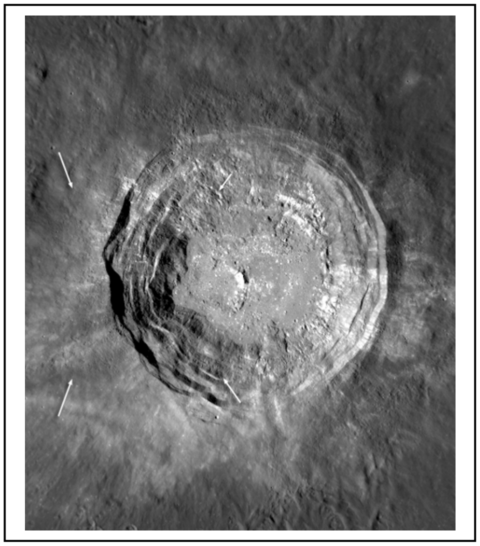
The volume of a lunar impact crater



Lunar craters have been excavated by asteroid impacts for billions of years. This has caused major remodeling of the lunar surface as the material that once filled the crater is ejected. Some of this returns to the lunar surface hundreds of kilometers from the impact site.

An example of a lunar crater is shown in the image to the left taken of the crater Aristarchus by the NASA Lunar Reconnaissance Orbiter (LRO).

Astronomers create models of the rock that was displaced that try to match the overall shape of the crater. The shape of a crater can reveal information about the density of the rock, and even the way that it was layered below the impact area.

One such mathematical model was created for a 17-kilometer crater located at lunar coordinates 38.4 °North, and 194.9 °West. For this particular crater, its depth, D, at a distance of x from its center can be approximated by the following 4th-order polynomial:

$$\mathbf{D = 0.0001x^4 - 0.0055x^3 + 0.0729x^2 - 0.2252x - 0.493}$$

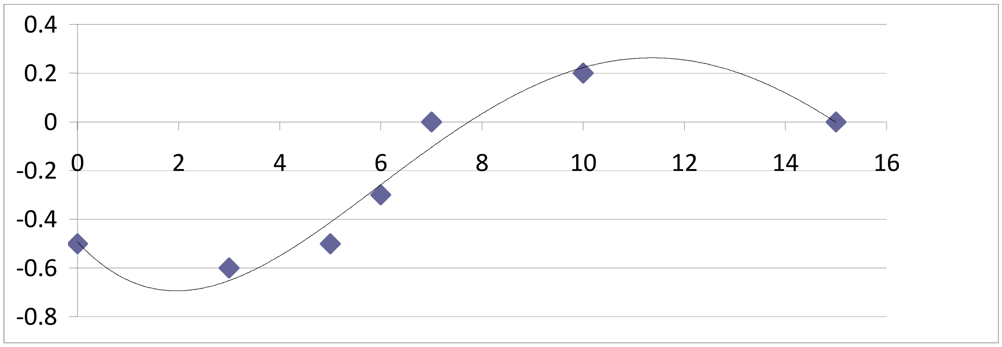
where D and x are in kilometers. The crater is symmetric around the axis x=0.

Problem 1 – Graph this function in the interval (0, +15.0) for which it provides a suitable model.

Problem 2 - To 2 significant figures, what is the approximate excavated volume of this symmetric crater using the method of inscribed and circumscribed disks?

Problem 3 – To 2 significant figures, what is the volume of this crater bounded by the function h(R) and the line y = +0.2 km, and rotated about the y-axis? (Use π = 3.141)

Problem 1 – Graph this function in the interval (0, +15.0) for which it provides a suitable model.



Problem 2 - To 2 significant figures, what is the approximate excavated volume of the symmetric crater using the method of inscribed and circumscribed disks? Answer; The volume of a disk is V = π R^2 h. For the crater, the inscribed disk has a radius of 5 kilometers and a height of 0.7 km, so its volume is Vi = 3.141 $(5)^2$ (0.7) = 55 km^3. The circumscribed disk has a radius of 10 km and a height of 0.8 km, so Vc = 3.141 $(10)^2$ (0.8) = 251 km^3. The estimated volume is then the average of these two or V = (Vc + Vi)/2 = 153 km^3. To 2 Significant Figure, the correct answer would be V = 150 km^3.

Problem 3 – To 2 significant figures, what is the volume of this crater bounded by the function h(R) and the line y = +0.2 km, and rotated about the y-axis? (Use π = 3.141) Answer: Using the method of shells, the volume differential for this problem is dV = 2π x [0.2 - h(x)] dx

The definite integral to evaluate is then $V = \int_0^{10} 2\pi x[0.2 - h(x)]dx$ Then :

$$V = 2\pi(0.2)\int_0^{10} x dx - 2\pi\int_0^{10} 0.0001x^5 - 0.0055x^4 + 0.0729x^3 - 0.2252x^2 - 0.493x\ dx$$

$$V = 2\pi(0.2)\frac{x^2}{2}\Big]_0^{10} - 2\pi\left(0.0001\frac{x^6}{6} - 0.0055\frac{x^5}{5} + 0.0729\frac{x^4}{4} - 0.2252\frac{x^3}{3} - 0.493\frac{x^2}{2}\right)_0^{10}$$

$$V = 2\pi(0.2)\frac{10^2}{2} - 2\pi\left(0.0001\frac{10^6}{6} - 0.0055\frac{10^5}{5} + 0.0729\frac{10^4}{4} - 0.2252\frac{10^3}{3} - 0.493\frac{10^2}{2}\right)$$

V = 2(3.141) [10 - 16.7 + 110 - 182.3 + 75.1 + 24.7] V = 6.24 [20.8]

V = 129.8 km^3 To 2 significant figures this becomes **V = 130 km^3**. So to check that Vi < V < Vo we have 55 km^3 < 130 km^3 < 251 km^3.