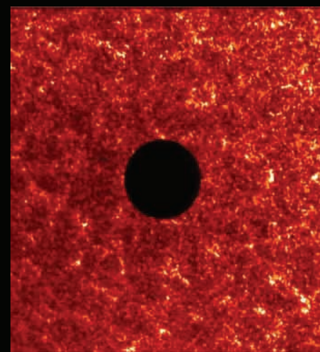
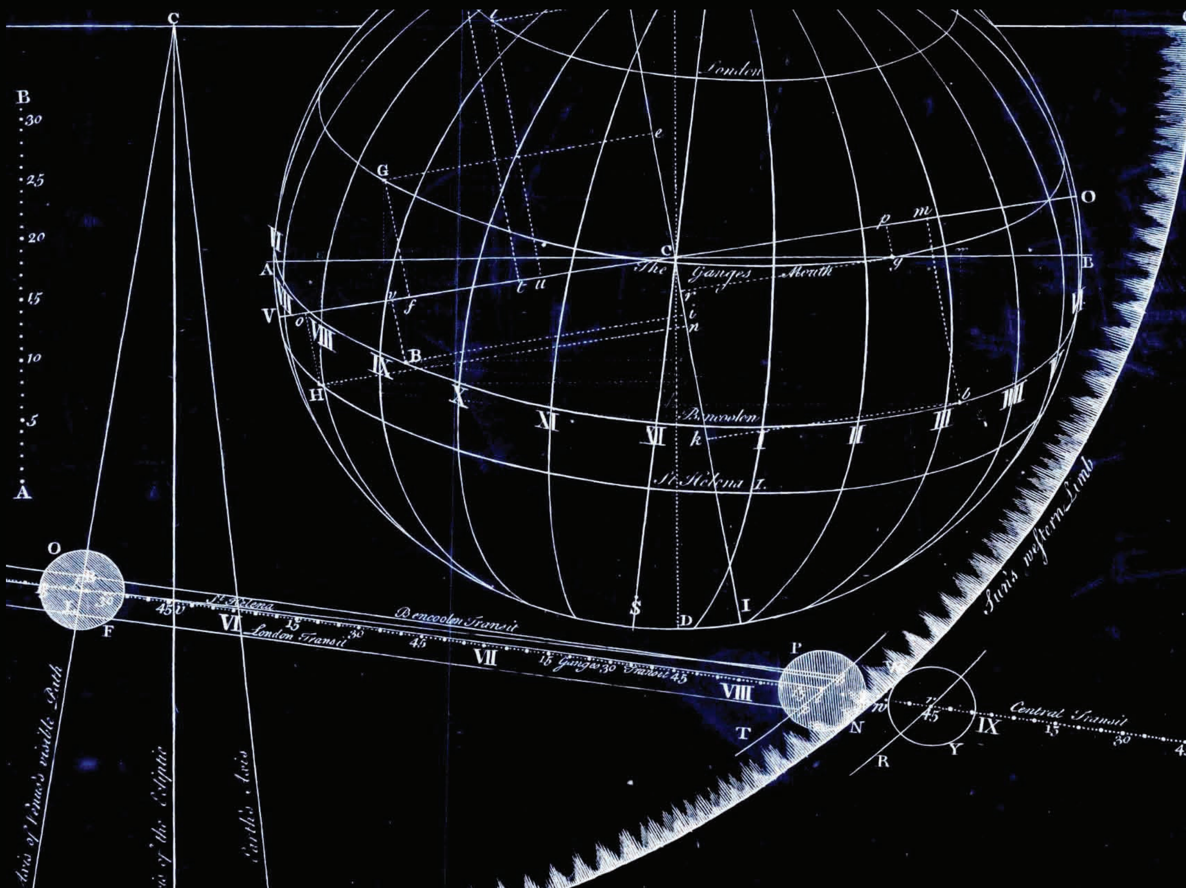
National Aeronautics and Space Administration

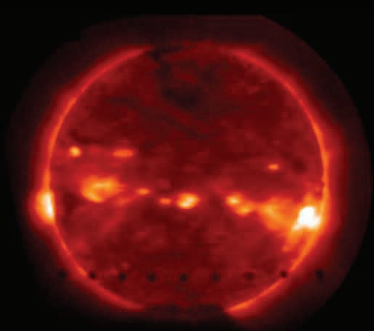
Transit of Venus Math

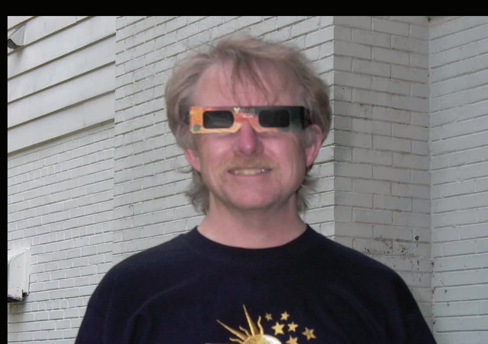


The Transit of Venus June 5/6, 2012



The Transit of Venus June 5/6, 2012





Top Row – Left image - Photo taken at the US Naval Observatory of the 1882 transit of Venus. Middle image - The cover of *Harpers Weekly* for 1882 showing children watching the transit of Venus. Right image – Image from NASA's TRACE satellite of the transit of Venus, June 8, 2004.

Middle - Geometric sketches of the transit of Venus by James Ferguson on June 6, 1761 showing the shift in the transit chords depending on the observer's location on Earth. The parallax angle is related to the distance between Earth and Venus.

Bottom – Left image **-** NOAA GOES-12 satellite x-ray image showing the Transit of Venus 2004. Middle image – An observer of the 2004 transit of Venus wearing NASA's Sun-Earth Day solar glasses for safe viewing. Right image – The Transit of Venus taken in 2004 by NASA's TRACE satellite.

During the 18th and 19th centuries, astronomers carefully measured the transits of Venus in order to determine a precise distance of the sun from Earth. This distance determined in units of kilometers would set the absolute physical scale for the entire solar system and beyond.

When do Transits of Venus happen?

Here is a table of the dates for some recent transits.

December 7, 1631	December 9, 1874
December 4, 1639	December 6, 1882
June 6, 1761	June 8, 2004
June 3, 1769	June 5, 2012

Math Puzzler 1 – To the nearest year, how often do transits of Venus occur? What kinds of numerical patterns can you find among the transit years? When will the next transits occur between 2012 and 2600?

How far away from Earth is Venus?

The diameter of Venus is known to be 12,000 kilometers. At the time of the transit of Venus, its angular diameter is measured as 0.016 degrees.

Math Puzzler 2 - Using trigonometry, what is the distance from Earth to Venus in kilometers at the time of the transit?

Math Puzzler 3 - The duration of the transit depends on the relative speeds between the fast-moving Venus in its orbit and the slower-moving Earth in its orbit. This speed difference is known to be 5.24 km/sec. If the June 5, 2012, transit lasts 24,000 seconds, during which time the planet moves an angular distance of 0.17 degrees across the sun as viewed from Earth, what distance between Earth and Venus allows the distance traveled by Venus along its orbit to subtend the observed angle?

Determining the Astronomical Unit

Based on the calculations of Nicolas Copernicus and Johannes Kepler, the distances of the known planets from the sun could be given rather precisely in terms of the distance between Earth and Sun - known as the Astronomical Unit (AU). For instance, Mercury was located at 0.39 AU, Venus was at 0.72 AU, Mars was at 1.5 AU and Jupiter was at 5.2 AU from the sun. But no one had an accurate measure of the distance from Earth to the sun in physical units such as kilometers, to the true physical size of the solar system was not very well known.

In the 1600's, astronomers realized that the transits of Venus could be used to make this measurement.

Math Puzzler 4 - At the time of the Transit of Venus, what is the distance between Earth and Venus in terms of Astronomical Units?

Math Puzzler 5 - From your answers to Math Puzzler 2 and 3, what is the average distance between Earth and Venus in kilometers using these two methods?

Math Puzzler 6 - Using a simple proportion, what is your estimate for the Astronomical Unit in kilometers?

Note to Educators: *The answers to these Math Puzzlers can be found by visiting the Educator's area at the SpaceMath@NASA website and clicking on the Transit of Venus button.*

Visit the SpaceMath@NASA website at

http://spacemath.gsfc.nasa.gov

to register for access to over 400+ problems and their solutions that apply math to all types of 'far out' space topics including NASA press releases and videos!

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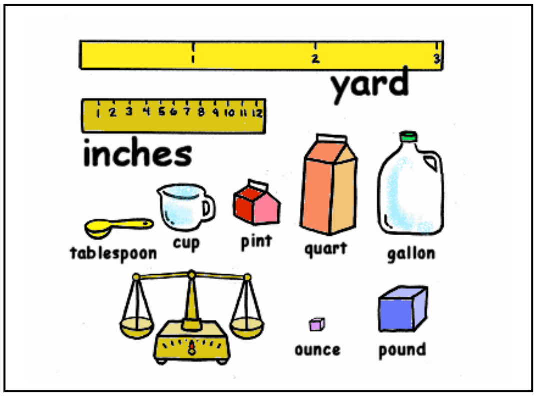
Transit of Venus - Unit Conversion Problems



For the Transit of Venus, astronomers use three types of units rather extensively, arcsecond, Astronomical Unit (AU) and kilometer.

If you take the time to develop a skill in working with these units, this will help avoid computational problems!

There really isn't much to remembering how to manipulate them. Just remember that:

1 radian = 57.2958 degrees
1 degree = 60 arcminutes
= 3600 arcseconds

1 AU = 149 million kilometers

Problem 1 - 1 microradian = ____________ arcseconds

Problem 2 - 1 square degree = __________ square arcseconds

Problem 3 - 0.72 Astronomical Units = _______________ kilometers

Problem 4 - 2.6 billion kilometers = _________________ Astronomical Units

Problem 5 - 25 milliarcseconds = _____________ degrees

Problem 6 - 0.01628 degrees = ____________ arcseconds

Problem 7 - 9 hrs 25 minutes 15 seconds = ______________ seconds of time

Problem 8 - 23003 seconds in time = ____ hrs ___ min ____ sec

Problem 9 – 4π square radians = ________ square degrees.

Problem 10 - 1888 arcseconds = ________ degrees

Problem 1 - 1 microradian = ______**0.206**______ arcseconds

1×10^{-6} radians x (57.2958 deg/1 radian) x (3600 arcsec / 1 degree)

Problem 2 - 1 square degree = ___**12,960,000**_______ square arcseconds
1 deg2 x (3600 asec/1deg)x(3600 asec/1deg)

Problem 3 - 0.72 Astronomical Units = __**107.3 million**_____________ kilometers
0.72 AU x (149 million km/1 AU)

Problem 4 - 2.6 billion kilometers = _____**17.45**____________ Astronomical Units
2.6 billion km x (1 AU/149 million km)

Problem 5 - 25 milliarcseconds = _____**0.00000694**________ degrees
25 milliasec x (0.001 asec/1 milli) x (1 degree/3600 asec)

Problem 6 - 0.01628 degrees = ______**58.61**______ arcseconds
0.01628 deg x (3600 asec/1 deg)

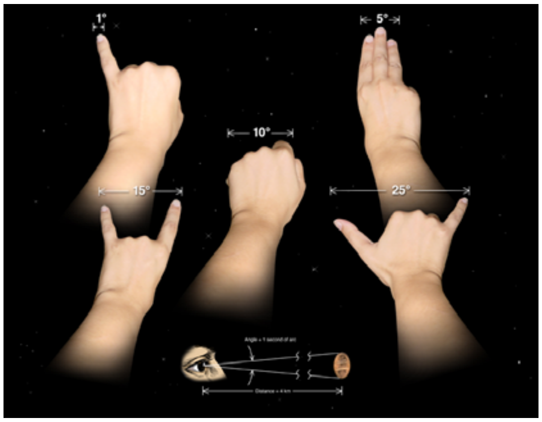
Problem 7 - 9 hrs 25 minutes 15 seconds = ______**33,915** _______ seconds of time
9 hr x (3600sec/1hr) + 25 m x (60s/1min) + 15s

Problem 8 - 23003 seconds in time = __**6**__ hrs _**23**__ min __**23**__ sec

Problem 9 – 4π square radians = ____**41,253**____ square degrees.
4 x (3.14159) rad^2 x (57.2958 deg/ 1rad) x (57.2958 deg / 1 rad)

Problem 10 - 1888 arcseconds = ____**0.524**____ degrees

1888 asec x (1 deg / 3600 asec)



Your hand can be used to measure large angular Separations in the sky when held at arms-length. Although suitable for navigating around the constellations ,individual planets, stars and galaxies are much smaller in angular size, so astronomers use different angle units. (Credit: NASA/CXC/M.Weiss)

We are all familiar with the degree as the basis for angle measure with a protractor, but astronomers rarely use angle measures in these units. Although astronomers study vast reaches of space, the objects they study are often hundreds or thousands of times smaller than a degree. Because no astronomer wants to use millidegrees or microdegrees in describing the apparent sizes of objects they see, instead we use the ordinary units of degrees smaller than a degree.

Thanks to the base-60 system we inherited from ancient Babylonians, we use the angle units of arc minutes and arcseconds where

1 degree = 60 arcminutes
1 arcminute = 60 arcseconds.

We also use radian units where

1 radian = 57.295833 degrees

Problem 1 – How many arcseconds are in 1.0 degrees?

Problem 2 – How many radians are in 360 degrees?

Problem 3 – The sun appears 0.5244 degrees in diameter as viewed from Earth. How many arcseconds is this?

Problem 4 – As viewed from the Voyager 1 spacecraft, the sun appears to have a diameter of 0.00441 degrees. How many arcseconds is this?

Problem 5 – The Transit of Venus in 2012 will travel along a chord on the solar disk that is 1528 arcseconds long. What percentage of the diameter of the sun's disk will this be, if the solar diameter at that time is 31.46 arcminutes?

Problem 1 – How many arcseconds are in 1.0 degrees?

Answer: 1.0 degrees x (60 minutes/ 1 degree) x (60 seconds / 1 minute
3600 arcseconds.

Problem 2 – How many radians are in 360 degrees?

Answer: 360 degrees x (1 radian / 57.295833 degrees) = **6.28318 = 2π**

Problem 3 – The sun appears 0.5244 degrees in diameter as viewed from Earth. How many arcseconds is this?

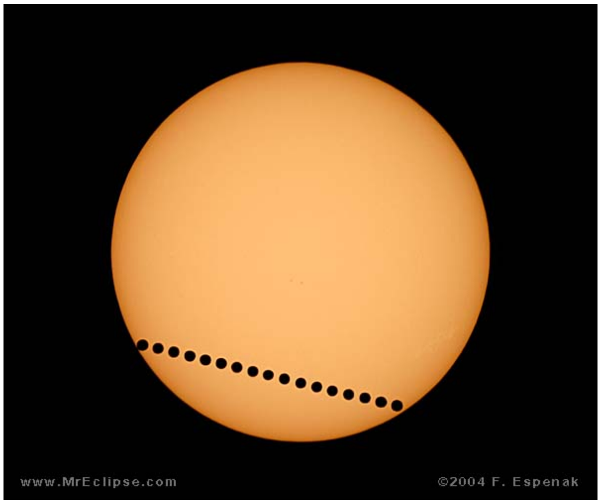
Answer: 0.5244 degrees x (3600 arcsec / 1 degree) = **1888 arcseconds.**

Problem 4 – As viewed from the Voyager 1 spacecraft, the sun appears to have a diameter of 0.00441 degrees. How many arcseconds is this?

Answer:
0.00441 degrees x (3600 arcsec / 1 degree) = **15.87 arcseconds**.

Problem 5 – The Transit of Venus in 2012 will travel along a chord on the solar disk that is 1528 arcseconds long. What percentage of the diameter of the sun's disk will this be, if the solar diameter at that time is 31.46 arcminutes?

Answer: 31.46 arcminutes x (60 arcsec / 1 arcmin) = 1888 arcsec.
Transit chord = 100% x (1528/1888) = **80.9 %**



2004 Transit of Venus showing different angular sizes For the sun and venus disks as viewed from Earth. (Courtesy Fred Espenak)

When you look at the sky, you do not know how far away the many different objects are so you cannot figure our how big they might be in meters or kilometers. What you can measure, however, is the angular size of objects and their distance apart in the sky in terms of angular measure. No matter what you are looking at, you can directly determine angular distances and angular sizes.

At the time of the Transit of Venus on June 5/6 2012, as viewed from the distance of Earth, the angular diameter of the sun will be 0.5244 degrees.

Problem 1 – To the nearest second of arc, what is the diameter of the sun, whose diameter is 0.5244 degrees?

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Problem 2 - The diameter of Venus is known to be 12,600 km. How far from Earth, D, in kilometers would Venus have to be so that its observed angular diameter is 58.6 arcseconds if the formula relating angular size and distance is given by:

$$\theta = \frac{722000}{D}$$

where θ is in degrees, and D is the distance from Earth to Venus in kilometers?

Problem 3 – In Kepler's model for the solar system, Earth would be at a distance from the sun of 1.0 Astronomical Units, and Venus would be at a distance of 0.28 Astronomical Units from Earth. From your answer to Problem 2, what is the distance corresponding to 1.0 AU in kilometers?

Problem 1 – To the nearest second of arc, what is the diameter of the sun, whose diameter is 0.5244 degrees?

Answer: 1 degree = 60 arc minutes, and 1 arc minute = 60 arcseconds, so the unit conversion is:

0.5244 degrees x (60 minutes/1 degree) x (60 seconds/1 minute) = 1887.84 arcseconds. Rounded to the nearest arcsecond you get **1888 arcseconds**.

Problem 2 - The diameter of Venus is known to be 12,600 km. How far from Earth, d, in kilometers would Venus have to be so that its observed angular diameter is 58.6 arcseconds if
Q = 722000/dkm. Where q is in degrees.

Answer: The angular diameter of Venus in degrees is just
Q = 58.6 arcseconds x (1 arcminute/60 arcseconds) x (1 degree / 60 arcminutes)
= 0.0163 degrees.
Then 0.0163 = 722000/d so d = **44.3 million kilometers**.

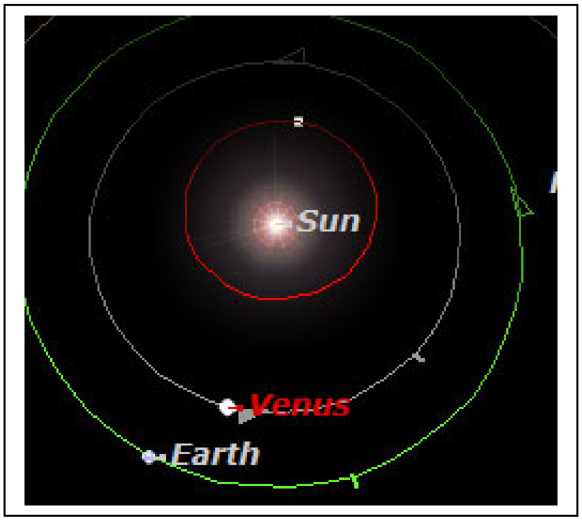
Problem 3 – In Kepler's model for the solar system, the Earth would be at a distance from the sun of 1.0 Astronomical Units, and Venus would be at a distance of 0.28 Astronomical Units from Earth. From your answer to Problem 2, what is the distance corresponding to 1.0 AU in kilometers?

Answer: Solve by using proportions:

44.3 million km / 0.28 AU = X / 1 AU so

X = **158 million kilometers**.

Transit of Venus – Relative orbit speed



Both Venus and Earth orbit the sun, but at different speeds along their orbits.

We can determine their orbital speeds from their distances from the sun and the length of each planet's 'year'. Here is the basic orbital data:

	Distance	Period
Earth	1.00 AU	365.24 days
Venus	0.72 AU	224.7 days

Problem 1 – Each planet has an orbit that is nearly a perfect circle centered on the sun, with a radius given in the table above. What is the circumference of each planet's orbit in kilometers if 1 AU = 149 million km?

Problem 2 – What is the orbit period for each planet in seconds?

Problem 3 – What are the orbital speeds of each planet along the orbit's circumference in kilometers per second?

Problem 4 – When Venus is exactly between the sun and Earth, how much faster is it traveling along its orbit relative to earth's speed at that moment in kilometers per second?

Problem 1 – Each planet has an orbit that is nearly a perfect circle centered on the sun, with a radius given in the table above. What is the circumference of each planet's orbit in kilometers if 1 AU = 149 million km?

Answer: Earth: $C = 2\pi R$, so
$C = 2 (3.141) \times (1.00\times 149$ million km)
C = 936 million km

Venus: $C = 2 (3.141) \times (0.72 \times 149$ million km)
C = 674 million km

Problem 2 – What is the orbit period for each planet in seconds?

Answer: Earth: P = 365.24 days x (24 h / 1 day) x (3600 seconds / 1 hour)
P = 31.6 million seconds

Venus: P = 224.7 days x (24 h / 1 day) x (3600 seconds / 1 hour)
P = 19.4 million seconds

Problem 3 – What are the orbital speeds of each planet along the orbit's circumference in kilometers per second?

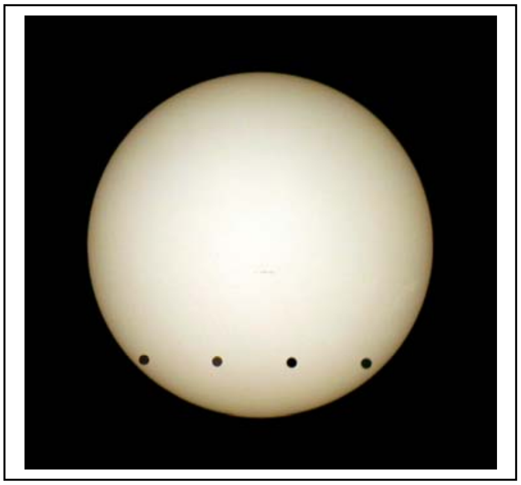
Answer; Venus = Distance/Time
= 674 million km / 19.4 million seconds
= **34.7 km/sec.**

Earth: Distance/Time
= 936 million km / 31.6 million seconds
= **29.6 km/sec.**

Problem 4 – When Venus is exactly between the sun and Earth, how much faster is it traveling along its orbit relative to earth's speed at that moment in kilometers per second?

Answer: 34.7 - 29.6 = **5.1 kilometers/sec.**

Transit of Venus – Duration of the Transit



Both Venus and Earth orbit the sun, but at different speeds along their orbits. We can determine their orbital speeds from their distances from the sun and the length of each planet's 'year'. Here is the basic data:

	D	P	V
Earth	1.00 AU	365.24 days	34.7
Venus	0.72 AU	224.7 days	29.6

where D is the distance to the sun in Astronomical Units; P is the orbit period in Earth days; V is the orbital speed in km/s

Problem 1 - As seen from Earth when Venus is directly between Earth and the sun, how much faster is Venus traveling than Earth?

Problem 2 – The photo above was taken by amateur astronomer Leif Moeller (EUC/ Syd) and is a combination of four images, each taken at 100 minute intervals. The composite picture shows how far Venus moved across the disk of the sun. If the diameter of the sun at that time was 1891 arcseconds, how many arcseconds did the disk of Venus travel in this time interval?

Problem 3 – From your answer to Problem 1 and 2, how many kilometers did Venus travel in this 1-hour time interval?

Problem 4 - The formula relating the angular size of an object to its actual diameter, d, and distance, D, is given by the formula

$$\theta = 57.296\frac{d}{D}$$

where θ is in degrees, and d and D are in kilometers. Use this formula, and your answers to problem 2 and 3 to estimate the distance from Earth to Venus.

Problem 5 – The distance to Venus from Earth at that time is given by Kepler's model for the solar system as 0.28 Astronomical Units. What is the length of an Astronomical Unit in kilometers? (Note 1.0 AU is the distance from the sun to Earth).

Problem 1 - As seen from Earth when Venus is directly between Earth and the sun, how much faster is Venus traveling than Earth?

Answer: The relative speed as viewed from Earth is just 34.7 - 29.6 = **5.1 km/s**

Problem 2 – The photo above was taken by amateur astronomer Leif Moeller (EUC/ Syd) and is a combination of four images, each taken at 100 minute intervals. The composite picture shows how far Venus moved across the disk of the sun. If the diameter of the sun at that time was 1891 arcseconds, how many arcseconds did the disk of Venus travel in this time interval?

Answer; Use a millimeter ruler to determine the angular scale of the image in arcseconds per millimeter, then measure the distance between Venus disk centers to get the angular distance traveled. When printed, the separation between discs is about 10 millimeters. The diameter of the sun is 46 mm, so the scale is 1891 arcsec/41 mm = 41.1 arcsec/mm. The separation between Venus discs is then 10 mm x 41.1 asec/mm = **411 asec**.

Problem 3 – From your answer to Problem 1 and 2, how many kilometers did Venus travel in this 1-hour time interval?

Answer: 5.1 km/s x 100 minutes x (60 sec/1 minute) = 30,600 km.

Problem 4 - The formula relating the angular size of an object to its actual diameter, d, and distance, D, is given by the formula

$$\theta = 57.296 \frac{d}{D}$$

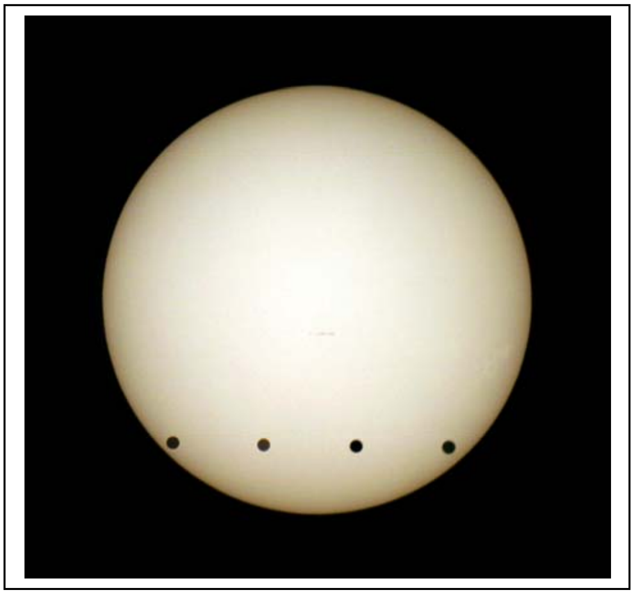
where θ is in degrees, and d and D are in kilometers. Use this formula, and your answers to problem 2 and 3 to estimate the distance from Earth to Venus.

Answer: q = 411 arcsec x (1 arcmin/60 arcsec) x (1 degree / 60 arcmin)
= 0.114 degrees.

Then from the formula, 0.114 = 57.296 x (30,600 km / D)
D = 57.296 x 30600/0.114
D = 15.4 million km.

Problem 5 – The distance to Venus from Earth at that time is given by Kepler's model for the solar system as 0.28 Astronomical Units. What is the length of an Astronomical Unit in kilometers? (Note 1.0 AU is the distance from the sun to Earth).

Transit of Venus – Angular distance and speed



As an object moves through the sky, astronomers can accurately measure the angular speed of the object; a quantity measured in terms of degrees per hour, arcminutes per second, or some other angle/time unit.

The photo to the left was taken by amateur astronomer Leif Moeller (EUC/ Syd) and is a combination of four images, each taken at 100 minute intervals. The composite picture shows how far Venus moved across the disk of the sun. The diameter of the sun at that time was 1891 arcseconds.

Problem 1 – From the angular information provided, what was the angular distance traveled by Venus between each pair of Venus disks, in units of arcseconds?

Problem 2 – How many seconds of time elapsed between the pairs of images?

Problem 3 – What was the angular speed of Venus as it passed across the face of the sun?

Problem 4 – Draw a line through the centers of the Venus disks that touches either edge of the sun's disk. How many minutes did the complete transit take from one edge of the solar dist to the other?

Problem 1 – From the angular information provided, what was the angular distance traveled by Venus between each pair of Venus disks, in units of arcseconds?

Answer: Use a millimeter ruler to measure the diameter of the sun and determine the scale of the image in arcseconds/mm. For typical copies, the diameter is about 57 mm, so the scale is 1891 arcseconds/57 mm = 33.2 arcsec/mm. Then the distance traveled by Venus is 12.5 mm or 12.5 x 33.2 = **415 arcseconds**.

Problem 2 – How many seconds of time elapsed between the pairs of images?

Answer: The images were taken 100 minutes apart, so the time interval is 100 x 60 = **6000 seconds**.

Problem 3 – What was the angular speed of Venus as it passed across the face of the sun in A) Arcseconds/second? B)Arcseconds/minute?

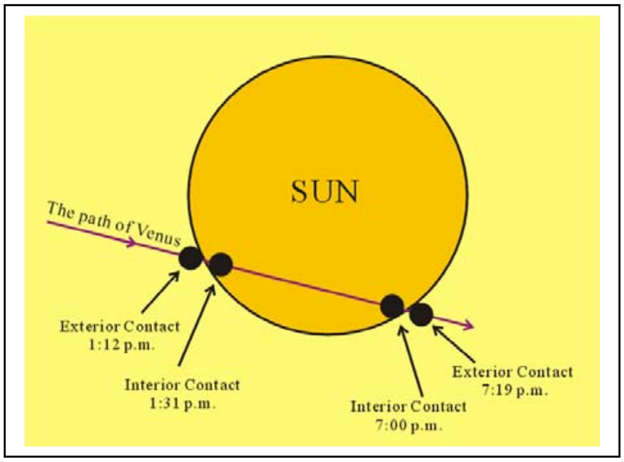
Answer: A) Angular speed = 415 arcseconds/6000 seconds
= **0.069 arcseconds/second.**

B) = 0.069 x (60 seconds/1 minute) = **4.1 arcseconds/minute**.

Problem 4 – Draw a line through the centers of the Venus disks that touches either edge of the sun's disk. How many minutes did the complete transit take from one edge of the solar dist to the other?

Answer; The line is about 42 millimeters long ,so L = 42 mm x 33.2 asec/mm and so L = 1394 arcseconds. At an angular speed of 4.1 asec/minute the duration was about T = 1394/4.1 = **340 minutes** (or 5.7 hours).

Transit of Venus – Transit durations



The time it takes for the Transit of Venus to begin and end depends on the angular speed of Venus across the face of the sun, and the angular length of the particular chord it takes.

Fill in the missing information in the table below for each of the transits of Venus that have been observed since 1631. Assume that the speed of Venus across the sun disk is the same in all calculations.

Transit	Start Time (UT)	End Time (UT)	Chord Length (Arcseconds)	Transit Duration
1631	04:35	06:13	412	
1639	14:06	21:45		7.5 hours
1761	02:11	08:27	1504	376 minutes
1769	19:24	01:26	1448	
1874	02:04	06:11		14820 seconds
1882	14:07	20:05	1411	5.88 hours
2004	05:23	11:17		0.246 days
2012	22:18	04:40	1528	382 minutes

Note: Universal Time (UT) is given in the 24-hour clock format so that '17:43' is 4:43 PM. Also, 1 hour = 3600 seconds, 1 day=24.0 hours, and 1 degree of arc = 3600 seconds of arc.

Note: In these calculations, the start of the transit for purposes of estimating chord lengths occurs half-way between First and Second Contact. The time of the end of the transit occurs mid-way between Third and Fourth Contact.

Transit	Start Time (UT)	End Time (UT)	Chord Length (Arcseconds)	Transit Duration
1631	04:35	06:13	412	**103 minutes**
1639	14:06	21:45	**1800**	7.5 hours
1761	02:11	08:27	1504	376 minutes
1769	19:24	01:26	1448	**362 minutes**
1874	02:04	06:11	**988**	14820 seconds
1882	14:07	20:05	1411	5.88 hours
2004	05:23	11:17	**1416**	0.246 days
2012	22:18	04:40	1528	382 minutes

Answer: Use the worked example for 2012 to find the transit angular speed:

1528 arcseconds/ 382 minutes = 4.0 arcseconds/minute.

Then 412 arcsec corresponds to 412/4 = **103 minutes**.

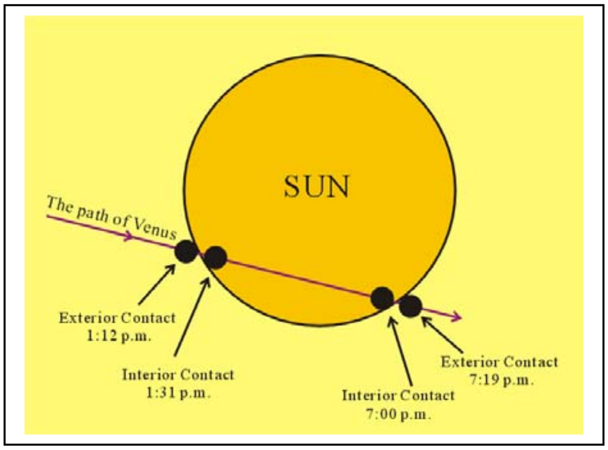
7.5 hours = 7.5 x 60 minutes = 450 minutes x 4 arcsec/min = **1800 arcsec**.

1448 arcseconds / 4 asec/min = **362 minutes**.

14820 seconds x 1/60 = 247 minutes x 4 asec/min = **988 arcseconds**.

0.246 days = 354 minutes x 4 asec/min = **1416 arcseconds**.

Transit of Venus – Working with Time



Astronomers who study planetary transits what to use a common clock to refer to events during the transit. For instance, from left to right in the diagram, we have First Contact, Second Contact, Third Contact and Fourth Contact, also referred to by the terms Exterior and Interior Contact.

A convenient time base is Universal Time, which is essentially the same as the local time (GMT) in Greenwich England at 0-degrees Longitude. This time-keeping is done on the 24-hour or 'Zulu' clock so, for example, 3PM GMT is just 15:00 UT.

Problem 1 – If First Contact for the transit started at 22:09 UT on June 5, 2012 and Fourth Contact ends at 04:49 UT on June 6, 2012, how many minutes did the transit last?

Problem 2 - An accurate estimate for the chord length of the transit is found by taking the average of the Third and Fourth contacts, and subtracting this from the average of the First and Second Contacts. If the Second and Third Contacts began at 22:27 and 04:32 UT, what is this transit duration in minutes?

Problem 3 - If the angular speed of Venus across the sun's disk is 4.0 arcseconds/minute, how many arcseconds long was the transit chord?

Problem 4 - The midpoint of the chord is connected to the center of the circular sun disk by a perpendicular line segment of length H. What is the length of H if the diameter of the sun at the time of the transit is 1888 arcseconds?

Problem 1 – If First Contact for the transit started at 22:09 UT on June 5, 2012 and Fourth Contact ends at 04:49 UT on June 6, 2012, how many minutes did the transit last?

Answer: We have to subtract 04:49 on June 6 from 22:09 on June 5

So add 24 hrs to the time on June 6 to get: 28:49 - 22:09, then do the time subtraction the normal way to get 6 hours and 40 minutes. Then 6 hours = 360 minutes, and so the total time is **400 minutes**.

Problem 2 - An accurate estimate for the chord length of the transit is found by taking the average of the Third and Fourth contacts, and subtracting this from the average of the First and Second Contacts. If the Second and Third Contacts began at 22:27 and 04:32 UT, what is this transit duration in minutes?

Average Third and Fourth: (04:32 + 04:39)/2 = 04:36 UT
Average First and Second: (22:09 + 22:27)/2 = 22:18 UT

Then 04:36 UT - 22:18 UT = 06:18 so 6 x 60 + 18 = **378 minutes**.

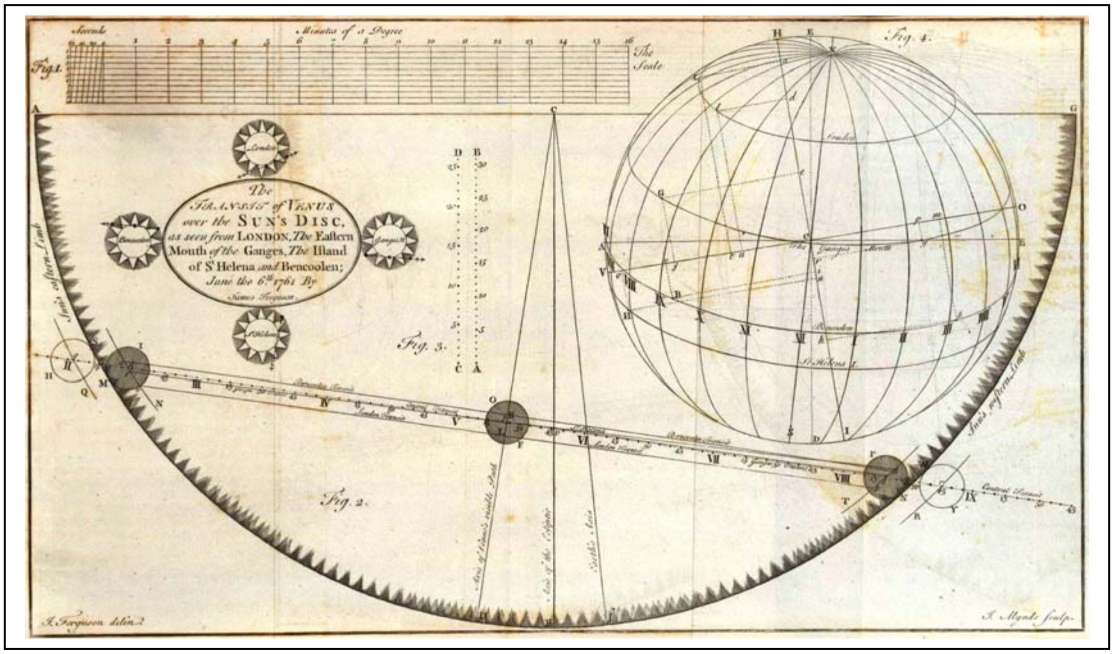
Problem 3 - If the angular speed of Venus across the sun's disk is 4.0 arcseconds/minute, how many arcseconds long was the transit chord?

Answer: The chord length L = 378 minutes x 4.0 arcseconds/minute
= **1512 arcseconds**.

Problem 4 - The midpoint of the chord is connected to the center of the circular sun disk by a perpendicular line segment of length H. What is the length of H if the diameter of the sun at the time of the transit is 1888 arcseconds?

Answer: The right triangle has a hypotenuse equal to the sun' radius of 1888/2 = 944 arcseconds. The length of the other side is 1512/2 = 756 arcseconds. Use the Pythagorean Theorem to determine H by solving:

$H^2 + (756)^2 = (944)^2$ so $H^2 = 319600$ and H = **565 arcseconds**.



When observers located in different parts of the world view the transit, they will see the path follow a slightly different chord on the solar disk. This is a result of the Parallax Effect.

If you place your thumb at arm's length and then alternately open one eye and close the other, you will see your thumb jump from side to side against the background furniture in your room. By measuring a similar effect as Venus transits the sun, as shown in the figure above from the 1761 transit, astronomers can determine the distance to Venus from earth. The first step in finding out the actual size of our solar system.

Problem 1 - Two teams of astronomers measure the start and end times of their two chords on the sun as: Team A: 11:25:30 UT and 16:40:11 UT and Team B: 11:30:10 and 16:38:15. If Venus travels across the sun at a constant angular speed of 4.0 arcseconds/minute, what are the lengths of the two transit chords observed by the two teams?

Problem 2 - If the diameter of the sun at the time of the transit is 1850 arcseconds, what is the length, in arcseconds, of the perpendicular bisector that connects each chord with the center of the sun?

Problem 3 - What is the difference in the lengths of the perpendicular chord bisectors?

Problem 4 - What percentage of the disk of Venus, with an angular diameter of 55 arcseconds, is the parallax difference in Problem 3?

Problem 1 - Two teams of astronomers measure the start and end times of their two chords on the sun as: Team A: 11:25:30 UT and 16:40:11 UT and Team B: 11:30:10 and 16:38:15. If Venus travels across the sun at a constant angular speed of 4.0 arcseconds/minute, what are the lengths of the two transit chords observed by the two teams?

Answer: Team A: 16:40:10 UT - 11:25:30 UT = 5:14:40 or 315 minutes.
Team B: 16:38:15 UT - 11:30:10 UT = 05:08:05 or 308 minutes.

Chord Lengths Team A = 315 minutes x 4 arcsec/min = **1260 arcseconds.**
Team B = 308 minutes x 4 arcsec/min = **1232 arcseconds.**

Problem 2 - If the diameter of the sun at the time of the transit is 1850 arcseconds, what is the length, in arcseconds, of the perpendicular bisector that connects each chord with the center of the sun?

Answer: Use the Pythagorean Theorem. Solar radius = 1850/2 = 925 arcsec.

Team A: $(925)^2 = (630)^2 + H^2$ so H = **677 arcseconds**.
Team B: $(925)^2 = (616)^2 + H^2$ so H = **690 arcseconds**.

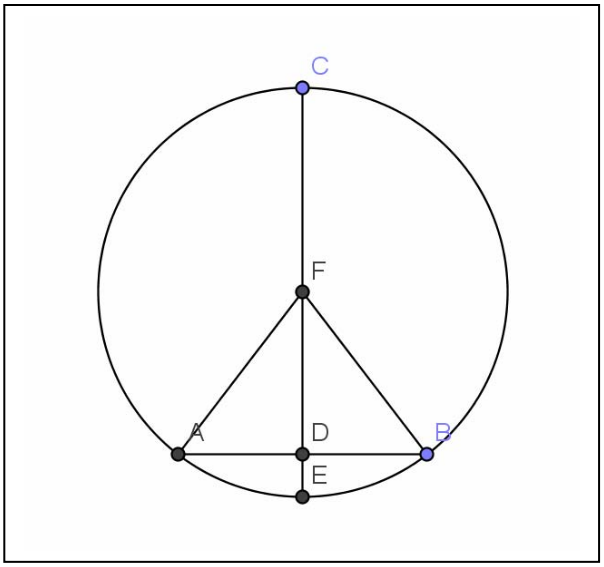
Problem 3 - What is the difference in the lengths of the perpendicular chord bisectors?

Answer: 690 - 677 = **13 arcseconds**.

Problem 4 - What percentage of the disk of Venus, with an angular diameter of 55 arcseconds, is the parallax difference in Problem 3?

Answer: This is only 100% x (13/55) = 24%, or one-quarter the diameter of the disk of Venus!

Note: This is why this parallax method requires great care in setting up the measuring equipment and taking the time measurements. At the speed of the disk of Venus of 4 arcseconds/minute, this 13 arcseconds is equal to about 3 minutes of time, so time errors this large can make it hard to see this small shift.



To measure the parallax shift observed between two observers watching the same transit, an important theorem in geometry needs to be proven.

"Any chord of a circle is connected to the center of its circle by a perpendicular bisector."

Once we have proved this, we can determine the exact parallax shift of any transit chord just by measuring the distance between its midpoint and the center of the solar disk, which in the diagram to the left is just segment FD.

Problem 1 - Prove that Triangle ADF is similar to Triangle FDB.

Problem 2 - Prove that Angle FDA = Angle FDB.

Problem 3 - Prove that Angle FDA = 90 degrees

Quad Erat Demostratum!

Problem 1 - Prove that Triangle ADF is similar to Triangle FDB.

AF = FB because they are the radii of the circle.
DF = DF because they are an identity.
AD = DB because the chord is bisected and these are the resulting segments

Using Side-Side-Side ,the triangles are, therefore, similar.

Problem 2 - Prove that Angle FDA = Angle FDB.

Because the triangles are similar, all of the corresponding angles are equal to each other, so Angle FDA must equal Angle FDB.

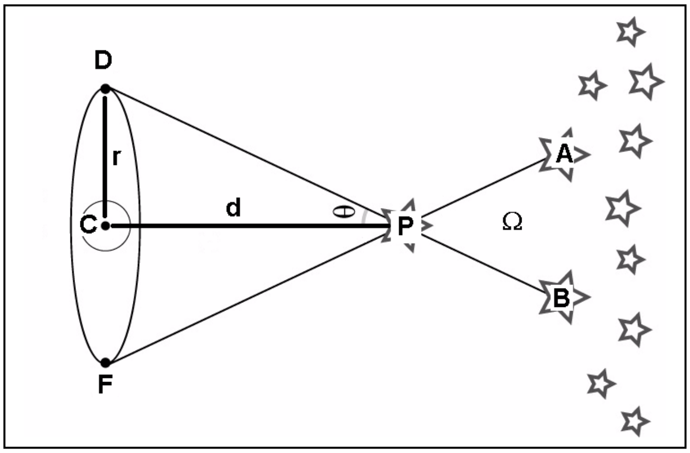
Problem 3 - Prove that Angle FDA = 90 degrees

Because mAngle FDA = mAngle FDB, and
mAngle FDA + mAngle = 180 degrees, Angle FDA = 90.

So, if we erect a perpendicular bisector of a chord, it will pass through the center of the circle

Conversly, any bisector of a chord that passes through the center of a circle will be a perpendicular bisector.

And also, any perpendicular line from a chord that passes through the center of its circle will be perpendicular to its chord.



If you place your thumb out at arm's length, and alternately open and shut each eye, you will see it move from side to side. This is called the Parallax Effect. The simple geometry of this effect is a powerful means for determining the distance to an object if you know what the Parallax Angle is, and the length of the baseline between the two observations.

In the figure above, the parallax angle **Ω** = mAPB that you measure between images A and B is related to other elements of the triangle formed by sightings from Station D and F, and the foreground object, P. To make this method work, you must be sure that, as you are blinking your eyes, or making the two sightings shown in the diagram at Stations D and F, that the background reference objects are so far away they do not move. If they do, you will measure a **smaller** angle than the true parallax shift, Ω, and so you will think the object is **farther away** than it really is.

Important note: It is almost always the case that when discussing the 'parallax angle' that you have to make certain you know whether it is the angle, θ, or the parallax shift angle $\Omega=2\theta$ that is being discussed!

Problem 1 - Suppose an object, P, is at a distance of d kilometers from the mid-point, C, between the two stations, and you observe it from two stations separated by 2r kilometers. What is the trigonometric relationship between the parallax shift angle Ω, d and r?

Problem 2 - Suppose that object P was very far away from C so that the angle, θ, was very small. Prove that for the 'skinny triangle limit', that the following two formulae are equivalent if d and r are in the same distance units:

Problem 3 - An astronomer spots an asteroid with a diameter of 100 kilometers, at a distance of 20 million kilometers, and a 10-meter long Near-Earth asteroid at a distance of 2,000 kilometers. Show that the apparent angular diameters of these bodies is about the same.

Problem 1 - Suppose an object, P, is at a distance of d kilometers from the mid-point, C, between the two stations, and you observe it from two stations separated by 2r kilometers. What is the trigonometric relationship between the parallax shift angle Ω, d and r?

Answer: $Tan\left(\frac{\Omega}{2}\right) = \frac{r}{d}$ or $Tan\theta = \frac{r}{d}$

Problem 2 - Suppose that object P was very far away from C so that the angle, θ, was very small. Prove that for the 'skinny triangle limit', that the following two formulae are equivalent if d and r are in the same distance units:

$\theta = 57.2958\frac{r}{d}$ for θ in degrees, $\theta = 206265\frac{r}{d}$ for θ in arcseconds

Answer:

Begin with $Tan\theta = \frac{r}{d}$ and re-write as $\frac{\sin\theta}{\cos\theta} = \frac{r}{d}$

In the limit where θ in **radians** becomes very small, we have cosθ = 1.0 and sinθ = θ

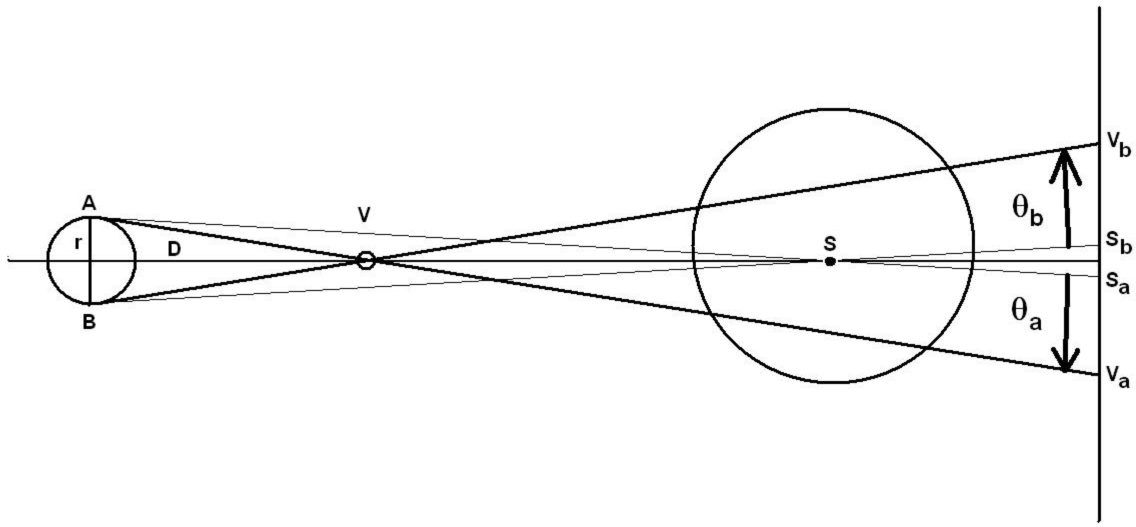
Then $\theta = \frac{r}{d}$ where θ is in radians. Since there are 2π radians in 360 degrees of arc measure, we have 1 radian = 57.2958 degrees, so that the first equation obtains from this.

For the second equation, 1 degree = 3600 arcseconds, so
57.2958 degrees x (3600 arcsec/1 degree) = 206264.88 or 206265 rounded ,and so the second equation is obtained.

Problem 3 - An astronomer spots an asteroid with a diameter of 100 kilometers, at a distance of 20 million kilometers, and a 10-meter long Near-Earth asteroid at a distance of 2,000 kilometers. Show that the apparent angular diameters of these bodies is about the same.

Answer: Body 1: θ = 206265 x (100 km/20 million km) = **1.0 arcseconds.**

Body 2: θ = 206265 x (10 meters /2,000,000 meters) = **1.0 arcseconds**.



When viewing a transit of an inferior planet such as Mercury or Venus from Earth, we see the planet pass across the face of the solar disk. An important method for determining the distance to these planets is to use the Parallax Method. By observing the planet from two distant places on Earth, the apparent location of the planet will shift relative to the center of the solar disk. This angular shift, Ωo = mAngle $V_b V V_a$ can then be used, together with the known distance between the observers, r, to solve for the distance to the planet, D, using the formula

$$\mathrm{Tan}(\Omega/2) = r/D$$

The answer you would get would be wrong, because the position of the Sun's center is also subject to the Parallax Shift, which is indicated by the angle Ωs=mAngleS_b S S_a in the diagram, and you have to take this into account in determining the actual parallax angle Ωa.

The diagram above shows the geometry of the Transit of Venus. Venus is located D=0.28 Astronomical Units from Earth. The Earth is located exactly 1.00 Astronomical Units from the sun.

Problem 1 - Suppose that two observers are located one full Earth diameter apart (r = 6378 km). If the parallax angle for the center of the sun given by Ωs as viewed from Earth between these two stations is Ωs = 17.658 arcseconds, and the apparent parallax angle, Ωo, for Venus is 45.4 arcseconds, what would the true parallax angle, Ωa, be with respect to the center of the sun?

Problem 1 - Suppose that two observers are located one full Earth diameter apart (r = 6378 km). If the parallax angle for the center of the sun given by Ωs as viewed from Earth between these two stations is Ωs = 17.658 arcseconds, and the apparent parallax angle, Ωo, for Venus is 45.4 arcseconds, what would the true parallax angle, Ωa, be with respect to the center of the sun?

Answer: Let's first consider what Observer A at the North Pole will be seeing. They will see the disk of Venus shift its position southwards on the face of the sun by 31.53 arcseconds while the center of the sun also shifts southwards by 8.829 arcseconds. Its location at that point will be at a distance of q = 31.53 – 8.829 = 22.70 arcseconds south of the center of the sun.

Observer B at the South Pole will see Venus move northwards by 31.53 arcseconds, while the center of the sun moves northwards by 8.829 arcseconds. The position of the disk of Venus on the sun will be 31.53 – 8.829 = 22.70 arcseconds north of the center of the sun.

The total observed parallax angle, Ωo, will then appear to be 22.70 + 22.70 = 45.4 arcseconds when the observers combine their data and calculate the total shift they saw. The true parallax angle, if the sun did not move (was at a very, very great distance from Earth so that θs = 0.0) would be Ωa = 31.53 + 31.53 = 63.06 arcseconds.

So, to correct their measurements to allow for the solar parallax, the astronomers have to ADD an additional 63.06 – 45.4 = 17.66 arcseconds (= 2 x θs) to their observed value for Ω so Ωa = Ωo + 2θs

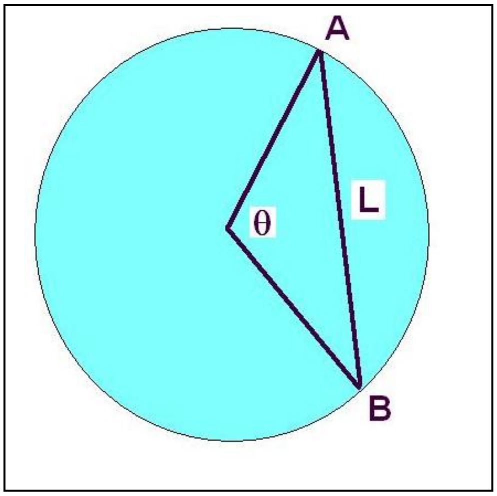
This means that the astronomers have to follow these steps.

Step 1 – Before the transit occurs, observe the solar disk from the same two stations and determine the solar parallax angle, θs. Since 1.0 AU = 149 million km, and the stations are separated by 2r = 12,756 km, the solar parallax angle is just θs = 206265 x (6378/149 million) = 8.83 arcseconds. If you do not know the solar distance before hand, compare the observations of the sun and determine this angle observationally. This will be hard unless, for instance, you can see background stars at the time of a total solar eclipse.

Step 2 - From each station, measure the distance of the disk of Venus from the center of the sun to get Ωo.

Step 3 – Add twice the solar parallax angle, θs = 8.83, to Ωo to get the true parallax angle Ωa.

Answer: Ωa = 45.4 + 2(8.83) = **63.06 arcseconds**, so θv = 31.53 arcseconds.



An essential ingredient of a parallax measurement is determining the distance between the two observers along an axis that is perpendicular to the line connecting the center of Earth and the center of the object. The formula for the length of the chordal distance between the observers is

$$L = 12756 \sin\left(\frac{\theta}{2}\right) \text{ kilometers}$$

where θ is the angle from the center of Earth separating the two observers on the surface, and r = 6378 km.

Problem 1 - Prove that the formula is correct from your knowledge of trigonometry.

Problem 2 - Suppose that two observers are located at the same longitude. Observer A is located at a latitude of θ_a in the Southern Hemisphere, and Observer B is located at a latitude of θ_b in the Northern Hemisphere. What is the formula for their chordal separation?

Problem 3 – Observer A is located in Fairbanks, Alaska (64d 54' North) while Observer B is located in Hilo, Hawaii (21d 18' North). What is the estimated baseline distance between these observers?

Problem 4 - Two observers want to be able to detect a Venus parallax shift of 35 arcseconds in the transit chords on the solar disk. To do this, they need a baseline that is about 10,000 kilometers long. If Observer A is located in Hilo, Hawaii (latitude +21d 18') at what latitude should Observer B be located?

Problem 1 - Prove that the formula is correct from your knowledge of trigonometry.

Answer: Construct a diagram showing a circle for Earth with R = 6378 km, and the locations of two points on the circumference. Connect the two points to the center of Earth. Draw a radial bisector of the arc angle θ formed by AB. Then

$$\frac{L}{2} = R\sin\left(\frac{\theta}{2}\right) \quad \text{so} \quad L = 2R\sin\left(\frac{\theta}{2}\right) \quad \text{and so} \quad L = 12756\sin\left(\frac{\theta}{2}\right) \text{ km}$$

Problem 2 - Suppose that two observers are located at the same longitude. Observer A is located at a latitude of θ_a in the Southern Hemisphere, and Observer B is located at a latitude of θ_b in the Northern Hemisphere. What is the formula for their chordal separation?

Answer. Southern hemisphere latitudes are negative so:

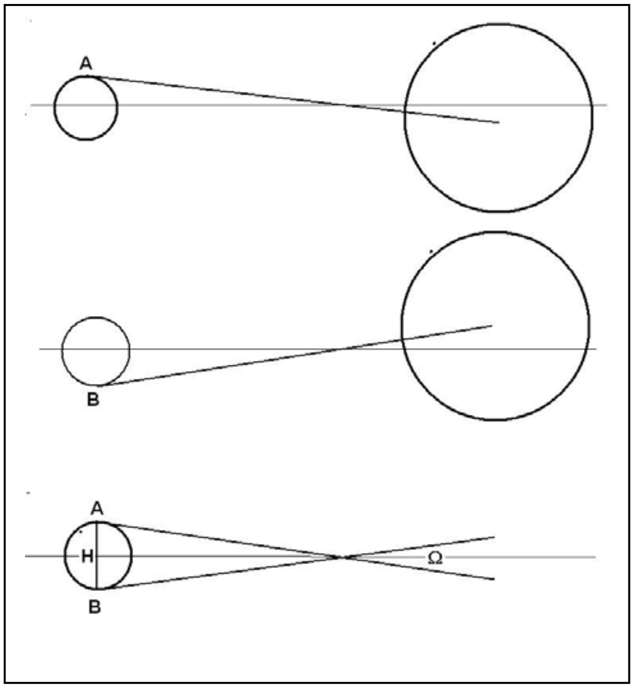
$$L = 12756\sin\left(\frac{\theta b - \theta a}{2}\right)$$

Problem 3 – Observer A is located in Fairbanks, Alaska (64d 54' North) while Observer B is located in Hilo, Hawaii (21d 18' North). What is the estimated baseline distance between these observers?

Answer; θ = 64.9 - 21.3 so q = 43.6 degrees, so $L = 12756\sin\left(\frac{43.6}{2}\right)$ and **L = 4,737 km.** (Note air travel distance is about 5038 km)

Problem 4 - Two observers want to be able to detect a Venus parallax shift of 35 arcseconds in the transit chords on the solar disk. To do this, they need a baseline that is about 10,000 kilometers long. If Observer A is located in Hilo, Hawaii (latitude +21d 18') at what latitude should Observer B be located?

Answer: Solve $10000 = 12756\sin\left(\frac{\theta}{2}\right)$ to get the latitude difference of q = 51.6 degrees, then latitude = 21.3 -51.6 = **30.3 South**, and 21.3 + 51.6= **72.9 North**.



As you look at the sun from two different locations on Earth, A and B, you will see the position of the sun in the sky shift by a very small angle, Ω, called the Parallax Angle.

For small angles less than a degree, the relationship of the Parallax Angle, Ω, to the separation of two observers A and B on Earth, H, is given by the formula:

$$\Omega = 206265\frac{H}{D}$$

where D is the distance from Earth to the Sun, and Ω is in seconds of arc.

Problem 1 – Two observers are located on the same longitude, but Observer A is at a latitude of 45d North, and Observer B is at a latitude of 45d South. If the radius of Earth is 6,378 km, and the sun is located 149 million km from Earth, A) what is the length of the baseline, H, between the two observers? B) What is the solar parallax angle, Ω, in arcseconds that they would observe?

Problem 2 - An observer is located in Fairbanks, Alaska at (64d 49' North, 147d 43' West) and Observer B is located in Hilo, Hawaii at (19d 43' North, 155d 05' West). A) What is the baseline distance between the two observers? B) What is the Solar Parallax angle in arcseconds?

Problem 3 – What is the maximum possible Parallax Angle from Earth's surface?

Problem 1 – Two observers are located on the same longitude, but Observer A is at a latitude of 45d North, and Observer B is at a latitude of 45d South. If the radius of Earth is 6,378 km, and the sun is located 149 million km from Earth, A) what is the length of the baseline, H, between the two observers? B) What is the solar parallax angle, Ω, in arcseconds that they would observe?

Answer:

A) h = 6378 sin(latitude) so H = 2 x 6378 sin(45) and so H = **9020 kilometers**.
B) Ω = 206265 (9020/149 million) so Ω = **12.5 arcseconds**.

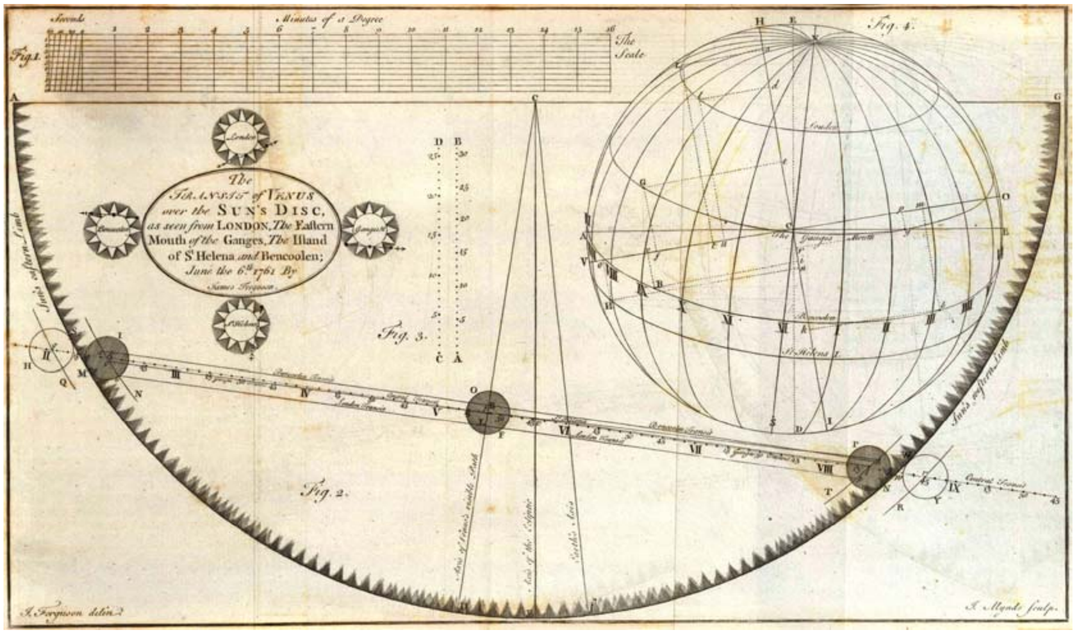
Problem 2 - An observer is located in Fairbanks, Alaska at (64d 49' North, 147d 43' West) and Observer B is located in Hilo, Hawaii at (19d 43' North, 155d 05' West). A) What is the baseline distance between the two observers? B) What is the Solar Parallax angle in arcseconds?

Answer:
A) Fairbanks: h = 6378 sin(latitude) so h = 6378 sin(64.8) and so h = 5770 km. Hilo: h = 6378 sin(19.7) so h = 6378 sin(19.7) and so h = 2150 km, and so **H = 7920 km**

B) Ω = 206265 (7920/149 million) so Ω = **11.0 arcseconds**.

Problem 3 – What is the maximum possible Parallax Angle from Earth's surface?

Answer H = 2 x 6378 with observers at the poles, then Ω = 206265 (12756/149 million) so Ω = **17.7 arcseconds**.



The apparent Parallax Angle, Ωo, for Venus at the time of the Transit of Venus is simply the perpendicular angular distance between the two transit chords observed by two observers stationed on Earth.

The lengths of the transit chords can be determined through the careful timing of the start and end of the transit, and the measured angular speed of Venus across the solar disk, or by drawing a line through the disks of Venus in photographs taken several hours apart.

The sketch above shows the two transit chords observed for the Transit of June 6, 1761.

Problem 1 – If the lengths of the two transit chords were determined to be 1471 and 1398 arcseconds respectively, and the solar diameter at the time of the observation was 1850 arcseconds, what is the angular difference, Ωo, in the perpendicular distances of the chords from the center of the sun? (Hint. Use the Pythagorean theorem)

Problem 2 – If the solar Parallax Angle for these two observers was Ωs=15 arcseconds, and the baseline between them was 11,000 kilometers, A) what was the true Parallax Angle for Venus, and B) the distance to Venus?

Problem 1 – If the lengths of the two transit chords were determined to be 1471 and 1398 arcseconds respectively, and the solar diameter at the time of the observation was 1850 arcseconds, what is the angular difference, Ωo, in the perpendicular distances of the chords from the center of the sun? (Hint. Use the Pythagorean theorem)

Answer: Solar radius = 1850/2 = 925 arcseconds
$(925)^2 = ha^2 + (1471/2)^2$ so ha = 561 arcseconds.

$(925)^2 = hb^2 + (1398/2)^2$ so hb = 606 arcseconds.

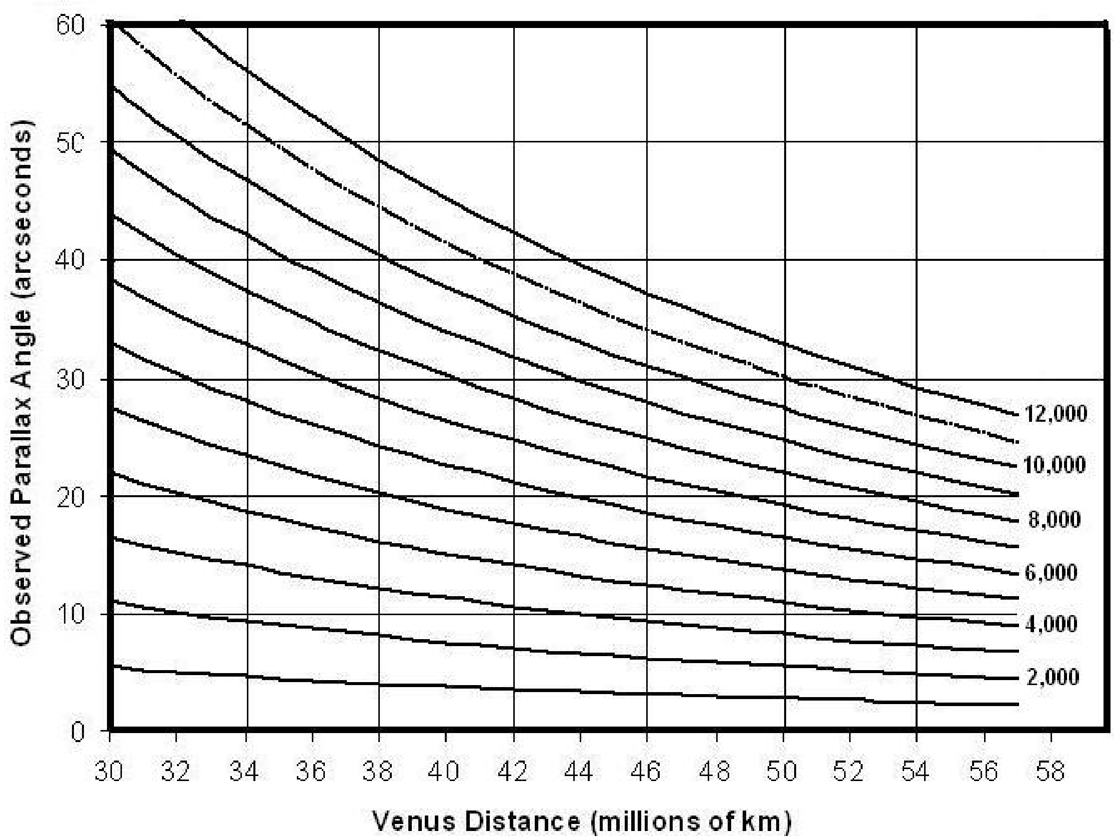
Then Ωo = 606 – 561 = **45 arcseconds**.

Problem 2 – If the solar Parallax Angle for these two observers was Ωs=15 arcseconds, and the baseline between them was 11,000 kilometers, A) what was the true Parallax Angle for Venus, and B) the distance to Venus?

Answer: A) Recall that $\Omega a = \Omega o + \Omega s$ so Ωa = 45.0 + 15.0 = **60 arcseconds**.

B) From the parallax formula: Ωa = 206265 (H/D) and since Ωa = 60 arcseconds, and H = 11,000 km, we get **D = 38 million km**.

Note: Although we do not have the exact numbers used by the observers of the 1761 transit of Venus, the estimated distance to Venus of 38 million kilometers, is only 10% different than the true distance of 42 million km.

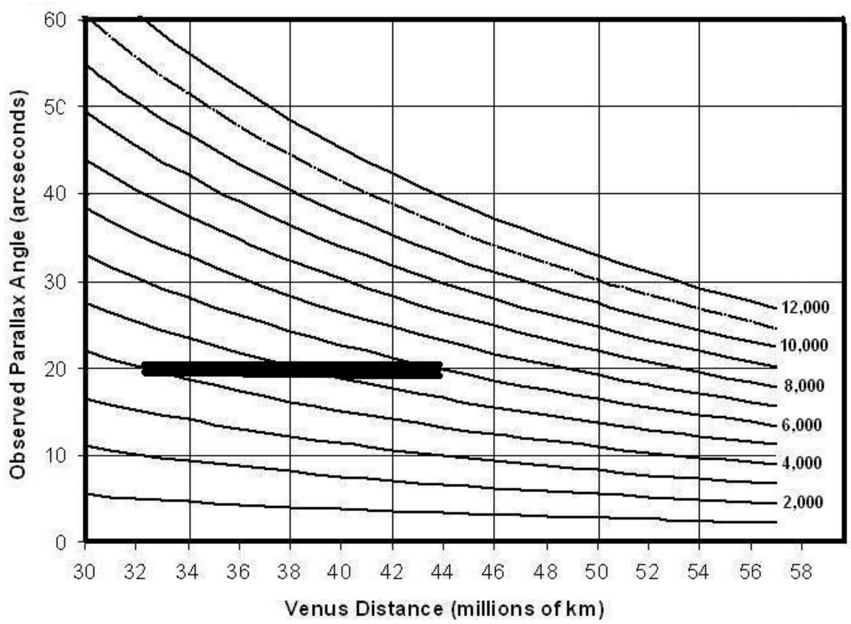


Depending on the distance between observers on Earth, the observed parallax angle, Ωo, will change depending on the distance between Earth and Venus. The graph above shows the relationship between baseline distance and observed Venus disk parallax on the solar disk for a range of baseline distances from 1,000 km to 12,000 km. For example, if two observers are separated by 9,000 km, and measure a parallax angle of about $\Omega o=50$ arcseconds between the transit chords, the distance to Venus will be about 30 million kilometers.

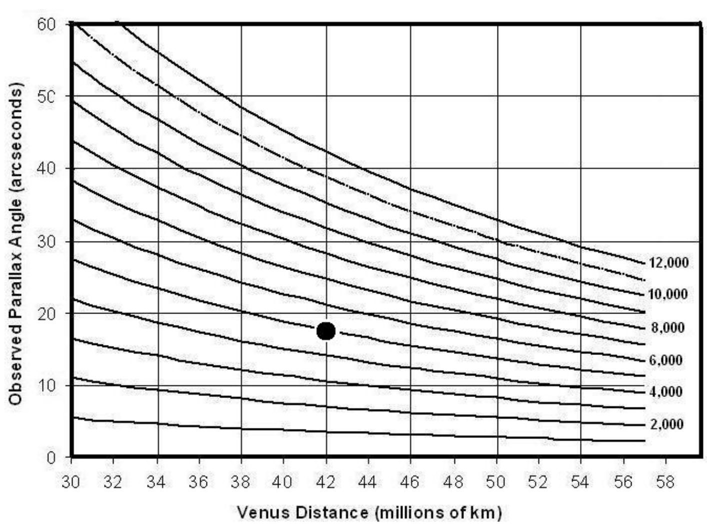
Problem 1 - Two observers can reliably measure a parallax angle between the transit chords of 20 arcseconds, and can observe over baseline separations from 4,000 to 6,000 kilometers. What is the range of distances to Venus that they can estimate using this observing set-up?
.

Problem 2 - Two observers are located in Fairbanks, Alaska and Hilo, Hawaii. The surface distance between these two locations is about 5000 kilometers. Using this as an estimated baseline distance, about what would they estimate for the distance to Venus for a parallax angle of 17 arcseconds?

Problem 1 - Two observers can reliably measure a parallax angle between the transit chords of 20 arcseconds, and can observe over baseline separations from 4,000 to 6,000 kilometers. What is the range of distances to Venus that they can estimate using this observing set-up? Answer: The horizontal bar shows this range to be **32 million to 42 million kilometers** for Y = 20 arcseconds and the ends falling on the curves for 4,000 km and 6,000 km.



Problem 2 - Two observers are located in Fairbanks, Alaska and Hilo, Hawaii. The surface distance between these two locations is about 5000 kilometers. Using this as an estimated baseline distance, about what would they estimate for the distance to Venus for a parallax angle of 17 arcseconds? Answer: **42 million kilometers.**



When two observers are located at the same longitude, the baseline distance between them is just the length of the chord whose angle subtends the sum of the observers latitudes. When observers are located at different longitudes, which is usually the case, the angle of the arc between them, called the Great Circle, can be calculated using spherical trigonometry.

For two observers located at the latitudes, α, and longitudes, β, of Observer A = (α_a, β_a) and Observer B = (α_b, β_b), the formula for the length of the surface arc, θ, between them is

$$\cos\theta = \sin\alpha_a \sin\alpha_b + \cos\alpha_a \cos\alpha_b \sin(b_b - b_a)$$

Once θ is known, we can use the chordal formula $L = 2r\cos\left(\frac{\theta}{2}\right)$

to estimate the baseline length where r is the radius of Earth in kilometers (r=6378 km).

Problem 1 - Observer A in Tokyo (α= 35d 42' N, β= 139d 48' E) wants to team up with Observer B in Rio de Janeiro (α= 22d 54 S, β= 43d 12' W) to determine the Venus parallax angle. About what is their baseline length?

Problem 1 - Observer A in Tokyo (α= 35d 42' N, β= 139d 48' E) wants to team up with Observer B in Rio de Janiero (α= 22d 54 S, β= 43d 12' W) to determine the Venus parallax angle. About what is their baseline length?

Answer: Observer A decimal degrees (+35.7, 139.8) Observer B (-22.9, 43.3)

We have to convert the Tokyo longitude into degrees West of the Prime Meridian to match the Rio de Janiero longitude naming convention so β = 360 - 139.8 = 220.2 West.

$$\cos\theta = \sin(35.7)\sin(-22.9) + \cos(35.7)\cos(-22.9)\sin(43.3 - 220.2)$$

So cosθ = -0.227 - 0.0405 and so θ = 74.8 degrees

Now we need to check for the 90 degree ambiguity because sin (176.9) is same as sin (3.1), so θ = 74.8 + 90 = 164.8 degrees

This angle measured in the surface of Earth is 164.8/360 = 0.457 of the circumference of Earth and corresponds to an arc distance (Great Circle distance) of 18,341 kilometers. Then from the chordal formula we get:

$$L = 12756\sin\left(\frac{164.8}{2}\right)$$

and so **L = 12,643 kilometers** is the chordal baseline separation.

Answer Key

Math Puzzler 1
The transit years are

1631		
1639	(1639-1631)	= 8 years
1761	(1761-1639)	= 122 years
1769	(1769 - 1761)	= 8 years
1874	(1874-1769)	= 105 years
1882	(1882-1769)	= 8 years
2004	(2004-1882)	= 122 years
2012	(2012-2004)	= 8 years

There are pairs of transits separated by 8 years, and then the pairs repeats in alternating patterns every 105 years and 122 years. The months alternate between June and December between pairs. Following this pattern, the next transits will be in the years 2117, 2125, 2247, 2255, 2360, 2368, 2490, 2498, 2603, 2611. During these years, the transit months will alternate December, December, June, June, December, December, etc.

Math Puzzler 2
Distance = 12000 km / sin(0.016) = **43 million km**.

Math Puzzler 3
Orbital speed of Earth = 29.78 km/s
Orbital speed of Venus = 35.02 km/s
Difference = tangential speed across sky viewed from Earth = 5.24 km/s
Distance traveled in 24,000 seconds along Venus orbit = 5.24 x 24000 = 126,000 km.
Apparent angular size of this distance = 0.17 degrees
Distance to Venus = 126,000 km / sin(0.17) = **42 million km**

Math Puzzler 4
Distance = 1.00 – 0.72 = 0.29 AU

Math Puzzler 5
Average Earth-Venus distance = (43 million + 42 million)/2 = **43 million km**

Math Puzzler 6

$$\frac{43 \text{ million}}{0.29 \text{ AU}} = \frac{X}{1.0 \text{ AU}}$$

so X = **148 million km**
actual value = 149 million km