The Mathematics Teacher

Press Release Math: Discovering the math behind the science

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"By using NASA press releases, students will discover how mathematics is used by scientists"

Press releases, on the face of it, seem an odd place to find math. This is generally true for the ones that typically lead to the stories in your daily paper or your favorite on-line news service. Fortunately, NASA press releases contain a wealth of quantitative information, and understated mathematics, but easily recovered if you know how to look for it. It is the rare NASA press release that doesn't provide at least some implicit mathematical content, and don't forget that '*A picture is worth a thousand equations*'! The often spectacular imagery that accompanies these press releases can be as evocative of mathematics as it is of the beauty and mysteries of space.

Why go to the all effort of perusing press releases about discoveries in space science just to find mathematics? Students often profess an interest and curiosity about space themes (e.g. the search for extraterrestrial life, black holes, space exploration). Science is a natural arena in which mathematics plays a crucial role at all different levels, from simple arithmetic, to the most complex calculus. Space science in particular provides the teacher with the perfect answer to the perennial question '*What am I ever going to use this for?*" Not all of your students are likely to become astrophysicists, but they will surely encounter the many math-based themes of topics such as global warming.

Since 2004, *SpaceMath@NASA* has delivered hundreds of math problems, and problem guides, into the hands of teachers and students. Every week during the academic year, several new math problems are posted in a one-page PDF format as new press releases are posted by NASA. Over 400 individual problem files and 25 special-topic problem books currently exist at the *SpaceMath@NASA* website. The interest in this resource continues to be intense. In February 2011, the 3 millionth math resource file was downloaded.

Independent surveys of *SpaceMath@NASA* end-users continues to show that students enjoy these problems and the topics they cover; are productively challenged; ask questions that demonstrate elevated curiosity and interest; have measurably improved their academic performance; and look forward to new problems based upon press releases.

How it is done.

NASA press releases often have numerical data, and almost always have some dramatic imagery. Let's see how to work a few press release problems.

NASA Space Telescope Discovers Largest Ring Around Saturn (October 6, 2009)

"…The new belt lies at the far reaches of the Saturnian system, with an orbit tilted 27 degrees from the main ring plane. The bulk of its material starts about six million kilometers (3.7 million miles) away from the planet and extends outward roughly another 12 million kilometers (7.4 million miles). One of Saturn's farthest moons, Phoebe, circles within the newfound ring, and is likely the source of its material. Saturn's newest halo is thick, too -- its vertical height is about 20 times the diameter of the planet. It would take about one billion Earths stacked together to fill the ring. " [note Saturn's diameter is 130,000 km]. **figure 1**

Students should review that the volume of a ring with an inner radios of r and an outer radius of R and a thickness h is just $V = \pi h (R^2 - r^2)$. The press release says that $r = 6.0 \times 10^6$ km, $R = 1.2 \times 10^7$ km, and $h = 20 \times (130{,}000) = 2.6 \times 10^6$ km. Then performing the volumes calculation we get $V = 8.1 \times 10^{20}$ km^3 . At this point it is a good time to introduce the concept of significant figures (SF). Note that the smallest number of significant figures in the numbers entering the calculation is 2, so we quote the final answer to 2 SF.

The article asserts that the volume of this ring is 1 billion times the volume of Earth. Since Earth is a spherical body with a radius of r = 6,378 km, we have from $V = 4/3\pi R^3$ that for Earth, $V = 1.086 \times 10^{12}$ km^3. In this case, since we provide r to 4 significant figures, we may give the answer to four significant figures. We can now divide the computed volume of the Saturn ring with the volume of Earth to get ratio = 7.5×10^8. In this case we only quote the answer to 2 significant figures because that is the minimum number available among the numbers that went into the quotient. The press release claimed that this factor is about one billion, and we can certainly round-up our computed answer to 'about' one billion in order to be appropriate to the generally non-technical style of the article.

LRO Sees the Apollo Landing Sites (July 17, 2009)

"*NASA's Lunar Reconnaissance Orbiter, or LRO, has returned its first imagery of the Apollo moon landing sites. The pictures show the Apollo missions' lunar module descent stage (height 3.5 meters) sitting on the moon's surface, as long shadows from a low sun angle make the modules' locations evident. (Image width 282 meters)"*

This dramatic image in **figure 2** shows the Lunar Landing Module at the center, which is casting a long shadow to the right. The width of the image, according to the press release, is 282 meters. From this information, what is the elevation angle of the sun over the lunar horizon?

Step 1 - We have to determine the length of the shadow in **figure 2**. With a millimeter ruler, students would take this image and measure its width in millimeters. From the given information about its actual width of 282 meters, they can determine the image scale in meters per millimeter. For example, if the image measures 95 mm wide, then the scale factor S = 282 m/95mm so S = 2.968. Since ‘95’ is 2 SF and ‘282’ is 3 SF, the final answer for S should be rounded to no more than 2 SF to get S = 3.0 m/mm.

Step 2- Students will measure the length of the shadow in **figure 2** from the center of the ‘dot’ that is the image of the Lander. This measurement, in millimeters, would be converted to actual meters by using the scale factor, S, determined in Step 1. In our example based on **figure 2**, we obtain L = 25 meters. Students should be able to obtain a value for L close to 25 meters. (Note 1mm measuring accuracy corresponds to a 3-meter uncertainty in L).

Step 3 – Method 1: The elevation angle of the sun can be found using basic trigonometry since $\mathrm{Tan}(\theta) = 3.5$ meters/25 meters so $\theta = \mathrm{Arctan}\ (0.14)$, and so $\theta = 8^{o}$. Method 2: The angle can be determined by having students draw a scaled model of right-triangle ABC, such as the one shown in **figure 3**, representing the Lander as segment AB, and the shadow as segment BC. A protractor can then be used to determine m$\angle ACB$ as 8^{o}. In this step, because we can only measure angles to about 1^{o} accuracy (1 SF), students only need to quote θ to the nearest degree.

The image in **figure 2**, and others similar to it obtained by LRO, can also be used to determine the frequency of craters of different diameters. Simply use the scale factor from Step 1 and measure the physical diameters (in meters) of each of the craters in the image. Tally the number of craters in 4 or 5 convenient size ranges (bins) then construct a histogram from the tally. This is a very important ‘statistic’ for studying surfaces in the solar system because crater frequency data can be used to estimate an age for the surface, the amount and rate of erosion, and other important geological factors.

NASA Research finds 2010 Tied for Warmest Year on Record (January 12, 2011)

'*The analysis found 2010 approximately 1.13 F warmer than the average global surface temperature from 1951 to 1980. To measure climate change, scientists look at long-term trends. The temperature trend, including data from 2010, shows the climate has warmed by approximately 0.36 F per decade since the late 1970s. 'If the warming trend continues, as is expected, and if greenhouse gases continue to increase, the 2010 record will not stand for long,' said James Hansen, the director of the NASA, Goddard Institute for Space Studies.*"

Let's look at **figure 4**, which was provided in the press release. This graph shows the change in degrees Celsius between 1880 and 2010 relative to the global average temperature between 1951-1980 indicated by the horizontal line. It shows that between 1880 and 1920 the average global temperature was about 0.3^{o} C below the average from 1951-1980, while between 2000 and 2010 it was about 0.60^{o} C warmer than the average. Students can explore this graph using the data in **table 1**.

Verifying numerical claims

One of the most basic things one may do with a press release is to verify, as best as possible, the numerical claims that are made. In this case, the press release says that 2010 was 1.13^{o} F warmer than average. We can convert Celsius to Fahrenheit using F = 9/5(C) + 32^{o}, so the temperature change $\Delta F = 9/5\Delta C$, $\Delta F = 9/5(0.6^{o})$ and so $\Delta F=1.08^{o}$ F which is similar to the quoted press release value.

The press release also says that the climate has warmed at a rate of $+0.36^{o}$ F per decade since the late 1970's. From the tabulated data, the slope of the curve in degrees per decade from 1970 to 2010 is just m = $(+0.61^{o} - 0.01^{o})/(13-9)$ so m = $+0.15^{o}$ C /decade. This can be converted to Fahrenheit degrees by multiplying by 9/5 to get m = $+0.27^{o}$ F/decade. This is a significantly different number than given in the press release. If we use the lowest value in the graph from the late 1970's of approximately -0.15^{o} C and the highest value from the decade of the 2010s, which occurred in the year 2010 of $+0.61^{o}$ C, we get m = $+0.19^{o}$ C/decade or $+0.34^{o}$ F/decade, which is indeed similar to the stated decadal rate.

Linear Fit: The graph in **figure 4** suggests that the temperature data could be approximated by a linear regression. What would the predicted temperature change be in 2050 if we used a linear extrapolation?

To represent an average linear trend over the entire interval [1880, 2010] we would select Point 1 at $(1880,-0.28^{o})$ and Point 2 at $(2010, +0.61^{o})$ and use the 2-point form of the equation for a straight line:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \qquad \text{to get} \qquad y + 0.28 = \frac{0.61 + 0.28}{2010 - 1880}(x - 1880)$$

This can be simplified to the more convenient slop-intercept form:

$$y = +0.0068x - 13.06$$

For the year 2050 we have x = 2050, so y = +0.0068(2050)-13.06 and so y = $+0.88^{o}$ C. Note that, since a Fahrenheit degree is 9/5 of a Celsius degree, the warming in 2050 would be 1.6^{o} F, compared to the average global temperature between 1951-1980. This also means that, relative to 2010, the 2050 global temperature from a linear extrapolation might be about 9/5 (0.88-0.61) = 0.5^{o} F warmer than in 2010, which is already on the world records as the warmest decade in several centuries!

Note, it may be helpful to review with the students that the general form of a linear equation, y = mx + b, has two free parameters, m and b, so we need two points on a line to determine them uniquely. For example, in anticipation of the method we will next use in determining the quadratic fit, we can insert the values for Point 1 to get the first equation (-0.28) = m (1880) + b and the values for Point 2 to get the second equation (+0.61) = m (2010) + b, then we have 2 equations in 2 unknowns, which we can solve easily by substitution:

$$b = -1880m - 0.28$$
$$+0.61 = 2010m + (-1880m - 0.28)$$
$$+0.89 = 130m$$

so m = +0.0068 and by substitution, b = -13.06, which as before yields the linear equation

$$y = +0.0068x - 13.06$$

At this point, the problem is appropriate for students that have had, or are taking Algebra 1 and are exploring the properties of linear equations. To make this problem applicable for Algebra 2 students, we ask whether a quadratic interpolation to the data might not be a 'better' fit, and what would be the extrapolated global warming in the year 2050 for a quadratic extrapolation?

Quadratic Fit: We are looking for a fitting function of the form $y = ax^2 + bx + c$ Here we will need to select three points, and from their values for x and y, create a system of three equations in three unknowns. We can choose the three approximately equally-spaced points Point 1 ($1880, -0.28^{o}$), Point 2 (1950, -0.04) and Point 3 at ($2010, +0.61^{o}$). To make the calculation easier, let is define x as the number of decades from 1880; x = (t-1880)/10. We would then get Point 1 = (0, -0.28), Point 2 = (7, -0.04) and Point 3 = (13, +0.61) and the system of equations:

$-0.28 = a\,(0)^2 + b(0) + c$ for x = 0

$-0.04 = a\,(7)^2 + b(7) + c$ for x = 7

$+0.61 = a\,(13)^2 + b(13) + c$ for x = 13

Since from the first equation we trivially get c = -0.28, we are then left with 2 equations in 2 unknowns:

$+49a + 7b = +0.24$

$+169a + 13b = +0.89$

We could then continue with the method of substitution to eliminate b, and then get the solution a = +0.0057 and b = -0.0056. We now have the desired quadratic approximation for this data as

$$y = 0.0057x^2 - 0.0056x - 0.28$$

For t = 2050, we have and x = 17 decades from 1880, and our prediction to 2 significant figures is that y = 1.3^{o} C. In terms of Fahrenheit degrees we get y = 9/5(1.3^{o} C) so y = +2.3^{o} F.

To make the problem a bit more challenging, let's use the method of matrices. We can write this system of equations in matrix form as:

$$\begin{pmatrix} 0.0 & 0.0 & 1.0 \\ 49 & 7 & 1.0 \\ 169 & 13 & 1.0 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} -0.28 \\ -0.04 \\ +0.61 \end{pmatrix}$$

To solve this system, we need to compute the inverse matrix. We first check that det(A) exists. Its value is -546, which is non-zero, so a unique solution for this system does indeed exist. The inverse of A can be found by using the 'long-hand' method of cofactors, by using a Texas Instruments, TI-83 calculator, or by using one of the many online matrix inversion calculators. We obtain

$$A^{-1} = \begin{pmatrix} 0.011 & -0.024 & 0.013 \\ -0.220 & 0.310 & -0.090 \\ 1.0 & 0.0 & 0.0 \end{pmatrix} \qquad \text{so that} \qquad \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} +0.0058 \\ -0.0057 \\ -0.28 \end{pmatrix}$$

We now have the desired quadratic approximation for this data as

$$y = 0.0058x^2 - 0.0057x - 0.28$$

For t = 2050, we have x = 17 decades, and our prediction to 2 significant figures is that y = +1.3^{o} C (or 2.3^{o} F). Note that this prediction is significantly different than our linear extrapolation of y = +0.88^{o} C (or +1.6 F). **Figure 5** shows the two solutions superposed on the plotted decadal data from Table 1.

At this point, one might open the discussion to the topic of how do we decide which of the two extrapolations to trust; the linear version that gives a 2050 temperature change of +1.6^{o} F, or the quadratic extrapolation which gives the higher value of +2.3^{o} F? Students would probably realize that the best fit is the one that 'follows the data better'. This could also lead to extension activities in which the student performs a two-tailed hypothesis test on the difference between the predicted and actual data, and determines whether the variances in each model are consistent with a random distribution to a set confidence level of 96% (3-sigma). The model with the worse variance would then be 'rejected'. Examining **Figure 5**, we see that over the full time interval [1880-2010], the quadratic fit provides a better interpolation to the data, and possibly a more reliable extrapolation to 2050, which is only 40 years beyond the upper bound to the actual data.

Conclusions

NASA press releases can be successfully 'reverse-engineered' to demonstrate how mathematics is used in actual scientific discoveries. Students respond well to working with actual data, especially when the topics appear on the Evening News. You do not have to be a rocket scientist to bridge the gap between press release and mathematics application. All you need is a bit of aggressive curiosity to see the possibilities. *SpaceMath@NASA* has created hundreds of math problems, often tied to press releases, to serve as template for the many possible math applications you and your students will find at NASA.

References

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Biography

Sten F. Odenwald, sten.f.odenwald@nasa.gov, is an astronomer at the NASA Goddard Spaceflight Center in Greenbelt, Maryland. An active science popularizer, he has turned his attention in the last 10 years to enhancing mathematics education resources at NASA as part of their STEM Initiative.

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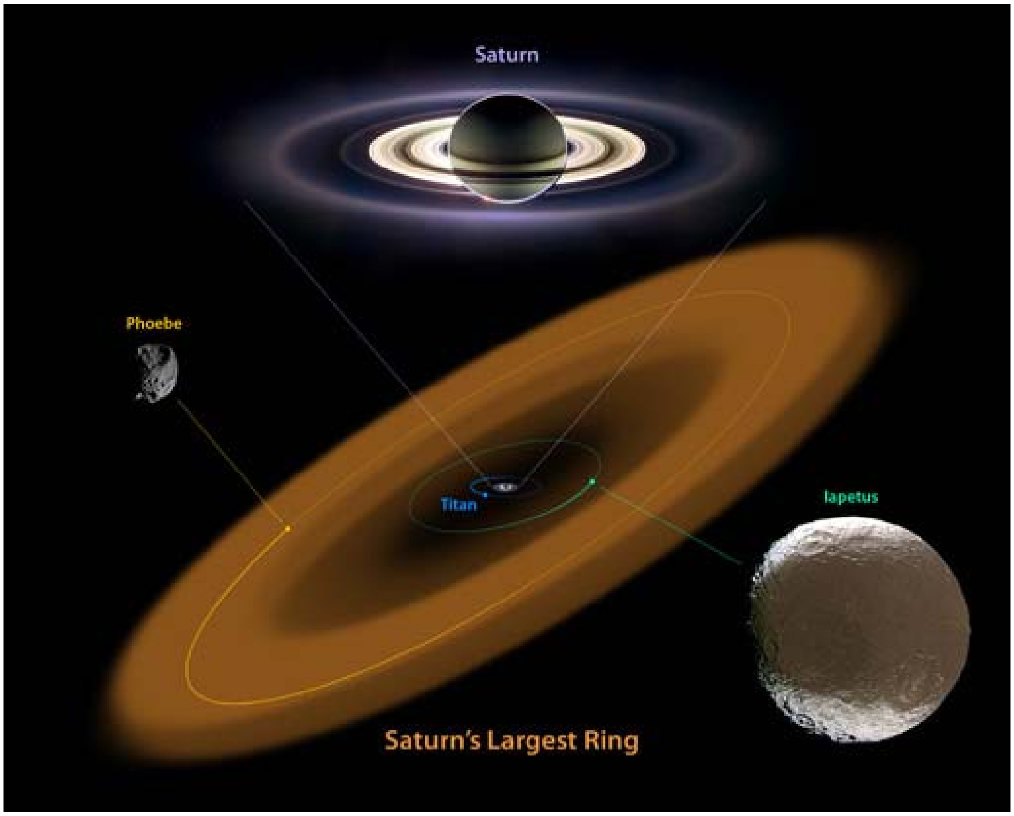

Figure 1 - An artist's sketch of the new dust and ice ring centered on the planet Saturn, discovered by NASA's Spitzer Infrared Observatory.

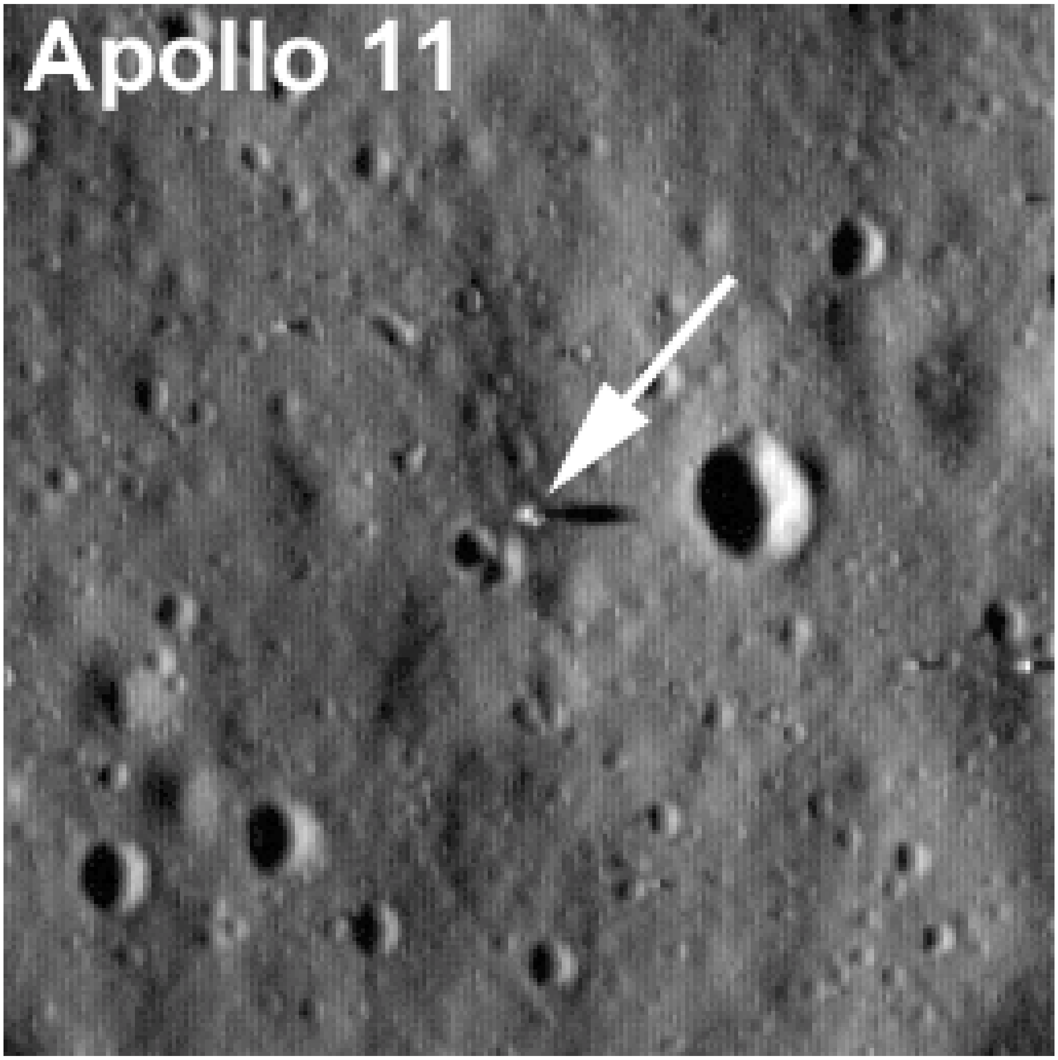

Figure 2 - The area surrounding the Apollo-11 landing site on the moon, imaged by NASA's Lunar Reconnaissance Orbiter (LRO). Features as small as one meter can be seen by the satellite from its orbit 60 km above the lunar surface.

Figure 3 - This is a scaled model of the right-triangle, which can be directly measured with a protractor to determine the solar elevation angle ACB.

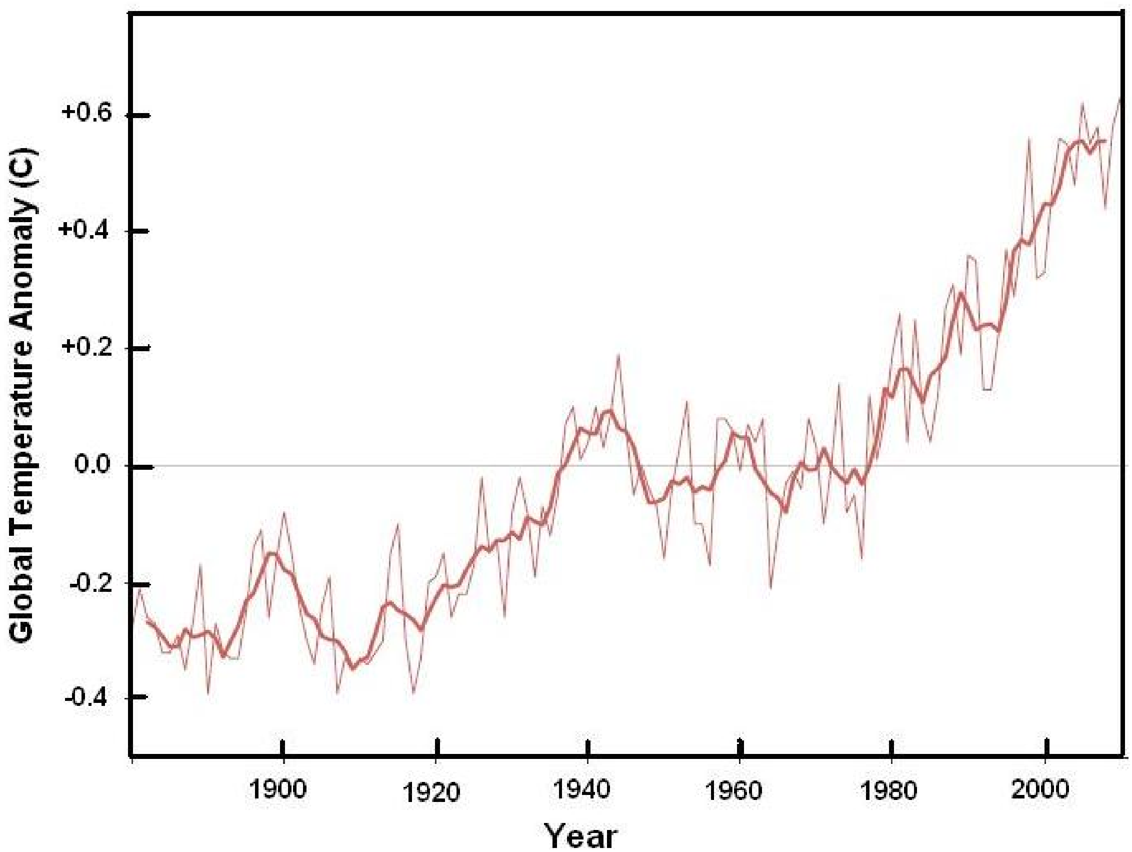

Figure 4 - This graph shows the difference in the global average temperature for the years 1880 to 2010, relative to the average temperature between 1951 and 1970 indicated by the horizontal line.

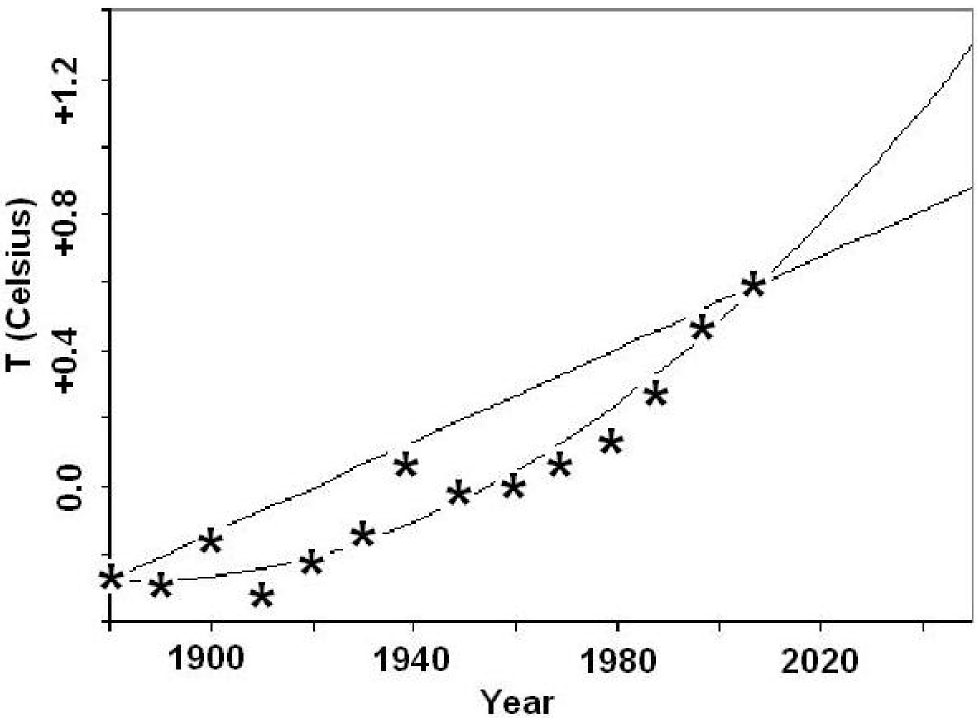

Figure 5 - The linear and quadratic interpolation models superposed on the global temperature data (asterisks) each decade. Note the divergence of the forecasts for ca 2050 at the far right of the graph.

Table 1 - Representative temperature differences at 10-year intervals

Decade	Year	T (^{o}C)	Decade	Year	T (^{o}C)	Decade	Year	T (^{o}C)
0	1880	-0.28	5	1930	-0.13	10	1980	+0.09
1	1890	-0.30	6	1940	+0.04	11	1990	+0.24
2	1900	-0.17	7	1950	-0.04	12	2000	+0.43
3	1910	-0.32	8	1960	-0.03	13	2010	+0.61
4	1920	-0.23	9	1970	0.01			