

## Answer Key:

This NASA, Mars Orbiter image of the Mars Rover, Spirit, landing area near Bonneville Crater. The width of the crater is 200 meters. (Credit: NASA/JPL/MSSS). It shows the various debris left over from the landing, and the track of the Rover leaving the landing site.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the width of the image is 895 meters.

Step 1: Measure the width of the image with a metric ruler. How many millimeters wide is it?

Answer: 157 millimeters.

Step 2: Use clues in the image description to determine a physical distance or length. Convert to meters.

Answer: 895 meters.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to two significant figures.

Answer:  $895 \text{ m} / 157 \text{ mm} = 5.7 \text{ meters / millimeter}$ .

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters.

Problem 1: About what is the diameter of Bonneville Crater rounded to the nearest 10 meters?

Answer: Students answers for the diameter of the crater in millimeters may vary, but answers in the range from 30-40 mm are acceptable. Then this equals  $30 \times 5.7 = 170$  meters to  $40 \times 5.7 = 230$  meters. Students may average these two measurements to get  $(170+230)/2 = 200$  meters.

Problem 2: How wide, in meters, is the track of the Rover?

Answer:  $0.2 \text{ millimeters} = 1 \text{ meter}$ .

Problem 3: How big is the Rover?

Answer:  $0.3 \text{ millimeters} = 1.7 \text{ meters}$  but since the measurement is only 1 significant figure, the answer should be 2 meters.

Problem 4: How small is the smallest well-defined crater in meters?

Answer:  $2 \text{ millimeters} \times 5.7 = 11.4 \text{ meters}$ , which to the nearest meter is 11 meters.

Problem 5: A boulder is typically 5 meters across or larger. Are there any boulders in this picture?

Answer: Students answers may vary and lead to interesting discussions about what features are real, and which ones are flaws in the printing of the picture. This is an important discussion because 'image artifacts' are very common in space-related photographs. 5 meters is about 1 millimeter, and there are no obvious rounded objects this large or larger in this image.

# A note from the Author:

*Hi again!*

*I hope you and your students are enjoying this collection of unusual math problems!*

*Astronomers use a variety of math skills each day, and we don't always use the most advanced forms of mathematics on a regular, day-to-day basis, either!*

*We keep a close eye on our research budgets, and have to determine how much money we have to pay our salaries, and anticipate costs such as travel expenses, purchasing equipment, and paying the page charges for publishing our work. This involves addition, subtraction, multiplication and division. It's like balancing a check book, or in some cases like filling your state and federal tax forms.*

*Our research can involve many different forms of mathematics and problem-solving skills. For example, when we read research articles by other scientists, there is often a detailed undercurrent of statistics and algebra to work through, so you have to have these skills at your immediate recall each time you read or write articles that describe the subjects you are working on. If you are involved in more theoretical work, you can be sure that a complete knowledge of algebra, calculus and in some cases even tensor analysis, will be at-play. This is because you are starting from very basic laws in physics and working your way up to the level where your particular object or phenomenon occurs. This will involve differential equations to determine rates-of-change in the many physical parameters that define your system. It will involve integration to 'sum-up' a relationship over some spatial region, like the interior of a star, planet or an entire galaxy or cluster of galaxies.*

*If you are working with data and images, we often use programs like IDL or FORTRAN that take arrays of data and perform operations on them to 'clean up' an image, or render its pixelized data into some other form. These computer languages are algebraic, so that if you have two images, A and B, you can create a new image, C, by saying ' $C = A + B$ ' or ' $C = A/B$ ' or even ' $C = 3A + 2B$ ' or re-scale the image by a factor of two to display it on a TV screen by calculating ' $C = 2A$ '. You can also create mathematical procedures for image processing that are very complex and convert the raw data in the image to a map of temperature, density, color or luminosity.*

*And of course, we use advanced math every day if we happen to be tutors or professors that are teaching students, or family members! College teaching also requires that you be able to derive, mathematically, all of the essential tools of astronomy and to be able to explain in great detail the various physical theories that have been created in stellar structure, galaxy dynamics and cosmology. These are all intensely mathematical relationships in nature.*

*I find mathematics incredibly enjoyable, whether it is algebra, calculus or even more advanced mathematics. It is always a thrill to be able to see how things are logically related to each other, and mathematics lets us see how this comes about in a very elegant and beautiful way, and makes predictions about our world that we could never have imagined otherwise!.*

Sincerely,  
Dr. Sten Odenwald  
NASA Astronomer





**National Aeronautics and Space Administration**

**Space Math @ NASA**  
**Goddard Spaceflight Center**  
**Greenbelt, Maryland 20771**  
**[spacemath.gsfc.nasa.gov](http://spacemath.gsfc.nasa.gov)**

**[www.nasa.gov](http://www.nasa.gov)**